2-бөлім Раздел 2 Section 2

Механика Механика Mechanics

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PHYSICOMATHEMATICAL MODEL OF CALCULATING CONTINUOUS BEAMS WITH ELASTIC YIELDING SUPPORTS

In this work there has been studied the rod system operation dependence on the action of external force effects. The system presents a multi-span continuous beam with elastic yielding supports. To identify the stress-strain state of the object under study, a precise analytical method of forces is used. The method of five moments is used as the resolving (canonical) equations. The final results are the parameters of the stress-strain state for a five-span continuous beam with variable compliance coefficients on 5 intermediate supports and an absolutely rigid left extreme support, deflections, bending moments, shear forces, and support reactions.Theoretical provisions and practical results can be used in the design of load-bearing beam structures in buildings and various engineering structures.

Key words: continuous beams, support, vertical displacements of beam nodes, compliance coefficient, force method, equations of five moments, deflection diagrams.

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Серпімді-икемді тіректері бар тұтас арқалықтарды есептеудің физика-математикалық моделі

Бұл жұмыста стержінді жүйеге, яғни серпімді-икемді тіректері бар көпаралықты тұтас арқалыққа сыртқы күштер әсерінің әрекеті зерттелінді. Зерттелетін объектінің кернеулі-деформацияланған күйін анықтау үшін нақты аналитикалық күштер әдісі пайдаланылды. Шешуші (канондық) теңдеулер ретінде бес моменттер әдісі қолданылды. Соңғы нәтижелер ретінде сол жағы қатаң бекітілген және бесаралық тіректерде өзгермелі коэффициенттері бар бесаралықты тұтас арқалықтың кернеулі-деформацияланған күйінің параметрлерінің иілу, иілу моменттері, көлденең күштері, тірек реакциялары келтірілген.
Алынған теориялық және практикалық нәтижелер ғимараттар мен әртүрлі инженерлік құрылыстардағы тұтас арқалықтар құрылымын жобалау кезінде қолданыла алады.

Түйін сөздер: Тұтас арқалықтар, тіректің икемділік коэффициенті, арқалықтың түйіндерінің тік жылжуы, икемділік коэффициенті, күштер әдісі, бес моменттердің теңдеулері, майысу эпюрасы.

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Физико-математическая модель расчета неразрезных балок с упруго-податливыми опорами

В данной работе выполнено исследование работы стержневой системы на действие внешних силовых воздействий, представляющей многопролетную неразрезную балку, опоры который является упруго-податливыми. Для выявления напряженного-деформированного состояния изучаемого объекта применяется точный аналитический метод сил. В качестве разрешающих (канонических) уравнений использован метод пяти моментов. В качестве конечных результатов приведены параметры напряженного-деформированного состояния для пятипролетной неразрезной балки с переменными коэффициентами податливости на 5-ти промежуточных опорах и абсолютно жесткой левой крайней опорой, прогибы, изгибающие моменты, поперечные силы, опорные реакции. Теоретические положения и практические результаты могут использоваться при проектировании несущих балочных конструкций в зданиях и различных инженерных сооружениях.

Ключевые слова: Неразрезные балки, коэффициент податливость опор, вертикальные смещения узлов балок, коэффициент податливости, метод сил, уравнения пяти моментов, эпюры прогибов.

1 Introduction

Multi-span continuous beams are widely used in bridge, industrial and civil construction [1-3] in the form of load-bearing beams, columns of multi-storey frames of high-rise buildings. They are also the base of the carriageway of pontoon bridges.

Intermediate supports of such structures can have elastic displacements at the joints of adjacent structures, which significantly affects their stress-strain state.

Continuous or multi-span beams are statically indeterminate beams that span several spans (two or more) and do not have intermediate hinges. Continuous beams constitute an important class of statically indeterminate rod systems and are often found both in construction and in other branches of present day technology. The most widely used general method of calculating statically indeterminate systems in practice is the method of forces. It is covered in detail in the scientific literature on the mechanics of structures [4-6].

The method of calculating continuous beams using the equations of five moments also has a number of significant advantages and is only a modernization of the method of force transformation [2, 7].

When calculating the main beams as a part of various beam cells, it becomes necessary to identify the parameters of their stress-strain state, that is, the values of displacements (deflections), internal forces, etc. In power engineering, such structures are used as load-bearing structures for mechanisms and machines [7, 8].

