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**ANALYSIS AND SYNTHESIS OF TOPOLOGICAL RADAR DETECTORS
OF LOW-CONTRAST TARGETS AGAINST THE BACKGROUND
OF HIGH INTENSITY NOISE AS A NEW BRANCH OF RADIOLOCATION
AND THE THEORY OF STATISTICAL SOLUTIONS**

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The new kind and approach of up-to-date radiolocation: fractal-scaling or scale-invariant radiolocation has been proposed. The main ideas and strategic directions in synthesis of fundamentally new topological radar detectors of low-contrast objects have been considered. The objects detection is conducted against the background of reflections from the ground, sea and precipitations. The new topologic signs and methods of detection of low-contrast objects against the background of high-intensity noise are presented. The methods are based on the textural and fractal analysis and also on the theory of deterministic chaos. The main purpose of the work is to interpret the main directions of radio physics, radio engineering and radio location in "fractal" language that makes new ways and generalizations promising radio systems in future.

Keywords: texture, fractal, lacunarity, Hurst exponent, signals detector, low-contrast target, radar, fractal - frequency MIMO – Systems.

Introduction

Radar detection of low observable and small objects near the surface of the earth and the sea, as well as at meteorological precipitation is an extremely difficult problem [1, 2]. In addition, sea and vegetation clutters are of non-stationary and multiscale character, especially at ϑ shallow angles. A variety of underlying coverings, radar observation conditions and following the above mentioned objects very often lead to the fact that the signal to noise ratio q_0^2 for such problems almost always fills the area of negative values (in decibels), i.e., $q_0^2 < 1$ dB [1,2]. This makes classical radar detection methods and detection algorithms inapplicable in most cases, that is, use of energy detectors (when the likelihood ratio is determined solely and only by the received signal energy) becomes essentially impossible.

What is to be done? There is a way! Detection of low-contrast targets against the background of mentioned above high-intense natural noise inevitably requires to offer, and then calculate some fundamentally new characteristic that is different from the functional related to noise and signal energy, and is determined by the topology and the dimension of the received signal.

Introduction of the concepts "texture", "deterministic or dynamic chaos", "fractal" and "fractal dimension" to the scientific use in radio physics and radiolocation, enabled the author to be the first to offer and then apply new dimension and topological (not energy!) characteristics (invariants), which the author combined under the general concept "sampling topology" [1-12].

The work objective is to give a brief analytical review of the development and improvement of the author's methods and algorithms for new topological, including fractal textural detectors of low-contrast radar targets against the background of high-intense ground and sea clutters, as well as meteorological precipitation ones.

1. Fractal-scaling or scale-invariant radiolocation and fractal MIMO-radars as a new kind of radiosystem

For further instantiation of the problems of detection of weak radar signals, we believe initial information to come from a variety of radio systems in the form of a one-dimensional signal and / or a radar image (RI) - Fig. 1. The simplified scheme of primary radiosystems and investigation of radar image and one-dimensional signal in millimeter wave band (MWB) were represented by the author much earlier. Currently, fractal radar, a MIMO-radar and a fractal MIMO radar as well as unmanned aerial vehicles (UAVs) are added to the scheme in Fig. 1. The concept of fractal radar is presented in [1-5, 8, 11, 12], the concept of fractal MIMO-radar is considered in [1-3, 5, 11, 12].

