DOI 10.31489/2019M2/101-107
UDC 51

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Initial values of pores' pressure and stress in the problems for soil consolidation

According to the basic model of V. Florin a method was developed for determining the initial excess pore pressure p_0 at all points of the soil mass. Here the sum of the main stresses in the skeleton of the soil was found. Moreover, the additional pressure due to the application of external load, with an instant consolidation of the soil goes to zero. As a design scheme, a two-phase soil cylinder compaction with the radius r, the height h with a permeable bottom and walls is assumed. A uniformly distributed load with intensity q is applied on some upper part $a < R$ of the cylinder area. In connection with the symmetry of the problem under investigation with respect to z axis, it is investigated in the cylindrical coordinates. On the basis of this result, the solution of the problem for the concentrated force is determined.

Keywords: pore pressure, stress, consolidation, soil, compaction, porosity.

Now the country has ambitious tasks for the development of science and capital construction. The solution of these tasks requires the most rational use of material and financial resources allocated for this construction. In this regard, the problem of designing and building high-rise buildings and large hydraulic structures on water-saturated clay soils poses a number of tasks for its complete solution. Studies of the causes of the deformation of many buildings and structures built in various regions of Kazakhstan, in particular, the South Kazakhstan region and beyond, have shown that water saturated soils, formed by the increase in groundwater, often lie at the base of the structures. At the same time, the deformation of the soil, determined by the influence of external pressure on it, reached unacceptable values.

The correct approach to solving these issues in all cases can facilitate the construction, speed up the time frame for its implementation and reduce the cost of the structure itself. However, any errors in this regard can be fraught with consequences. Here the main thing is to set the task correctly, especially in cases of erection of large and important structures erected on water-saturated clay soils, taking into account all the necessary conditions in which they find themselves. To achieve the greatest effect in solving these problems in all cases, you should strive to assess and predict the rate of sediment of the foundations of structures.

The process of compressing the soil layer usually occurs gradually over time. Therefore, there is a final sediment and a sediment changing over time. In clay soil, the compression process under static load occurs mainly due to the displacement of clay particles with partial destruction of the natural structure of the soil and its connections. The low permeability of the clay soil mainly determines the rate of its compression. At the same time, the slow destruction of its bonds under load determines the slow change in precipitation over time. At the same time, the duration of the precipitation of a layer of clay soil depends on the thickness of the compacted massif and reaches its final value after a long period of time.

For the calculation of the consolidation of heterogeneous soil bases in the device vertical drains need to know: structural properties of clay soils; basic assumptions for studying soil compaction; arrangements of the vertical drains in the soil foundations; properties of the heterogeneity of weak soils; equations of the state of the skeleton of the soil; the basic resolving equations of mechanics of compacted inhomogeneous soils; boundary conditions of problems; typical settlement schemes. Wherein:
– the excess pore pressure equals zero at $t > 0$ on the surface of a vertical drain with a radius r_0, i.e.

$$p_0(r_0, 0, t) = 0; \tag{1}$$

– the movement of water does not occur in the zones of influence of a vertical drain with a radius R, i.e.

$$\frac{\partial p}{\partial r} = 0 \ \text{ at } \ r = R; \tag{2}$$

– the excess pore pressure is zero on the horizontal surface of the soil mass, i.e.

$$p(r,0,t)=0; \tag{3}$$

– the lower horizontal boundary of the soil mass is impenetrable. Consequently,

$$\frac{\partial p}{\partial z}=0 \ \text{ at } \ z=h. \tag{4}$$

Expressions (1)–(4) are used as boundary conditions for solving problems of the theory of consolidation of earth masses. To determine the initial excess pore pressure p_0 at all points of the soil mass, we use the basic model of V.A. Florin. At the same time, bearing in mind the unchangeability of the porosity coefficient at the initial moment of application of an external load, we find that the sum of the main stresses in the soil skeleton $\theta^{(0)}=0$. In addition, the additional pressure p^* due to the application of external load, with the instant consolidation of the soil goes to zero. Considering all this in accordance with the expression $\theta^{(0)}=\theta^*=n(p_0-p^*)$, we get

$$p_0=\frac{\theta^*}{n}, \tag{5}$$

where n takes one of the values 1, 2, 3 depending on the dimensionality of the studied problems.