The purpose and the tasks of the study. The purpose of this work is to study the stress-strain state of rod beam systems with a high degree of static uncertainty with supports of both rigid and elastic yielding types using the precise analytical method of forces for their calculation.

At this, the following tasks are realized: developing the resolving canonical equations of the method of forces; obtaining expressions for calculating their elements taking into account the compliance coefficients of the supports; developing a methodology of calculating the compliance coefficients of the supports of a five-span continuous beam, and their effect on the parameters of the stress-strain state.

2 Theoretical provisions and calculation methods

This paper is dealing with the stress-strain state of a five-span continuous beam with elastic yielding supports under the action of concentrated nodal forces P_1, P_2,...,P_5 (Fig. 1, a) with variable bending stiffness of the spans.

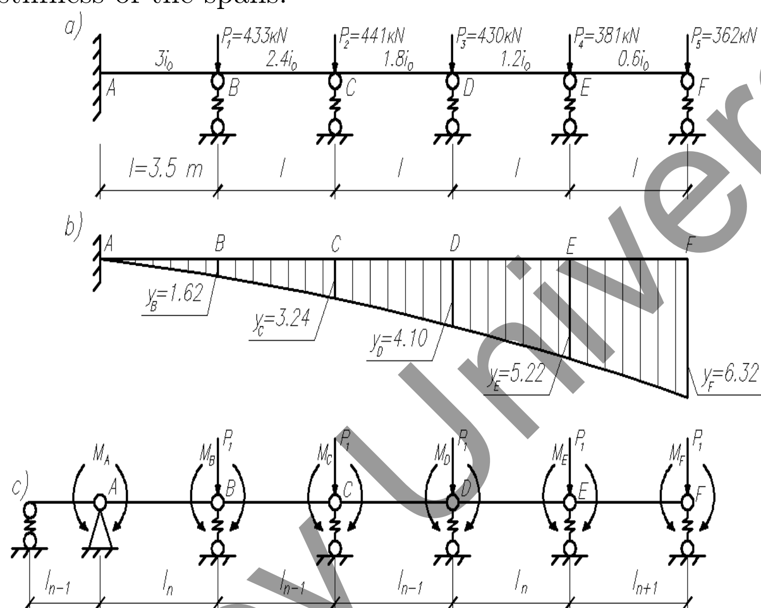

Figure 1: Towards the calculation of a continuous beam: a) preset continuous beam; b) the curve of initial (preset) deflections of the beam nodes; c) the main system of the force method

The support elasticity is determined by the coefficient of their compliance: C_i (i=1,2,...,m) is the movement of the "i–th" support caused by the action of a unit force ($\overrightarrow{P}_i = 1$) on it; the unit of measurement of C_i is cm/kg. Let the supports of the beam (except for the support "A") receive some initial displacements (Figure 1, b), then the compliance coefficients of the supports have the following values:

$$C_B = \frac{y_B}{P_1} = \frac{1.62}{43.3 \cdot 10^3} = 3.74 \cdot 10^{-6} cm/kg; \quad C_C = \frac{y_C}{P_2} = \frac{3.24}{44.1 \cdot 10^3} = 73.47 \cdot 10^{-6} cm/kg;$$

$$C_D = \frac{y_D}{P_3} = \frac{4.1}{43 \cdot 10^3} = 95.35 \cdot 10^{-6} cm/kg; \quad C_E = \frac{y_E}{P_4} = \frac{5.22}{38.1 \cdot 10^3} = 137 \cdot 10^{-6} cm/kg;$$

$$C_F = \frac{y_F}{P_5} = \frac{6.32}{36.2 \cdot 10^3} = 174.59 \cdot 10^{-6} cm/kg.$$

We take the exact analytical method of forces [9-11] as the method of calculating such continuous beams. The main system is taken a multi-span beam divided by hinges in the supports into single-span beams. The main unknowns are the support moments (Figure 1, c). The canonical equation for the n-th support will have the form of the equation of five moments [9, 10].

$$\delta_{n,n-2}M_{n-2}+\delta_{n,n-1}M_{n-1}+\delta_{n,n}M_{n}+\delta_{n,n+1}M_{n+1}+\delta_{n,n+2}M_{n+2}+\Delta_{np}=0,\ (i=A,B,C,D,E). \tag{1}$$