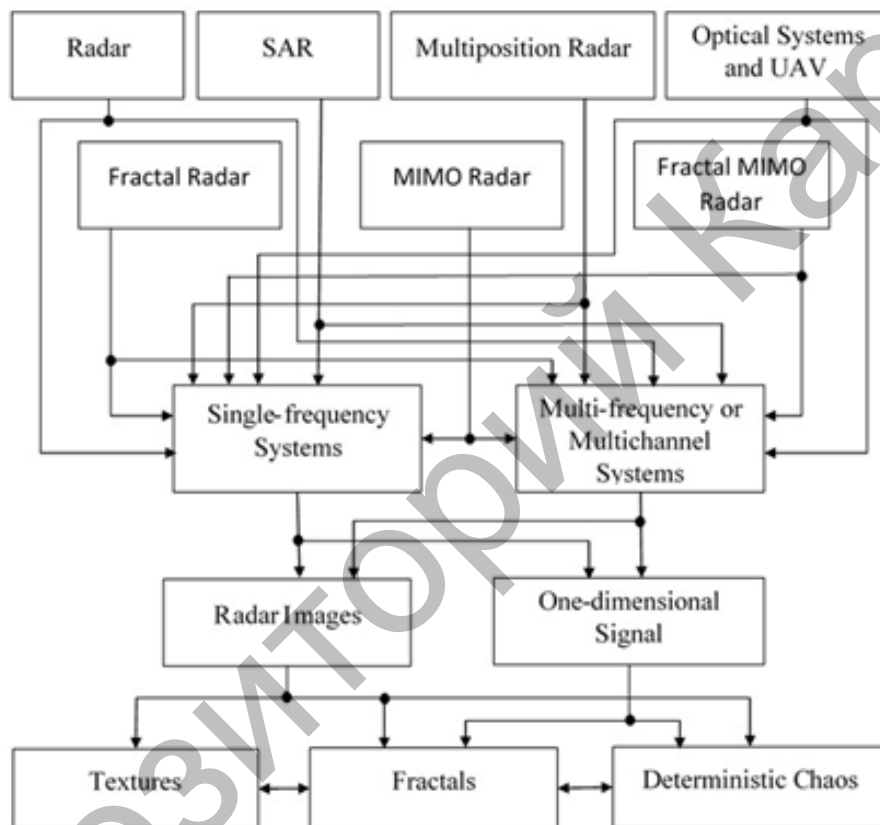


Fig.1. Radio systems of initial information

In general, the technology of MIMO systems implies that each wireless device involved in data exchange, has several spatially distributed receiving and transmitting antennas. The basic idea of fractal MIMO-radars is using fractal antennas and fractal detectors [1-12]. The capability of fractal antennas to work on several frequencies simultaneously or to radiate broadband sounding signal provides a sharp increase in the number of degrees of freedom that defines many of the important advantages of this type of radiolocation and significantly expands adaptation possibilities.

To represent these specifics, in [1-3, 5, 11, 12] a new term «fractal-frequency MIMO systems (FF MIMO)» was introduced, which reflects their physical properties much better. MIMO technologies related to the spatial multi-channel systems provide great opportunities for the application of the author's global fractal-scaling method for signal processing, various algorithms and technologies of fractal detectors [1-12] at all stages of the synthesis of information MIMO systems. The idea of a fractal radar station (Fig. 1 and 2) is based on the concept of fractal radio systems developed by the author - Fig. 3 [4, 8, 9].

