$t = \theta_i$, $i = \overline{1,m}$; B, C and $D$are constant $(n \times n)$-matrices, and $0 = \theta_0 < \theta_1 < \cdots < \theta_m < \theta_{m+1} = T$, $\|x\| = \max_{i=\overline{1,n}} |x_i|$.

The argument $\gamma(t)$ is a step function defined as $\gamma(t) = \xi_{i-1}$if $t \in [\theta_{i-1}, \theta_i)$, $i = \overline{1,m+1}$; $\theta_{i-1} < \xi_{i-1} < \theta_i$for all $i = \overline{1,m+1}$; where$0 = \theta_0 < \theta_1 < \cdots < \theta_m < \theta_{m+1} = T$.

A function$x(t)$ is called a solution to problem (1), (2) if:

(i) $x(t)$is continuous on$[0,T]$;

(ii) $x(t)$is differentiable on $[0,T]$with the possible exception of the points θ_j, $j = \overline{0,m}$, where the one-sided derivatives exist;

(iii) $x(t)$satisfies(1) on each interval (θ_{i-1}, θ_i), $i = \overline{1,m+1}$; at the points θ_j, $j = \overline{0,m}$, Eq. (1) is satisfied by the right-hand derivatives of $x(t)$;

(iv) $x(t)$satisfies the boundary condition (2).

Boundary-value problems for differential equations with piecewise constant argument have been under intensive investigation of researchers in mathematics, biology, engineering, and other fields for the last 30 years [1-3]. Many problems for loaded differential equations and methods for solving them were investigated in [4-8] and the references therewith.

Main goal in this paper is to extend the Dzhumabaev parametrization method [9] to loaded differential equations with piecewise constant argument of generalized type. For this purpose, we have developed computational method solving a boundary-value problem for loaded differential equations with piecewise constant argument of generalized type.

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INTEGRATION OF THE LOADED SINE-GORDON EQUATION WITH SOURCE OF THE INTEGRAL TYPE.

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The sine-Gordon equation arises in applications as diverse as the description of surfaces of constant mean curvature.

In the work [1],[2] were shown that the sine-Gordon equation with a self-consistent source and loaded sine-Gordon equation with a self-consistent source can be solved using the inverse scattering problem method.

In this paper, we are consider the system of equations

$$\begin{cases} u_{xt} = \sin u + \gamma(t) u_x(0,t) u_{xx} + \int_{-\infty}^{\infty} \left(\phi_1^2 - \phi_2^2\right) d\eta \\ L(t)\phi = \eta\phi \end{cases} \tag{1}$$

with the initial condition

$$u(x,0) = u_0(x), \quad x \in R, \tag{2}$$

where $\gamma(t) -$ given continuous function,

$$L(t) = i\begin{pmatrix} \frac{d}{dx} & \frac{u_x}{2} \\ \frac{u_x}{2} & -\frac{d}{dx} \end{pmatrix}.$$

The initial function $u_0(x)$ $\left(-\infty < x < \infty \right)$ has the following properties:

1. $u_0(x) \equiv 0 (\mathrm{mod}\, 2\pi)$ at $x \to \infty$;

$$\int_{-\infty}^{\infty} \left((1+|x|))|u_0'(x)| + |u_0''(x)|\right) dx < \infty$$

2. The operator $L(0)$ has not spectral singularities and in the upper half of the comlex plane it has N simple eigenvalues $\xi_1(0), \xi_2(0), ..., \xi_N(0)$ lying in the upper semi-plane of the complex plane.

In the problem, the vector function $\phi = \left(\phi_1(x,\eta,t), \phi_2(x,\eta,t)\right)^T$ is the solution of the system of equations $L(t)\phi = \eta\phi$ determined by the asymtotics

$$\phi \to M(\eta,t)\begin{pmatrix} \exp(-i\eta x) \\ \exp(i\eta x) \end{pmatrix}, \; at \; x \to \infty$$

Where $M(\eta,t)$ - is a initially given continous function that satisfies the conditions

$$M(-\eta,t) = M(\eta,t), \; \int_{-\infty}^{\infty} |M(\eta,t)|^2 d\eta < \infty \tag{3}$$

for all non-negative values of t.

The solution $u(x,t)$ of problem (1)-(3) is sought for a class of functions that have sufficient smoothness and rather quickly tends to its limits at $x \to \pm\infty$,

$$\begin{aligned} & u(x,t) \equiv 0 (\mathrm{mod}\, 2\pi), \; |x| \to \infty; \\ & \int_{-\infty}^{\infty} \left((1+|x|))|u_x(x,t)| + |u_{xx}(x,t)|\right) dx < \infty \end{aligned} \tag{4}$$

Equation (1) refers to the class of so-called loaded equations[2].

In this work, we obtained representations for the solutions $u(x,t), \phi(x,\eta,t)$ of problem (1)-(4) with in the framework of the inverse scattering problem method for the operator $L(t)$.

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