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FLUCTUATION-DISSIPATION CORRELATION OF THE SIGNAL OF STARS OF THE FS CMA TYPE

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Abstract. *Due to the importance of stellar evolution, new theories and approaches to the study of stars are constantly being developed. This study presents novel results obtained through the application of the fluctuation-dissipation theorem. According to this theorem, fluctuations within a system give rise to dissipation in the form of thermal equilibrium. The spectral correlation function of fluctuations is related to the degree of photon dissipation. In this work, the evolutionary stage of stars is determined by analyzing the relationship between dissipation and fluctuation in complex FS CMa-type systems.*

Keywords: Fluctuation-dissipation analysis, FS CMa-type stars, binary stars, photon dissipation.

1. Introduction

Stellar evolution is an extremely complex and lengthy process that plays a key role in the dynamics and development of the Universe. Stars form from dense clouds of gas and dust and pass through several stages of their existence, each accompanied by unique physical phenomena. Stars are primarily composed of hydrogen and helium, which are the main elements present in their cores and atmospheres. Understanding stellar evolution is crucial for interpreting observational data, refining theories of stellar development, and understanding the complex interactions between stars under various astrophysical conditions [1]. Thus, the study of stellar evolution contributes to a broader understanding of the formation, evolution, and dynamics of stars and galaxies in the Universe.

FS CMa-type stars represent a distinct subclass of eruptive objects that exhibit characteristics of both B[e]-type stars and systems with signs of accretion and mass loss. These objects are characterized by strong emission lines (notably *Hα*), often with double-peaked profiles indicative of rotating disks or collimated outflows, and a pronounced excess of radiation in the near- and mid-infrared ranges, pointing to the presence of circumstellar dust [2]. Unlike classical B[e] stars, FS CMa objects are not associated with high-mass supernova progenitors or young stellar clusters, which complicates their classification within standard evolutionary frameworks [3]. It is assumed that most FS CMa objects are binary systems, in which the interaction between the components plays a key role in shaping the observed spectral features and energy distributions [4]. The mechanisms of dust formation, particularly near hot B-type stars, can be explained by interactions within close binary systems, where matter transferred from one component to the other cools and condenses into a dust shell or disk. Thus, the study of FS CMa-type stars opens a new avenue for investigating interacting binaries, transitional stages of stellar evolution, and the conditions for dust formation in extreme environments.

There are various methods used to study stellar evolution, such as the observation of star clusters, asteroseismology, numerical modeling and the construction of evolutionary tracks, spectroscopy and photometry, the study of variable stars, and so on [5–8]. In our study, statistical methods will be applied to astrophysical processes. One of the key tools of statistical physics is the fluctuation-dissipation relation (FDR). The FDR emerged within the framework of nonequilibrium statistical physics and kinetic theory, and its formalism was thoroughly developed in the works of Klimontovich [9, 10], L.D. Landau and E.M. Lifshitz [11], and Kubo [12]. These theoretical foundations form the basis of modern analysis techniques for open systems, including astrophysical applications. The use of the FDR allows for the extraction of physically meaningful parameters from fluctuation spectra, and has found wide application in plasma physics, quantum optics, biophysics, and astrophysics [13, 14]. In addition, the information-entropy method for astrophysical phenomena, as used in the works of Zhanabaev Z.Zh. [15–17], will also be considered. In our article, we will analyze the evolutionary stage and physical properties of binary stars and FS CMa-type systems using the FDR to quantitatively describe the connection between radiation fluctuations and dissipative processes in stars with complex structures. These methods provide insight into the dynamics of binary star systems through the equilibrium between fluctuations and dissipation.

The stellar data used in this study were obtained from the PolarBase database, as it operates with high-resolution telescopes employed for spectro-polarimetric observations of stars, specifically the Bernard Lyot Telescope (since 2006) and the Canada-France-Hawaii Telescope (since 2005) [18].

2. Spectral correlation functions of diffusion of concentration in a fluctuating medium.

We will use terms and notations of books (article) of Y.L.Klimontovich "Statistical Physics", "Physics of Open Systems" and others [9, 10]. The Einstein-Smoluchowski equation with random force $\mathrm{y(t)}$ (Langevin source) for the sensor output voltage $\delta\mathrm{V}(\mathrm{t},\vec{\mathrm{r}})$ (denoting fluctuations through δ means also δ is Dirac function):

$$\frac{\delta \mathrm{V}(\mathrm{t},\vec{\mathrm{r}})}{\partial \mathrm{t}} = \mathrm{D}\frac{\partial^2}{\partial \mathrm{r}^2}\delta(\mathrm{t},\vec{\mathrm{r}}) + \mathrm{y}(\vec{\mathrm{r}},\mathrm{t}) \tag{1}$$

where D is the diffusion coefficient. From equation (1) through Fourier-transformation in frequency ω and in wave number $\vec{\mathrm{k}}$ we have

$$\mathrm{y}(\omega,\vec{\mathrm{k}}) = \delta\mathrm{V}(\omega,\vec{\mathrm{k}})(\mathrm{i}\omega + \mathrm{Dk}^2). \tag{2}$$