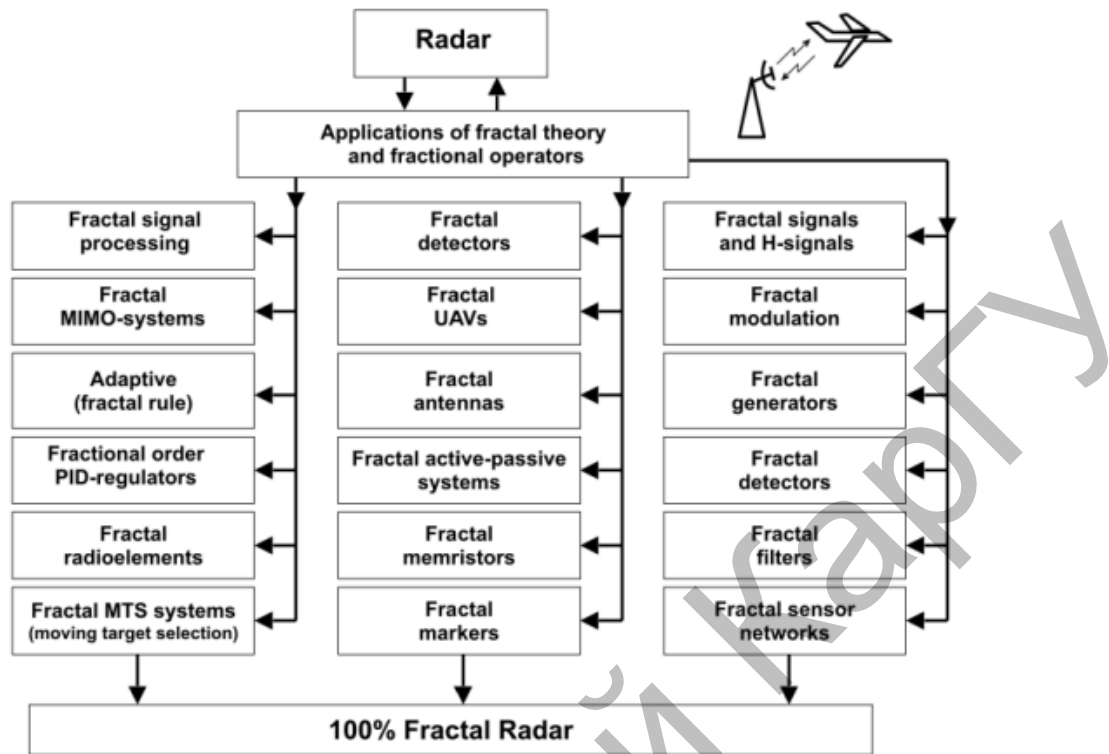


Fig.2. A fractal radar

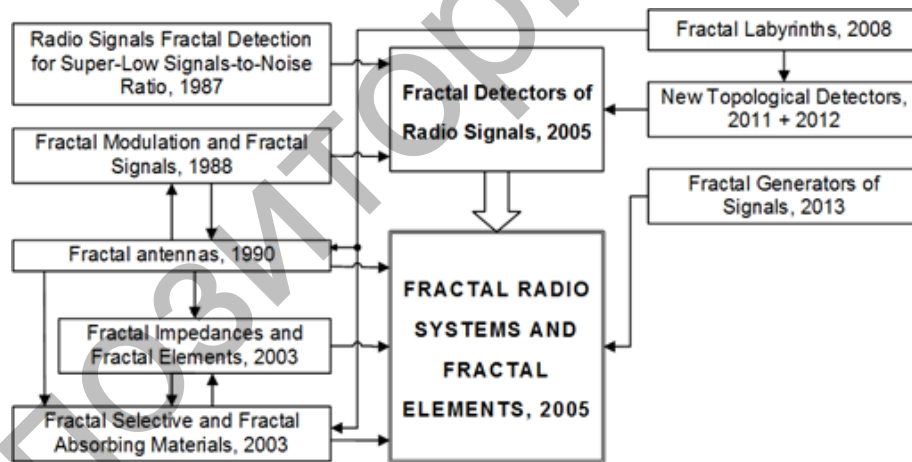


Fig.3. Fractal radio systems

2. New features and topological methods for the detection of low-contrast targets against the background of high-intensity noise

All currently existing and used by the author methods and topological features of low observable objects detection against the background of the high-intensity sea, ground and meteorological phenomena clutters are compactly represented in Fig. 4. The relationship between a variety of features and methods are also marked there. The work on the classification of such methods, algorithms and features started in May 2015 in China during the defense of the project «Leading Talents of Guangdong Province» and finished at the beginning of 2016 in China. That

was also when the author essentially completed the work on the book [2], where some of presented here results were introduced for the first time ever.