The initial stresses in the soil mass skeleton according to the basic design model are found using the following formula:

$$\sigma_{ij}^{(0)}=\sigma_{ij}^*-\delta_{ij}p_0, \tag{6}$$

where i,j — take values $1,2$ for the flat task; i,j - values $1,2,3$ for the spatial problem; index (0) means that the values of these quantities correspond to the moment of instantaneous application of the load; δ_{ij} is the Kronecker symbol.

Moreover, expressions (6) for a two-dimensional problem with regard to dependence (5) can be represented as follows:

$$\sigma_{11}^{(0)}=-\sigma_{22}^{(0)}=(\sigma_{11}^*-\sigma_{22}^*)/2; \quad \sigma_{12}^{(0)}=\sigma_{12}^*. \tag{7}$$

For a three-dimensional problem, it has the form

$$\sigma_{ij}^{(0)}=\sigma_{ij}^*-\delta_{ij}\theta^*/3. \tag{8}$$

Here, the additional shear stresses arising at the initial moment of instantaneous load application are equal to their final values. The corresponding normal stresses are much smaller than the final values, and at each point of the ground environment, according to expressions (7), are equal in magnitude and inverse in sign.

It should be noted that in the case of a flat problem and a flat boundary surface, the determination of the above initial stresses in the soil skeleton after finding the values of p_0 is not difficult. Indeed, if the values θ^* are known, then the calculation of the stresses $\sigma_{11}^*,\sigma_{22}^*$ and σ_{12}^* are easily done using the expressions:

$$\sigma_{11}^*=\left(\theta^*+y\frac{\partial\theta^*}{\partial y}\right)/2; \quad \sigma_{22}^*=\left(\theta^*-y\frac{\partial\theta^*}{\partial y}\right)/2; \quad \sigma_{12}^*=-y\frac{\partial\theta^*}{\partial x}/2. \tag{9}$$

Considering expressions (7) and (9) we find:

$$\sigma_{11}^{(0)}=-\sigma_{22}^{(0)}=y\frac{\partial\theta^*}{\partial y}; \quad \sigma_{12}^{(0)}=-y\frac{\partial\theta^*}{\partial x}. \tag{10}$$

Expressions (5)–(10) applied to the axially symmetric problem are of the form:

$$p_0=\frac{\sigma_{11}^*+\sigma_{22}^*}{3}; \quad \sigma_{11}^*=\sigma_{rr}^*; \quad \sigma_{22}^*=\sigma_{33}^*=\sigma_{zz}^*; \tag{11}$$

$$\sigma_{ij}^{(0)}=\sigma_{ij}^*-\delta_{ij}p_0; \tag{12}$$

$$\sigma_{rr}^{(0)}=-\sigma_{zz}^{(0)}=(\sigma_{rr}^*-\sigma_{zz}^{(0)})/2; \quad \sigma_{rz}^{(0)}=\sigma_{rz}^*. \tag{13}$$

As can be seen from (11)–(13), the determinations of the initial and final stresses depend on the distribution of the instantaneous pressures in the soil. If the continuous function $p_0(r,z,t)$ for the axially symmetric problem was previously found in some way, then the initial stresses in the soil skeleton in relation to a limited compaction region can be expressed as

$$\sigma_{rr}^{(0)}=-\sigma_{zz}^{(0)}=-\frac{z}{2}\frac{\partial p_0(r,z)}{\partial z};\quad \sigma_{rz}^{(0)}=-z\frac{\partial p_0(r,z)}{\partial r}, \tag{14}$$

where $p_0(r,z,t)$ is the initial distribution of pore pressure; r is a radius of the soil cylinder; $\sigma_{ij}^{(0)}$ is the initial pressures at the studied point.

Thus, in order to calculate the values of pore pressure for any moment of time and coordinates, it is necessary to know the values of these quantities for the initial $p_0(r,z,0)=p_{ini}$ point of time $t=0$.

We proceed to the determination of the pressure in the pore fluid for the initial moment of time. To do this, we consider the compaction of a two-phase soil cylinder of radius r, height h with a permeable bottom and walls. Let at the same time on some upper part of the area $a<R$ a uniformly distributed load with intensity q is applied.

In view of the symmetry of the problem under study with respect to the z axis, it is convenient to study it in cylindrical coordinates.