The correlation function (correlator) of white noise, the random force $y(t)$, does not depend on frequency. Therefore,

$$\langle y_i y_j \rangle_{\vec{k}} = 2Dk^2 \tag{3}$$

where the coefficient 2 takes into account the modulus delta-collision function $|\mathrm{y}|^{\mathrm{n}} = 2\delta(\mathrm{y})$, where n is any number.

From formula (2), the observed values $\delta\mathrm{V}(\omega,\vec{\mathrm{k}})$ represent correlators (i.e., matrix products) involving the modulus $(\mathrm{i}\omega + \mathrm{Dk}^2)$. The correlator is given by:

$$\langle \delta\mathrm{V_i}\delta\,\mathrm{V_j}\rangle_{\omega_\mathrm{i}\vec{\mathrm{k}}}, (\omega^2 + \mathrm{D^2k^4}). \tag{4}$$

Considering (3) from (2), (4) we write down

$$\langle \delta\mathrm{V_i}\delta\,\mathrm{V_j}\rangle_{\omega_\mathrm{i}\vec{\mathrm{k}}} = \frac{2\mathrm{D}^2\vec{\mathrm{k}}^2}{\omega^2 + \mathrm{D}^2\vec{\mathrm{k}}^4} \tag{5}$$

The expression $D\vec{\mathrm{k}}^2$ has the frequency dimension $[\mathrm{D}] = \frac{\mathrm{m}^2}{\mathrm{c}}, [\mathrm{k}^2] = \frac{1}{\mathrm{m}^2}$. Therefore, in the description of experiments $\overrightarrow{\mathrm{kk}}^2 = \Delta\omega$ is used as a measure of the broadening of the spectrum due to collisions of gas and air particles.

For air, the frequency dimension is approximately $D \sim 10^{-4}\frac{1}{c}$, while for semiconductor materials, $D \sim (10^{-4} \div 10^{-9})\frac{1}{c}$. The fundamental (resonant) frequency ω_0, highlighted in formula (5), is used in the so-called Lorentz spectrum, defined as

$$f(\omega, \omega_0, \Delta\omega) = \frac{2(\Delta\omega/2)}{(\omega-\omega_0)^2+(\Delta\omega/2)^2} \tag{6}$$

The function $f(\omega, \omega_0, \Delta\omega)$ is one of the representations of the Dirac delta function δ, satisfying the normalization condition:

$$\frac{1}{\pi}\int_{-\infty}^{\infty} f(\omega, \omega_0, \Delta\omega)d\omega = 1. \tag{7}$$

$\Delta\omega$ is determined from the experiment as "half-width of the spectrum" i.e. the value of $\Delta\omega = \omega_0 - \omega(f = 1/2)$, equal at the point where $f = 1/2$. For modelling, $Dk^2 = \Delta\omega$ is used as a frequency-independent parameter.

For modelling $f(c, \omega, \omega_0)$ considering gas concentration C_0, the parameter $\omega_0 C_0$ is introduced instead of $\Delta\omega$. And also $\omega_0 C$ is also frequency independent. We will now consider the Lorentz spectrum as a function of C. In dimensionless form $\omega = \omega_0 \frac{C}{C_*}$, where C_* is the characteristic concentration by fluctuations, equation (6) has the following form:

$$f(\omega, \omega_0, C/C_*) = \frac{1}{\pi}\frac{\frac{\omega_0 C}{2C_*}}{(\omega-\omega_0)^2+\left(\frac{\omega_0 C}{2C_*}\right)^2} \tag{8}$$

With a change in the impurity concentration C_0, the mass density within the volume also changes. Fluctuation interactions between particles in the volume lead not only to oscillations, but also to rotational motions. Based on the fluctuation-dissipation relation, the following equation can be written:

$$f_Л(\omega, \omega_0, C) = \alpha(\omega_0) * \mathrm{cth}\left(\frac{C\hbar\omega}{2kT}\right) \tag{9}$$
$$\alpha(\omega_0) = \mathrm{const}$$

Let us introduce the notation:

$$C_* = \frac{2kT}{\hbar\omega_0}, \text{ for the power spectrum } C = \frac{C}{C_*} \tag{10}$$