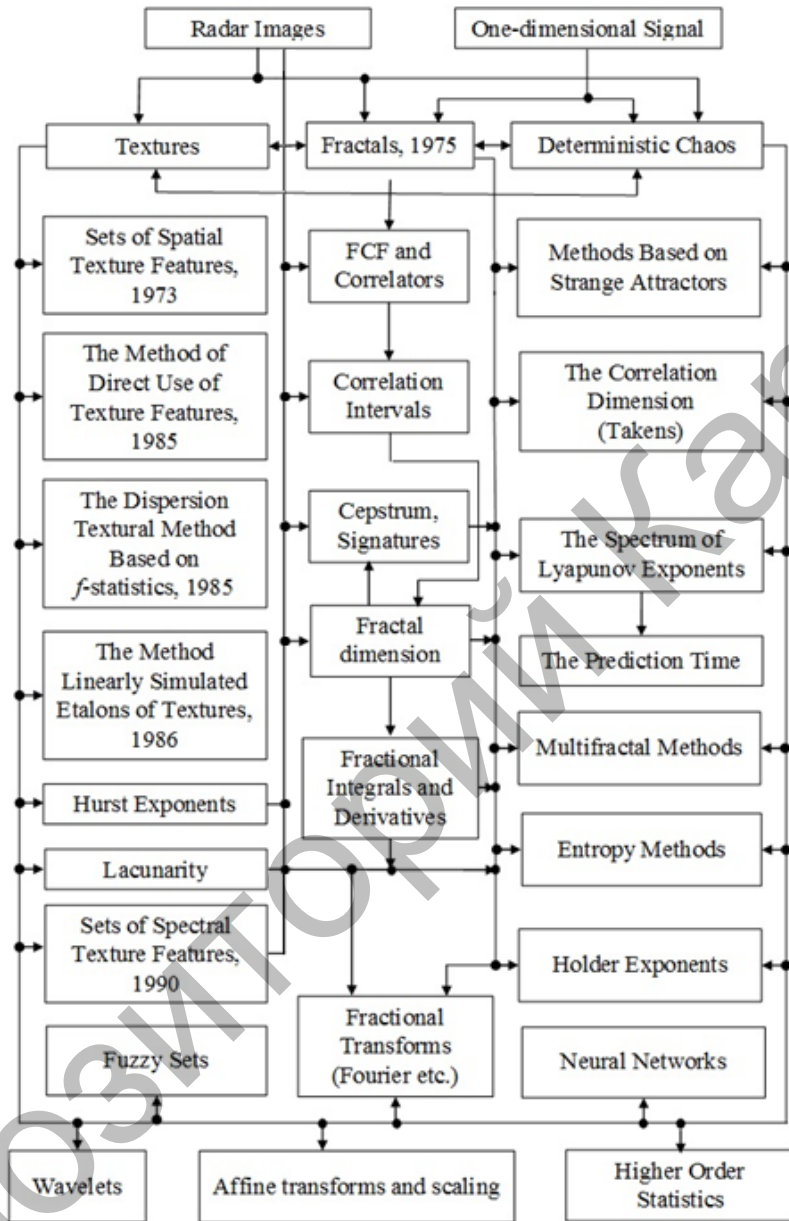


Fig. 4. Topological features and methods for detection of low-contrast targets against the background of high-intense noise.

Introduction of the concept of textural features ensemble to the US in 1973, [13], in the 1980-ies made it possible for the author to be the first to calculate complete ensembles of 28 textural features and to conduct their detailed synchronic analysis for real (optical aerial photography (OAP) and radar images within MMW range at a wavelength of 8.6 mm), as well as for synthesized textures based on autoregressive models, depending on the season [1, 2] 4).

Long-term field experiments were carried out by the author in cooperation with "Almaz" CCB and other leading industrial organizations of the USSR. All investigations were performed at wavelengths $\lambda=2.2$ and 8.6 mm (active radiation) and $\lambda=3.5$ mm (passive radiation). When extracting a MMW signal scattered by a variety of land covers, back in 1985 the author conducted first experiments on sorting areas of frequency and time scaling, the presence of which imply

certain fractal properties of the accepted sampling. At the same time, the problem of calculating textural features with account for the drift of their signatures under alternation of seasons was posed and solved. The assessments of the impact of window sizes on the accuracy of determining textural features for images of different land cover types were optimized.

For a long time the works on the study of radar images of the land cover at MMW using textural information have actually been carried out only in Russia and are of interest so far (especially now) [1, 2, 4]. After calculating ensembles of textural features based on optical and radar images, in 1985-1986 the author proposed methods and algorithms for the detection of low-contrast targets against the background of high-intense noise. Those included the method of direct use of textural features (1985), the dispersion method based on f -statistics (1985) and a detection method using linearly simulated patterns, i.e. textures (1986) [1-4, 8, 11]. The created methods of detection are quite valid at low signal to noise ratio of the order of or less than one (times). To the author's knowledge, no textural method for detecting low-contrast target has been proposed abroad. Moreover, an important advantage of the textural methods of processing is the possibility to neutralize speckles at coherent images of the Earth's surface, obtained by SAR.

The methods of deterministic chaos are widespread; they are shown in the right column of Fig. 4. It should be only noted that the algorithms of radar detection of low-contrast targets against the background of woodlands for the radar at a wavelength of 2.2 mm were tested by the experimenters in 2001. It was the first time when a strange attractor was reconstructed. It controlled the radar scattering of millimeter radio waves. Its dynamic and geometric characteristics were measured; D fractal dimensions, depending on the value of m embedding dimension were calculated as well. The most accurate estimate of D can be obtained at the breakpoint of $D(m)$ convex curve, at that paying no attention to reduction in scale ratio above and below.