Therefore, in order to determine p_{ini}, one should solve the equation

$$\nabla^2 p_0=\frac{\partial}{\partial r}\left(k_r\frac{\partial p_0}{\partial r}\right)+\frac{1}{r}\cdot\frac{\partial}{\partial r}(k_r p_0)+\frac{1}{r^2}\cdot\frac{\partial}{\partial\varphi}\left(k_\varphi\frac{\partial p_0}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial p_0}{\partial z}\right)=0. \tag{15}$$

If the pressure in the pore fluid does not depend on the angle, then instead of equation (15) we have

$$k_r\left(\frac{\partial^2 p_0}{\partial r^2}+\frac{1}{r}\frac{\partial p_0}{\partial r}\right)+k_z\frac{\partial^2 p_0}{\partial z^2}=0. \tag{16}$$

Here $p_0=p_{ini}$; k_r,k_z are the constant filtration coefficients in horizontal and vertical directions.

The solution of equation (16) when $k_r=k_z$ applied to the problem under consideration should satisfy the following boundary conditions:

$$\begin{gathered}\frac{\partial p_0}{\partial r}=0\ \text{ at }\ r=R;\\ \frac{\partial p_0}{\partial z}=0\ \text{ at }\ z=0;\\ p_0=q-p_{str}\ \text{ at }\ z=h, r\le a;\\ p_0=0\ \text{ at }\ z=h, r>a.\end{gathered} \tag{17}$$

Here p_{str} is the structural strength of compression.

This solution can be represented as follows:

$$p_0(r,z,0)=(q-p_{str})\left(\frac{a^2}{R^2}+\frac{a}{R}\right)\sum_{k=1}^{\infty}\frac{J_1(\mu_k\frac{a}{R})ch\frac{\mu_k}{R}z}{\mu_k J_k^2(\mu_k)ch\frac{\mu_k}{R}h}J_0\left(\mu_k\frac{r}{R}\right), \tag{18}$$

where μ_k is the countless positive roots of the transcendental equation of the form $J_1(\mu)=0$; $J_0(\mu)$, $J_1(\mu)$ are the Bessel functions of the first kind, respectively, of the zero and first orders.

From the expression (18) when $z=h$ we get

$$p_0(r,h,0)=\frac{qa^2}{R^2}+\frac{2aq}{R}\sum_{k=1}^{\infty}\frac{J_1(\mu_k\frac{a}{R})}{\mu_k J_k^2(\mu_k)}J_0\left(\mu_k\frac{r}{R}\right). \tag{19}$$

The sum of the series (19) at $r\le a$ is equal to q, and at $r>a$ equals zero. Consequently, the obtained solution (18) fully satisfies all the boundary conditions (17).

The solution of the problem for a concentrated force is obtained from the equation (16) under the following boundary conditions:

$$\begin{gathered}\frac{\partial p_0}{\partial r}=0\ \text{ at }\ r=R;\\ \frac{\partial p_0}{\partial z}=0\ \text{ at }\ z=0;\end{gathered} \tag{20}$$

$$p_0 = \infty \quad \text{at} \quad z = h, \quad r = 0;$$

$$p_0 = 0 \quad \text{at} \quad z = h, \quad r \neq 0.$$

This solution under the boundary conditions (20) has the form

$$p_0(r, z, 0) = \frac{Q}{\pi R^2} + \frac{Q}{\pi R^2} \sum_{k=1}^{\infty} \frac{J_0(\mu_k \frac{r}{R}) ch \frac{\mu_k}{R} z}{J_0^2(\mu_k) ch \frac{\mu_k}{R} h}. \tag{21}$$

The expression (21) can be obtained directly from the solution for a uniformly distributed load (18), assuming $Q = \pi a^2 q = const$ always at value of a that tends to zero.

Consider the more general case, i.e. the distributed external load varies in the time and for the boundary conditions of the form:

$$\alpha^{(oc)} \frac{\partial p_0}{\partial x_k} + \beta^{(oc)} p_0(x_k) = \gamma^{(oc)} q, \qquad k = 1, 2, 3. \tag{22}$$

Here $\beta^{(oc)} = 0$, $\gamma^{(oc)} = 0$, at $x = r = R$ or $z = 0$ and $\alpha^{(oc)} = 0, \gamma^{(oc)} = \beta^{(oc)}$ at $x = r \leq a, z \to h$ or $r > a, z \to h$ at $\alpha^{(oc)} = \gamma^{(oc)} = 0$. Note that the $\alpha^{(oc)}, \beta^{(oc)}, \gamma^{(oc)}$ are the boundary condition parameters.