The coefficients of canonical equation (1), taking into account elastic yield of the supports will take the form:

$$\delta_{n,n-2}=\frac{C_{n-1}}{l_{n-1}l_n};\delta_{n,n-1}=\frac{l_n}{6EJ_n}-\frac{C_{n-1}}{l_n}\left(\frac{1}{l_{n-1}}+\frac{1}{l_n}\right)-\frac{C_n}{l_n}\left(\frac{1}{l_n}+\frac{1}{l_{n+1}}\right);$$

$$\delta_{n,n}=\frac{l_n}{3EJ_n}+\frac{l_{n+1}}{3EJ_n l_{n+1}}+\frac{C_{n-1}}{{l_n}^2}+C_n\left(\frac{1}{l_n}+\frac{1}{l_{n+1}}\right)^2+\frac{C_{n+1}}{l_{n+1}}; \tag{2}$$

$$\delta_{n,n+1}=\frac{l_{n+1}}{6EJ_{n+1}}-\frac{C_{n+1}}{l_{n+1}}\left(\frac{1}{l_{n+1}}+\frac{1}{l_{n+2}}\right)-\frac{C_n}{l_{n+1}}\left(\frac{1}{l_n}+\frac{1}{l_{n+1}}\right);\delta_{n,n+2}=\frac{C_{n+1}}{l_{n+1}l_{n+2}};$$

$$\Delta_{np}=\frac{B_n^{\Phi}}{EJ_n}+\frac{A_{n+1}^{\Phi}}{EJ_{n+1}}+\frac{C_{n-1}}{\ell_n}R_{n-1}-C_n\left(\frac{1}{\ell_n}+\frac{1}{\ell_{n+1}}\right)R_n+\frac{C_{n+1}}{\ell_{n+1}}R_{n+1},$$

where R_{n-1}, R_n, R_{n+1} are respectively the support reactions $n-1$, n, $n+1$ in the composition of the basic system; B_n^D, A_{n+1}^D are respectively fictitious reactions of the support considered in the n and $n+1$ spans of the beam (for example under the action of the uniformly distributed load q_i for a span with the length l_i these reactions are equal to $B_i^D=A_i^D=q_i l_i^3/24$). The deflection on the n-th support of the beam, taking into account their elastic yield is equal to the total response of this support multiplied by C_n, that is:

$$y_n=C_n\left[\frac{M_{n-1}}{l_n}-\left(\frac{1}{l_n}+\frac{1}{l_{n+1}}\right)\right]M_n+\left(\frac{M_{n+1}}{l_{n+1}}\right)+R_n. \tag{3}$$

3 Results

Let's accept the proposed theory of calculation for the preset continuous beam (Figure 1,a).

1. Support "A" ($n=A$)

$$\delta_{A,A}M_A+\delta_{A,B}M_B+\delta_{A,C}M_C+\Delta_{A,P}=0. \tag{4}$$

Let's calculate the coefficients of equation (4):

$$\delta_{A,A} = \frac{0}{3EJ_0} + \frac{10^{-8}}{3 \cdot 3 \cdot 8.06} + \frac{3.74 \cdot 10^{-6}}{(350)^2} = 0.0445 \cdot 10^{-8};$$

$$\delta_{A,B} = \frac{10^{-8}}{6 \cdot 3 \cdot 8.06} - \frac{3.74 \cdot 10^{-6}}{350}\left(\frac{1}{35} + \frac{1}{35}\right) = -0.05417 \cdot 10^{-8};$$

$$\delta_{A,C} = \frac{3.74 \cdot 10^{-6}}{(350)^2} = 0.0305 \cdot 10^{-8}; \quad \delta_{A,P} = \frac{3.74 \cdot 10^{-6}}{350} \cdot 43.3 \cdot 10^3 = 4.627 \cdot 10^{-3}.$$