Based on the found maximum Lyapunov exponent $\lambda_1 > 0.6$ bit/s it has been shown that, when measuring the current conditions with an accuracy of up to 1 bit, we lose all predictive power over time during 1.7 seconds. Therefore, the prediction interval of echoed signal intensity is by about 8 times greater than classical correlation time τ ($\tau \approx 210$ ms at a wind speed of 3 m/sec). The prediction interval provides an opportunity to estimate roughly the amplitude of further samples in the sample collection and, as noted by the author, it can be used in radar practice. Calculations of Hurst exponent H showed that in two out of three cases, the scattering process of millimeter waves by woodlands corresponds to the persistent process with $H > 0.5$, i.e. to the process with maximal rank.

3. Fractal topological detectors of low-contrast targets in high-intensity noise

Currently, great interest is evinced in various fractal and scaling methods (Fig. 4). Those fractal investigations started almost simultaneously in Russia, the USA and China in the 1980s. [1-12]. And the global problem to detect a fractal object against the intensive fractal background with additive Gaussian and nongaussian noise and interference was once posed [1-12]. Distinguishing features and methodology of the author's approach differed so greatly and were unusual for that time, that it was followed by a number of foreign articles with references to his early works on fractal processing of signals and radar images (see, e.g., [14-16]), in which it was taken further.

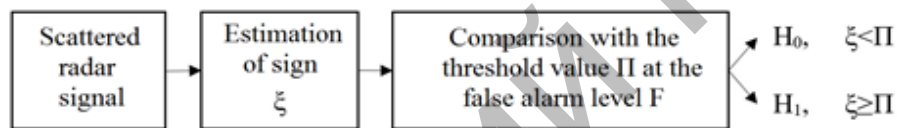
At the same time, the fractal dimension D or its signature $D(t, f, \bar{r})$ in different parts of the surface image is also a corresponding texture measure, i.e. the spatial correlation properties of scattering of radio waves from the respective surface areas. Moreover, the texture also determines lacunarity (Fig. 4), which uses second-order statistics for fractal images [1-12, 17-26]. Lacunarity is small for large dense texture and it is great when the texture is coarse-grained.

Lacunarity Λ (sensu Mandelbrodt) is defined by the formula

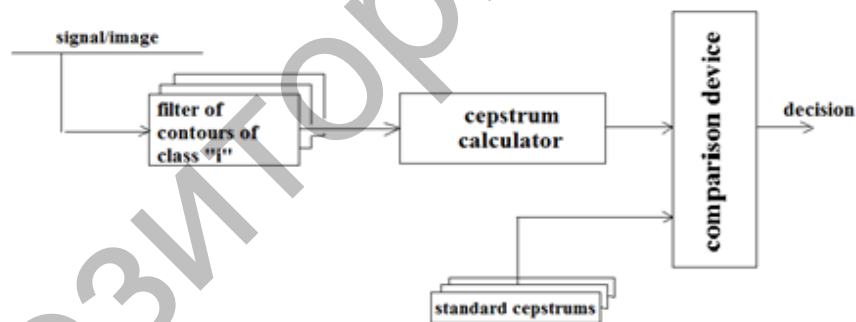
$$\Lambda = \left\langle \frac{M}{\langle M \rangle} \right\rangle^{-1}.$$

Here, M is a "mass" of a fractal formation, $\langle M \rangle$ is an anticipated "mass", the brackets $\langle \dots \rangle$ stand for data ensemble averaging.