The solution of equation (16) with the boundary conditions (22) can be represented as follows

$$p_0(r, z) = \frac{2}{R^2} \sum_{k=1}^{\infty} \frac{J_0(\mu_k \frac{r}{R}) ch \frac{\mu_k}{R} z}{J_k^2(\mu_k) ch \frac{\mu_k}{R} h} \int_0^a q(r, h) J_0 \left(\mu_k \frac{r}{R} \right) dr. \tag{23}$$

After finding the change in the instantaneous pressures in the soil, according to formulas (11)–(13), the initial and final stresses in the soil skeleton can already be determined. At the same time we have

$$-\sigma_{rr}^{(0)} = \sigma_{zz}^{(0)} = -\frac{z}{2} \frac{2}{R^3} \sum_{k=0}^{\infty} \frac{\mu_k J_0(\mu_k \frac{r}{R}) ch \frac{\mu_k}{R} z}{J_k^2(\mu_k) ch \frac{\mu_k}{R} h} \int_0^a q(r, h) J_0 \left(\mu_k \frac{r}{R} \right) dr;$$

$$\sigma_{rz}^{(0)} = -z \frac{2}{R^3} \sum_{k=0}^{\infty} \frac{\mu_k J_1(\mu_k \frac{r}{R}) ch \frac{\mu_k}{R} z}{J_k^2(\mu_k) ch \frac{\mu_k}{R} h} \int_0^a q(r, h) J_0 \left(\mu_k \frac{r}{R} \right) dr;$$

$$\sigma_{rr}^* = \left(1 + z \frac{\partial}{\partial z} \right) p_0; \quad \sigma_{zz}^* = \left(1 - z \frac{\partial}{\partial z} \right) p_0; \quad \sigma_{rz}^* = \sigma_{zr}^* = -z \frac{\partial p_0}{\partial x}.$$

Consider a few special cases related to the loading of the upper surface of a compacted soil cylinder.

Case 1. On a part of the outer area with the radius $a < R$, a uniformly distributed load with intensity q is applied. For this case, the expression (23) is reduced to the form:

$$p_0(r, z) = \frac{2a(q - p_{str})}{R^2} \sum_{k=1}^{\infty} \frac{J_0(\mu_k \frac{r}{R}) ch \frac{\mu_k}{R} z}{J_k^2(\mu_k) ch \frac{\mu_k}{R} h} J_0 \left(\mu_k \frac{r}{R} \right). \tag{24}$$

Expression (24) makes it possible to determine the distribution of pore pressure in a compacted two-phase soil cylinder for an initial point in time. Moreover, for a three-phase soil environment, it looks as follows:

$$p_0^t(r, z) = \frac{1}{\omega_0'} p_0(r, z).$$

Here ω_0' is the number that takes into account the three-phase soil. Using relations (9)–(14) we find the initial and final stresses in the soil skeleton.

Calculation of vertical drains. Existing baseline calculations with vertical drains are also based on the theory of filtration consolidation. The calculations are to determine the degree of compaction of the soil of the base under the influence of an external load at any time.

The use of vertical sand drains usually allows to reduce the time for consolidation of soil foundations composed of weak water-saturated clay soils in the construction of transport, industrial and hydraulic structures.

For the calculation of the vertical sand drains, the soil compaction is considered around a single drain. To do this, in a soil array with planes that limit the scope of one drain from another, cut a prismatic block of water-saturated clay soils that the drains were located along the vertical axis of the block. Then, in order to calculate the stress-strain state, the prismatic block is replaced by a soil cylinder of the same volume with a drain along the vertical axis of the cylinder.

In this case, the calculation of vertical drains also reduces to solving the axially symmetric spatial problem of the theory of the multiphase soils consolidation. In particular, determining the pressures in the pore fluid for the initial moment of time consists in solving equation (16) under appropriate boundary conditions. Moreover, in calculating the problems of filtration consolidation for the case of application of vertical drains, the boundary conditions are taken on the basis of the existing classical calculations of filtration consolidation [1–4].