2. Support "B" ($n = B$)

$$\delta_{B,A} M_A + \delta_{B,C} M_C + \delta_{B,D} M_D + \Delta_{B,P} = 0$$

$$\delta_{B,B} = \frac{10^{-8}}{3 \cdot 3 \cdot 8.06} + \frac{10^{-8}}{3 \cdot 2.4 \cdot 8.06} + (37.4 \cdot 10^{-6}) \cdot \left(\frac{1}{350} + \frac{1}{350}\right)^2 + \frac{73.47 \cdot 10^{-6}}{(350)^2} = 0.2131 \cdot 10^{-8};$$

$$\delta_{B,A} = \frac{10^{-8}}{6 \cdot 3 \cdot 8.06} - \frac{37.4 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = \delta_{A,B} = -0.05417 \cdot 10^{-8};$$

$$\delta_{B,C} = \frac{10^{-8}}{6 \cdot 2.4 \cdot 8.06} - \frac{73.47 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) - \frac{37.4 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = -0.1724 \cdot 10^{-8};$$

$$\Delta_{B,P} = -37.4 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) \cdot 43.3 \cdot 10^3 + \frac{73.47 \cdot 10^{-6}}{350} \cdot 44.1 \cdot 10^3 = 0.00322 \cdot 10^{-3};$$

$$\delta_{B,D} = -\frac{73.47 \cdot 10^{-6}}{350 \cdot 350} = 0.059976 \cdot 10^{-8}.$$

3. Support "C" ($n = C$)

$$\delta_{C,A} M_A + \delta_{C,B} M_B + \delta_{C,C} M_C + \delta_{C,D} M_D + \Delta_{C,P} = 0$$

$$\delta_{C,A} = \frac{37.4 \cdot 10^{-6}}{350 \cdot 350} = 0.03053 \cdot 10^{-8};$$

$$\delta_{C,B} = \frac{10^{-8}}{6 \cdot 2.4 \cdot 8.06} - \frac{37.4 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) - \frac{73.47 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = -0.1724 \cdot 10^{-8};$$

$$\delta_{C,C} = \frac{10^{-8}}{3 \cdot 2.4 \cdot 8.06} + \frac{10^{-8}}{3 \cdot 1.8 \cdot 8.06} + \frac{37.4 \cdot 10^{-6}}{(350)^2} + 73.47 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right)^2 +$$

$$+\frac{93.35 \cdot 10^{-6}}{(350)^2} = 0.386842 \cdot 10^{-8};$$

$$\delta_{C,D} = \frac{10^{-8}}{6 \cdot 1.8 \cdot 8.06} - \frac{93.35 \cdot 10^{-6}}{350}\frac{37.4 \cdot 10^{-6}}{(350)^2} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) -$$

$$-\frac{73.47 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = -0.26414 \cdot 10^{-8};$$

$$\delta_{C,E} = \frac{95.35 \cdot 10^{-6}}{350 \cdot 350} = 0.077837 \cdot 10^{-8};$$

$$\Delta_{C,P} = \frac{37.4 \cdot 10^{-6}}{350} \cdot 43.3 \cdot 10^3 - 73.47 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) \cdot 44.1 \cdot 10^3 +$$

$$+\frac{95.35 \cdot 10^{-6}}{350} \cdot 43.3 \cdot 10^3 = -2.1731 \cdot 10^{-3}.$$

4. Support "D" ($n = D$)

$$\delta_{D,B} M_B + \delta_{D,C} M_C + \delta_{D,D} M_D + \delta_{D,E} M_E + \Delta_{D,E} = 0$$

$$\delta_{D,B} = \frac{73.47 \cdot 10^{-6}}{350 \cdot 350} = 0.059976 \cdot 10^{-8};$$

$$\delta_{D,C} = \frac{10^{-8}}{6 \cdot 1.8 \cdot 8.06} - \frac{73.47 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) - \frac{93.35 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = -0.2652 \cdot 10^{-8};$$

$$\delta_{D,D} = \frac{10^{-8}}{3 \cdot 1.8 \cdot 8.06} + \frac{10^{-8}}{3 \cdot 1.2 \cdot 8.06} + \frac{73.47 \cdot 10^{-6}}{(350)^2} + 95.35 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right)^2 = 0.5471 \cdot 10^{-8};$$

$$\delta_{D,E} = \frac{10^{-8}}{6 \cdot 1.2 \cdot 8.06} - \frac{137 \cdot 10^{-8}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) - \frac{95.35 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right)^2 = -0.3622 \cdot 10^{-8};$$

