the Jonsson spectrum of K with respect to cosemanticness. Then $[T]$ denotes the cosemanticness class of a theory $T \in JSp(K)$.

Let us introduce the operations " $\wedge$ " and " $\vee$ " for arbitrary L-theories T and T' as follows (3). Let

$$T \wedge T' = \{\varphi \wedge \varphi' \mid \varphi \in T, \varphi' \in T'\},$$

if this theory is consistent. Similarly, let

$$T \vee T' = \{\varphi \vee \varphi' \mid \varphi \in T, \varphi' \in T'\}.$$

Each cosemanticness class $[T] \in JSp(K)_{/\bowtie}$ is a distributive lattice with respect to operations " $\vee$ " and " $\wedge$ ".

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WEIGHTED ESTIMATES FOR SOME CLASS OF QUASILINEAR OPERATORS

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Let $0 < q, p, r < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$. Let $u = \{u_i\}_{i=1}^{\infty}$ and $v = \{v_i\}_{i=1}^{\infty}$ be weight sequences, i.e., positive sequences of real numbers. We denote by $l_{p,v}$ the space of sequences $f = \{f_i\}_{i=1}^{\infty}$ of non-negative real numbers such that

$$\|vf\|_p = \left(\sum_{i=1}^{\infty}(v_i f_i)^p\right)^{\frac{1}{p}} < \infty.$$

For any non-negative $f \in l_{p,v}$ we consider the following iterated discrete Hardy-type inequality with three weights

$$\left(\sum_{n=1}^{\infty} u_n^q (K^{\pm} f)_n^q\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty}(v_i f_i)^p\right)^{\frac{1}{p}}, \tag{1}$$

where C is a positive constant independent of f and K is a quasilinear operators defined as follows

$$(K^{-}f)_n = \left(\sum_{k=n}^{\infty} a_{k,n} \left(\sum_{i=k}^{\infty} f_i\right)^r\right)^{\frac{1}{r}},$$

$$(K^{+}f)_n = \left(\sum_{k=1}^{n} a_{n,k} \left(\sum_{i=1}^{k} f_i\right)^r\right)^{\frac{1}{r}}.$$

where $a_{k,n}$ is a non-negative matrix. The aim of this paper is to characterize the inequality and its dual version for the case $0 < p \leq r < q < \infty$. Iterated inequalities involving matrix operators were studied in works [1] and [2] for the same parameters ratio.

Theorem 1. Let $0 < p \leq r < q < \infty$ and $a_{k,n}$ be a non-negative matrix. Then for any non-negative $f \in l_{p,v}$:

(a) If $0 < p \leq 1$, the inequality holds for operator K^{-} if and only if

$$D = \sup_{j\geq 1} v_j^{-1} \left(\sum_{n=1}^{j} u_n^q \left(\sum_{i=n}^{j} a_{i,n}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty$$

(b) If $1 < p < \infty$, the inequality holds for operator K^{-} if and only if

$$\mathcal{D} = \sup_{j\geq 1} \left(\sum_{i=j}^{\infty} v_i^{-p'}\right)^{\frac{1}{p'}} \left(\sum_{n=1}^{j} u_n^q \left(\sum_{i=n}^{j} a_{i,n}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty.$$

Moreover, $C \approx D$ in case (a) and $C \approx \mathcal{D}$ in case (b), where C is the best constant.

Theorem 2. Let $0 < p \leq r < q < \infty$ and $a_{n,k}$ be a non-negative matrix. Then for any non-negative $f \in l_{p,v}$:

(a) If $0 < p \leq 1$, the inequality holds for operator K^{+} if and only if

$$M = \sup_{j\geq 1} v_j^{-1} \left(\sum_{n=j}^{\infty} u_n^q \left(\sum_{i=j}^{n} a_{n,i}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty$$

(b) If $1 < p < \infty$, the inequality holds for operator K^{+} if and only if

$$\mathcal{M} = \sup_{j\geq 1} \left(\sum_{i=1}^{j} v_i^{-p'}\right)^{\frac{1}{p'}} \left(\sum_{n=j}^{\infty} u_n^q \left(\sum_{i=j}^{n} a_{n,i}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty.$$

Moreover, $C \approx M$ in case (a) and $C \approx \mathcal{M}$ in case (b), where C is the best constant.

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ON THE SOLUTION OF A BOUNDARY VALUE PROBLEM FOR A HYPERBOLIC EQUATION WITH FRACTIONAL LOADING

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Abstract. In this work investigates a boundary value problem for a fractional-loaded hyperbolic equation with one spatial variable that generalizes the classical equation of string vibrations. The equation contains the fractional derivative of Riemann Liouville, and the load is carried out at a time-dependent point. The purpose of the study is to construct an explicit solution to the problem. To find a solution, the initial problem is reduced to a Volterra type integral equation. A representation of the solution is obtained in the form of the sum of two integrals corresponding to different areas of variable definition. It is shown that there is a unique solution to the problem. The conditions for the functions included in the equation are given for which a solution exists. The results obtained can be used in modeling processes described by hyperbolic equations with fractional derivatives.

This paper studies a boundary value problem for a fractionally loaded hyperbolic equation with one spatial variable. The equation contains a Riemann-Liouville fractional derivative, and the load is applied at a time-dependent point [1, 2]. The aim of the research is to construct an explicit solution to the problem, which will provide deeper insight into the behavior of solutions to such equations and expand their applications in practical problems [3, 4].

Consider the equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \mu D_t^{\alpha} u(x,t)\Big|_{x=0} + f(x,t), \quad 0 \le x < \infty, \quad t > 0 \tag{1}$$

with the initial conditions

$$u(x,t)\Big|_{t=0} = g_1(x), \tag{2}$$

$$\frac{\partial u(x,t)}{\partial t}\Big|_{t=0} = g_2(x), \tag{3}$$

and the boundary condition

$$u(x,t)\Big|_{x=0} = h(t). \tag{4}$$