Let $\alpha(\omega_0) \sim \omega_0$. The proportionality coefficient is incorporated into the normalization condition, i.e., into the value of $f(c)_{\max}$, the Lorentz spectrum is summed over the frequency ω (with ω ranging from 100 to 200). The hyperbolic cotangent function describes the emission process, while the absorption is characterized by the hyperbolic tangent (i.e., the inverse function). Equation (9) therefore takes the following form:

$$f_Л(\omega, \omega_0, C/C_*) = \alpha(\omega_0)\mathrm{th}\left(\frac{C\hbar\omega_0}{2kT}\right) \tag{11}$$

The value of concentration, $C_{*_0} = C_*/2$ corresponding to the saturation of the sensor signal, we determine from the equality of fluctuation coherent quantum ($\hbar\omega_0/2$) and dissipation thermal (kT) factors:

$$C_{*_0} = 2kT/\hbar\omega_0. \tag{12}$$

Telescope observations are represented through the emission wavelength λ:

$$\omega_0 = k * c = 2\pi * c/\lambda, \tag{13}$$
$$x = C * \hbar\omega/2kT = C * \hbar kc/2kT = C * (\hbar c * 2\pi)/(\lambda_0 * 2kT) \tag{14}$$

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where c is the speed of light, C is the elemental concentration, T is the temperature of the star, $\hbar\omega_0 = 2.11\text{eV}$ is the energy of the He photon, kT is the thermal energy, k is Boltzmann's constant.

The algorithm of the fluctuation-dissipative stellar analysis has been demonstrated above, and the formulae necessary to fulfil the purpose of the study have been summarized. The next section reflects the results obtained from this work and draws a conclusion based on these results.

3. Results and Discussion

The fluctuation-dissipation method provides a relationship between the dissipation power and the fluctuation intensity of *He* elements in the stellar atmosphere. Equation (11) describes the equality between the probability density of the concentration and the probability of dissipation. The saturation values of dissipation at C/C_* may serve as an indicator of the evolutionary stage of the stars. Saturation curves as a function of relative concentration were modeled for each star, starting from zero. It was found that in early-type stars (e.g., HD 210839, type O9.5 Iab). *He* saturation is reached rapidly due to the high temperature and ionization potential. In main-sequence stars (e.g., Vega, A0 V), the saturation process is moderate, whereas in blue supergiants (e.g., P Cygni, B1-2 Ia-0ep), *He* saturates slowly and the $H\alpha$ intensity persists longer, which corresponds to a more rarefied atmosphere. Figure 1 shows the variation of the dissipation function (11) for HD 210839, Vega, and P Cygni, with their corresponding saturation points as follows: $C_* = 0.75$, $C_* = 1.2$, $C_* = 2$, respectively. As we can see, C_* is smaller for younger stars and increases for stars that are in the middle or later stages of their evolution.

For comparison, the corresponding Lorentzian functions are also shown in Figure 1 as dashed lines, using the same color scheme. The Lorentzian approximation exhibits a broader profile, especially in the case of P Cygni, which may be attributed to enhanced turbulence and pressure in its extended atmosphere.

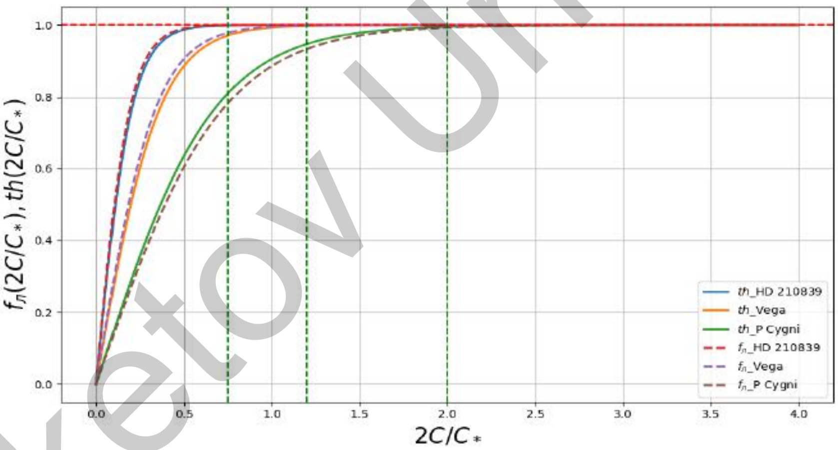

Fig.1. Dissipation function versus normalized concentration for the stars HD 210839, Vega, and P Cygni for the *He* (λ5875) spectral line, along with the integral of the Lorentzian function.

For the star HD 210839 (blue line), saturation is reached more rapidly (a steep rise in the function), reflecting the high level of ionization in the hot atmosphere of an O-type star. Vega (orange line) shows similar behavior but with a less pronounced rise, consistent with the cooler atmosphere of an A-type star. For P Cygni (green line), saturation occurs significantly more slowly, reflecting the typical characteristics of the rarefied atmosphere of a B-type supergiant. The green vertical dashed lines indicate characteristic threshold concentration values at which the transition to saturation occurs in the model. These values may correspond to different excitation regimes of *He* in the star's atmosphere.