5. Support "E" ($n = E$)

$$\delta_{E,C} M_C + \delta_{E,D} M_D + \delta_{E,E} M_E + \delta_{E,P} = 0$$

$$\delta_{E,D} = \frac{10^{-8}}{6 \cdot 1.8 \cdot 8.06} - \frac{93.35 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) - \frac{137 \cdot 10^{-6}}{350} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) = -0.3621 \cdot 10^{-8};$$

$$\delta_{E,E} = \frac{10^{-8}}{3 \cdot 1.2 \cdot 8.06} + \frac{10^{-8}}{3 \cdot 0.6 \cdot 8.06} + \frac{97.35}{(350)^2} + 137 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right)^2 +$$

$$+\frac{174.6 \cdot 10^{-6}}{(350)^2} = 0.77164 \cdot 10^{-8};$$

$$\delta_{E,P} = \frac{95.35 \cdot 10^{-6}}{350} \cdot 43 \cdot 10^3 - 137 \cdot 10^{-6} \cdot \left(\frac{1}{350} + \frac{1}{350}\right) \cdot 38.1 \cdot 10^3 +$$

$$+\frac{174.6 \cdot 10^{-6}}{350} \cdot 36.2 \cdot 10^3 = -0.539 \cdot 10^{-3}.$$

The system of linear algebraic equations (5) (SLAE) for the preset continuous beam will have the form:

$$A \cdot \vec{M} = \vec{P}, \tag{5}$$

where *A(5x5)* is the square matrix of the 5^{th} order (Table 1); $\vec{M} = |\{M_A, M_B,\ M_C, M_D, M_E\}|$ is the vector of the moments at support of the basic system; $\vec{P} = |\{P_B, P_C, P_D, P_E, P_F\}|$ is the vector of free members taking into account the load preset for the beam.

Table 1 – matrix of the force method for calculating 5-span continuous beam with elastic yielding supports

№	M_A	M_B	M_C	M_D	M_E	Right part
1	0.0445	-0.05417	0.03053	–	–	-4.627x10^5
2	-0.05417	0.2131	-0.1724	0.059976	–	0.00322x10^5
3	0.03053	-0.1724	0.386842	-0.2652	0.07784	2.1731x10^5
4	–	0.059976	-0.2652	0.05417	-0.3624	-0.74176x10^5
5	–	–	0.07784	-0.3624	0.77164	0.0539x10^5

By solving the system of equation (6), we obtain the moments at support of the continuous bean:

$$\vec{M} = A^{-1}\vec{P}, \tag{6}$$

where A^{-1} is the reverse matrix.

Then there is given solution for equation (6):

$$M_A = -1.513 \cdot 10^7\ (kg\,cm); \qquad M_B = -3.434 \cdot 10^6\ (kg\,cm); \qquad M_C = 7.982 \cdot 10^5\ (kg\,cm);$$

$$M_D = 8.398 \cdot 10^5\ (kg\,cm); \qquad M_E = 3.205 \cdot 10^5\ (kg \cdot cm).$$

Let's calculate the largest deflection (at the "F" point) by multiplying the curves M (Figure 2, b) and $\vec{M}_F$ (Figure 2, c) according to the Vereshchagin rule (7).

