The proof follows from Theorem 2 of [6].

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AFFINE AUTOMORPHISMS OF THE UNIVERSAL MULTIPLICATIVE ENVELOPING ALGEBRA OF THE TWO-DIMENSIONAL LEFT-SYMMETRIC ALGEBRA WITH ZERO MULTIPLICATION

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In this work we rewrite for the left-symmetric algebra the result of D. Kozybaev, U. Umirbaev [1] on the basis of the universal multiplicative enveloping algebra of the right-symmetric algebra. Also we describe the affine automorphisms of the universal multiplicative enveloping algebra of the two-dimensional left-symmetric algebra with zero multiplication. The question of describing all automorphisms of this algebra remains open. Although it was easy to notice that the automorphism groups of the left and the right universal multiplicative enveloping algebras of the two-dimensional left-symmetric algebra with zero multiplication are tame.

An algebra A over an arbitrary field k with a bilinear product $x \cdot y$ is called *a left-symmetric algebra*, if the identity

$$(xy)z - x(yz) = (yx)z - y(xz) \qquad (1)$$

is satisfied for any $x, y, z \in A$.

Recall that $U(A)$ is an associative algebra with 1 generated by the operators of left multiplication l_x and right multiplication r_x, where $x \in A$. The identity (1) directly implies the defining relations of the algebra $U(A)$:

$$l_x l_y - l_y l_x = l_{[x,y]}, \quad r_x l_y - l_y r_x - r_x r_y + r_{yx} = 0, \quad x, y \in A.$$

The linear basis of the algebra $U(A)$ is described by the following

Theorem 1. Let A be a left-symmetric algebra with linear basis $x_1, x_2, \ldots, x_k, \ldots$. Then the basis of the universal multiplicative enveloping algebra $U(A)$ of A consists of words of the form

$$l_{x_{j_1}} l_{x_{j_2}} \ldots l_{x_{j_t}} r_{x_{i_1}} r_{x_{i_2}} \ldots r_{x_{i_s}},$$

where $j_1 \le j_2 \le \ldots \le j_t, \quad s, t \ge 0.$

Denote by $RU(A)$ the right universal multiplicative enveloping algebra of the algebra A, i.e., subalgebra of the algebra $U(A)$ generated by the universal operators r_x, where $x \in A$. Similarly, we define the left universal multiplicative enveloping algebra $LU(A)$ as the subalgebra of the algebra $U(A)$ generated by the universal operators l_x, where $x \in A$.

Corollary 1. 1) Under the conditions of Theorem 1, words of the form

$$l_{x_{j_1}} l_{x_{j_2}} \ldots l_{x_{j_t}},$$

where $j_1 \le j_2 \le \ldots \le j_t, \quad t \ge 0,$ form the basis of the algebra $LU(A)$.

2) Algebra $LU(A)$ is the associative algebra with generators $l_x, \quad i \ge 1$, and defining relations $l_{x_j} l_{x_k} - l_{x_k} l_{x_j} - l_{[x_j, x_k]} = 0, \quad j > k.$

3) Under the conditions of Theorem 1, the algebra $RU(A)$ is the free associative algebra with free set of generators

$$r_{x_{i_1}} r_{x_{i_2}}, \ldots, r_{x_{i_s}}, \ldots .$$

Theorem 2. Let A_2 be the two-dimensional left-symmetric algebra over an arbitrary field k with linear basis e, f and $ef = fe = 0$. If φ is the affine automorphism of the universal multiplicative enveloping algebra $U(A_2)$ of A_2 then

$$\varphi(l_e) = \alpha l_e + \beta l_f + \nu,$$
$$\varphi(l_f) = \gamma l_e + \delta l_f + \mu,$$
$$\varphi(r_e) = \alpha r_e + \beta r_f,$$
$$\varphi(r_f) = \gamma r_e + \delta r_f,$$

where $\begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \ne 0, \quad \alpha, \beta, \gamma, \delta, \nu, \mu \in k.$

Corollary 2. 1) $LU(A_2)$ is the free associative commutative algebra with the free generators l_e, l_f over a field k;

In 1953, van der Kalk [2] proved that the automorphisms of a polynomial algebra in two variables over an arbitrary field are tame. A similar result for the free associative algebras was obtained byL. Makar Limanov [3] and A.G. Cherniyakievich [4]. Moreover, the automorphism groups of these algebras are isomorphic.

Corollary 3. All automorphisms of the algebras $LU(A_2)$ and $RU(A_2)$ are tame. Moreover,

$$Aut(LU(A_2)) \cong Aut(RU(A_2)).$$

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