A similar analysis was carried out for FS CMa-type stars, objects characterized by strong emission, dusty envelopes, and unstable circumstellar processes, as shown in Figure 2. Among these, MWC 645 exhibited the most rapid *He* saturation, likely due to the high density and temperature of its circumstellar environment. In contrast, 3 Pup shows a significantly slower saturation process, reflecting a more stable and evolved envelope. The characteristic *He* related dependencies observed in these objects are consistent with current understanding of their evolutionary stages, as well as with the differences in intensity and profiles of the corresponding spectral lines. On the fluctuation-dissipation plots for FS CMa-type stars MWC 645, MWC 728, and 3 Pup, the corresponding saturation points are: $C_* = 1, C_* = 1.4, C_* = 2.5$. A visual comparison of the tangent and Lorentzian curves highlights that the dissipation model provides a better description of sharp saturation transitions in hot stars, whereas the Lorentzian model may be useful for capturing smoother behavior in the atmospheres of stars.

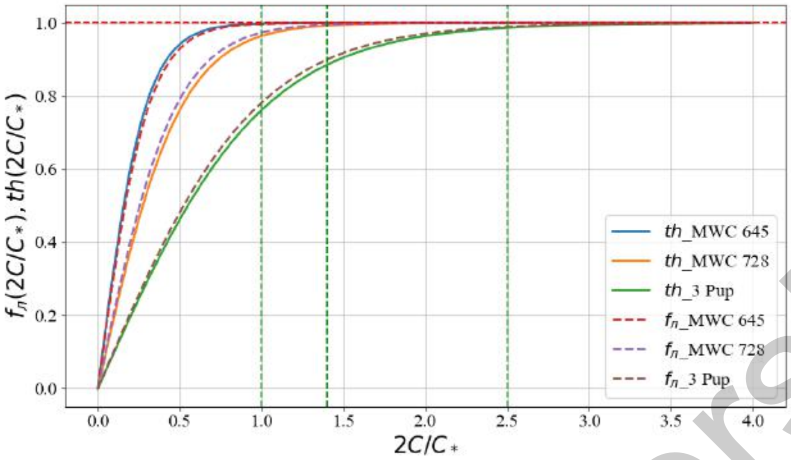

Fig.2. Dissipation function versus normalized concentration for the stars MWC 645, MWC 728, and 3 Pup for the *He* (λ5875) spectral line, along with the integral of the Lorentzian function.

Thus, the proposed method, based on the fluctuation-dissipation approach, provides a qualitative description of the saturation behavior of emission lines and can be applied to assess the evolutionary status of shell stars. This methodology opens up prospects for a quantitative analysis of elemental saturation and its comparison with results from spectrophotometric observations.

4. Conclusion

In the present study, a new methodology for analyzing the evolutionary state of stars was developed and tested using the fluctuation-dissipation relation. The core of the method lies in the relationship between spontaneous fluctuations in the stellar atmosphere and dissipative processes, manifested through the saturation of emission lines, particularly the *Hα* and *He* lines. The fluctuation-dissipation relation (FDR) is expressed as an integral of the Lorentzian function over frequency, taking into account the frequency shift caused by the presence of *Hα* and *He* element concentrations. Dissipation is represented as the absorption of photons, i.e., the inverse of the photon Bose condensation number, described via the hyperbolic tangent function (tanh).

Modeling results indicate that young, massive stars of early spectral types (e.g., HD 210839) reach saturation more rapidly than more evolved objects, such as Vega or P Cygni. Similarly, among FS CMa-type stars, the differences in C_* values are consistent with independent assessments of their structural properties, circumstellar environment density, and envelope activity levels. The observed patterns confirm that dissipation saturation serves as a sensitive indicator of the thermodynamic and dynamic state of a star's atmosphere.

The proposed fluctuation-dissipation analysis method represents an effective tool for determining the evolutionary characteristics of stars, particularly in cases where traditional approaches provide limited information. Future prospects include expanding the sample of studied objects, applying the method to spectral lines of other elements, and integrating the results with findings from astrophysical and polarimetric observations.

Conflict of interest statement
The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

CRediT author statement
Zhanabaev Z.Zh.: Conceptualization, Methodology, Supervision; **Imanbayeva A.K.:** Writing – review & editing, text correction and proofreading; **Akniyazova A.Zh.:** Investigation, Writing – original draft, Data validation; **Ashimov Ye.K.:** Formal analysis, Visualization, Writing – review & editing. The final manuscript was read and approved by all authors.

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