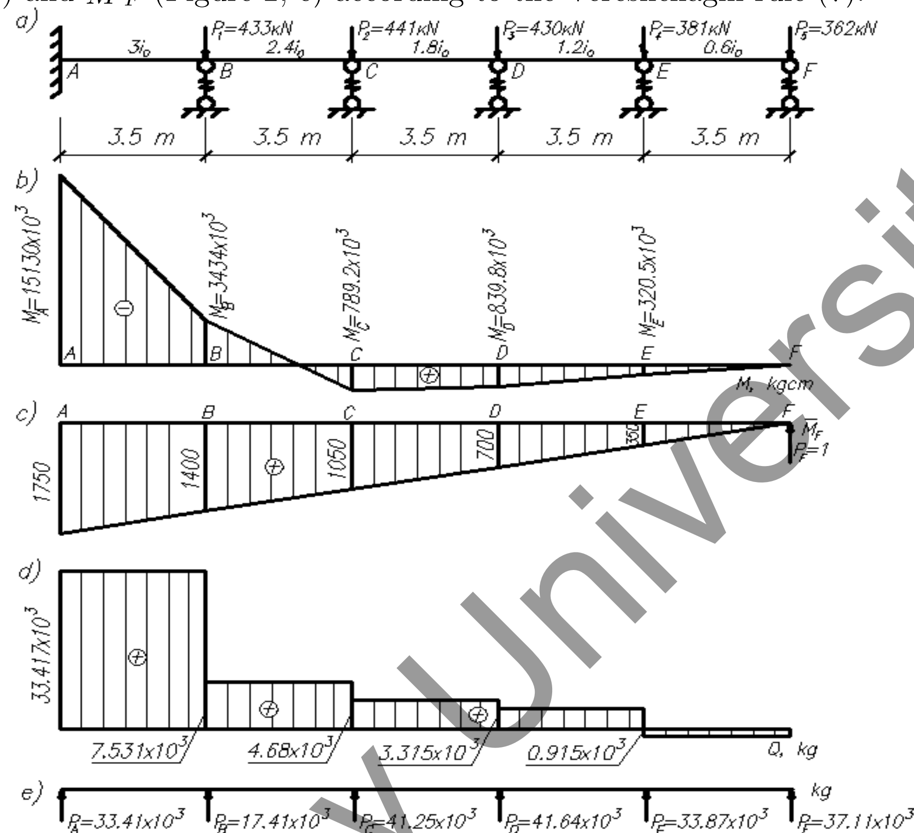

Figure 2: The results of calculating beams:a) preset continuous beam; b) the curve of moments; c) single curve of the moments; d) the curve of transverse forces; e) support reactions

$$\sum \int_0^S \frac{M \cdot \bar{M}_S}{EJ} dS = \frac{1}{EJ}(M) \times (\bar{M}_S) = y_n, \tag{7}$$

According to (7), we have

$$y_F = y_{\max} = 6.48\,cm.$$

The ($y_F = y_{\max} = 6.48\,cm$) value is close enough to the initial value (Figure 1, b; $y_F = 6.32\,cm$, the error is about 2.53 %), which confirms reliability of the theoretical provisions of this work, as well as the accuracy of practical calculations.

Using the ordinates of the obtained curves of moments (Figure 2, b), according to Zhuravsky rule, we determine the ordinates of the diagrams of transverse forces (Figure 2, d) and then the values of the support reactions (Figure 2, f).

Let's check the condition of equilibrium of the continuous beam.

$$\sum F_{k,y} = \sum P_i. \tag{8}$$

According to Figure 2, a, e, based on expression (8), we have:

$$R_A + R_B + R_C + R_D + R_E + R_F = P_1 + P_2 + P_3 + P_4 + P_5,$$

or $(33.417 + 17.14 + 41.25 + 41.64 + 33.87 + 37.116) \cdot 10^3 = 204.7 \cdot 10^3$; the error is 0.016%. The deflections are calculated by formula (3), [9].

$$y_n = \left[R_n^0 + \frac{1}{l_{n+1}} \left(M_{n+1} - M_n \right) - \frac{1}{l_n} \left(M_n - M_{n-1} \right) \right] \left(\frac{1}{\aleph_n} \right). \tag{9}$$

4 Conclusions

1. In this work, there has been studied the stress-strain state (SSS) of a five-span continuous beam with elastic yielding supports with different bending stiffness of the spans under the action of nodal forces (Figure 1, a).

2. The applied research method is the precise analytical method of forces; in this case, the unit and load coefficients are determined taking into account the coefficients of elastic compliance of the supports of the beams.

3. Based on the equation of five moments, the supporting bending moments M_A, M_B, M_C, M_D, M_E have been determined (Figure 2, b), and then, according to the Zhuravsky rule, the transverse forces on the spans (Figure 1, d) of the beam have been calculated and all the support reactions have been determined;

4. The greatest deflection has been calculated (at the "F"point); at this, sufficient proximity of its value to the initial value of the deflection in Figure 1, b has been established, which confirms the correctness of the theoretical provisions of the study with their practical implementation.

5. In the course of the study, it has been found that the presence of elastic yielding supports reduces the highest value of the corresponding parameters of the stress-strain state in comparison with the cantilever bar as follows:

   - for bending moments by 4.74 times;

   - for deflections by 14.92 times;

This suggests that in order to reduce the material consumption of a continuous beam, the presence of elastic yielding supports is advantageous.

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