Therefore, the boundary conditions for solving the problem in the case of application of vertical drains for the initial moment of time has the form (22). And $\alpha^{(oc)} = \gamma^{(oc)} = 0$ at $r = r_0$, i.e. the excess pore pressure or pressure function on the surface of a vertical drain of a radius r_0 is equal to 0; $\alpha^{(oc)} = \gamma^{(oc)} = 1$ at $t = \tau_1$, i.e. at the time of application of the load, the pore pressure on the surface of the soil layer is equal $\frac{1}{3}\theta^* + p^*$; $\beta^{(oc)} = 0, \ \ \alpha^{(oc)} = \gamma^{(oc)} = 0$, at $r = R$, i.e. through the surface of the cylinder of the zone of influence of the vertical drain with a radius R as a result of symmetry, the flow of water does not occur; $\beta^{(oc)} = 0, \gamma^{(oc)} = 0$, at $z = 0$, i.e. the lower horizontal boundary of the soil mass is impenetrable, and due to the symmetry of the flow there is no movement of water through the surface.

The solution of the equation (16) under these boundary conditions can be represented as

$$p_0(r,z) = \frac{\pi^2}{2}\sum_{i=0}^{\infty}\frac{\mu_i^2 J_1\left(\mu_i\frac{R}{\sqrt{k_r}}\right)\frac{1}{k_r}\int_{\frac{r_0}{\sqrt{k_r}}}^{\frac{R}{\sqrt{k_r}}}}{ch(\mu_i\frac{h}{\sqrt{k_z}})[J_0^2(\mu_i\frac{r_0}{\sqrt{k_r}}) - J_1^2(\mu_i\frac{R}{\sqrt{k_r}})]}V_0\left(\mu_i\frac{r}{\sqrt{k_r}}\right)ch\mu_i\frac{z}{\sqrt{k_r}}, \tag{25}$$

$$V_0(\mu_i\bar{r}) = J_0(\mu_i\bar{r_0})Y_0(\mu_i\bar{r}) - Y_0(\mu_i\bar{r_0})J_0(\mu_i\bar{r}).$$

Using the expression (25), we find the pressure in the pore fluid of the vertical drain for the initial moment of time.

The solution of the problem for a uniformly distributed force is determined from relation (25). In this case, in (25), the value $q(r,h,t)$ is replaced by q and we perform the integrations. It should be noted that the problems of soil consolidation were studied in [5–15] with regard to physical nonlinearity and heterogeneity of the earth masses, respectively.

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Қысымның кеуек қуысындағы бастапқы шарты және кернеудің топырақ консолидациясындағы есебі

Топырақтың жылжу қасиеті В.А.Флорин берген түрінде жазылған серпімді жылжымалы теориясына бойсындырып, кеуек қуысындағы бастапқы қысымды p_0 анықтау әдістері келтірілген. Мақалада биіктігі h радиусы r болатын, едені мен жандары су өткізетін екіфазалы топырақтан тұратын цилиндр тығыздалуының алғашқы уақыттағы кернеуі зерттелген. Цилиндр жоғары бетіне ауданы $a < R$ бетіне қарқындылығы q болған тең жайылған күш қойылған. Зерттеліп отырған есептің z өсіне симметриялы болуына байланысты цилиндрлік координаталарда қарастырылған. Осындай қойылымында топырақтың кеуегіндегі суға түсетін басым күшімен қатар, оның қаңқасындағы бастапқы кездегі кернеуді анықтайтын есептеу өрнектері табылған.

Кілт сөздер: кеуек қуысындағы қысым, кернеу, консолидация, топырақ, тығыздық, кеуектілік, цилиндрлік координаттар, шекаралық шарттар.

А. Дасибеков, А. Абжапбаров, Н.К. Аширбаев, М.Т. Шоманбаева

Начальные значения порового давления и напряжений в задачах консолидации грунтов

Согласно основной модели В.А. Флорина, разработана методика определения начального избыточного порового давления p_0 во всех точках грунтового массива. При этом найдена сумма главных напряжений в скелете грунта. Кроме этого, дополнительное давление, обусловленное приложением внешней нагрузки, при мгновенной консолидации грунта обращается в нуль. В качестве расчетной схемы принято уплотнение двухфазного грунтового цилиндра радиусом r, высотой h с водопроницаемым дном и стенками. На некоторой верхней части площади цилиндра $a < R$ приложена равномерно-распределенная нагрузка с интенсивностью q. В связи с симметрией данной задачи относительно оси z она исследована в цилиндрических координатах. На основе этого результата определено решение задачи для сосредоточенной силы.

Ключевые слова: поровое давление, напряжение, консолидация, грунт, уплотнение, пористость.

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