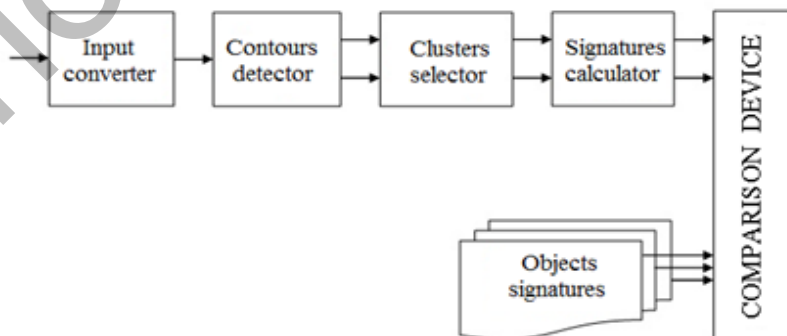
Lacunarity as a feature of objects detection was considered by the author in 1997. The introduction of fractional measures and scaling invariant makes it necessary to work with power-series probability distributions. The basic principles of a fractal detector were discovered and offered as early as in the 1980s; and for the first time ever (Fig. 2 and 3) the operation of the working model of fractal nonparametric detector of radar signals (FNDRS) was performed in 2003-2005. [1-12, 16-25]. In 2005 in the US it was exhibited as part of ISTC project with CCB "Diamond" and IRE RAS and earned a very high opinion of experts [1, 2]. The authors proposed unconventional algorithms of fractal-scaling detection, which offered high resistance. Some original versions of generalized structures of radar fractal detectors are presented in Fig. 5. The schematic view of the conjectured detector is shown in Fig. 5a. Based on a received radio signal or an image, a set of textural and fractal features ξ is determined. Then, in the threshold device at the threshold value of T and a certain level of false alarm probability F , a decision on obtained signal H_1 or its lack H_0 is issued. As ξ features, the value of the fractal dimension D , Hurst exponents $0 \leq H \leq 1$ for multiscale surfaces, p exponents, lacunarity values, etc. can be used. Hurst exponent is $H=3-D$ for RI and $H=2-D$ for a one-dimensional signal.



(a)



(b)



(c)

Fig.5. The initial (a) and detailed (b, c) structures of the first fractal detectors.

The integrated structural scheme of the fractal detector of radar signals is shown in Fig. 5 *b*. It consists of a circuit filter and a *fractal spectrum* computer. Further specification of the structural scheme of FNDRS is shown in Fig. 5*c*. An incoming signal (RI, a one-dimensional sample collection) arrives at the input converter.

It is intended to prearrange the analyzed sample collection. This preparation could include either an *induced noise contamination* (in the case of low resolving power of an analog-to-digital radar converter) or, for example, *contrast compression* in the case of sampling with a high dynamic range. Crucially, using the schemes in Fig. 5 it is definitely possible to synthesize absolutely all kinds of other fractal detectors in the future. (The author wishes to emphasize that "new" schemes of fractal detectors have been recently put forward by the authors who do not understand the fractal radiophysics and radiolocation at all and, not just knowing the basics of fractal analysis and dynamic chaos, they lay claim to some originality).

For a long time (over 35 years) the priority in this area in Russia and over the world strictly belongs to V.A. Kotelnikov IRE of RAS and to the author in particular. The author's concept (Fig. 3) of fractal radio systems and fractal devices makes the synthesis of other types of fractal detectors possible (Fig. 6 and 7). The detector based on the Hurst exponent works by using one or more search frequencies of radar (Fig. 6).

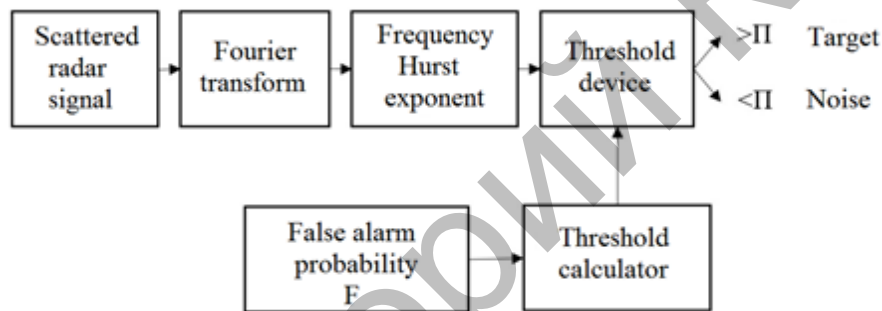


Fig.6. A fractal detector on the basis of the Hurst exponent.

Hurst exponent H reflects the irregularity of a fractal object. The smaller H exponent, the more irregular the fractal object is. So when an object occurs, the Hurst exponent grows. Fig. 7 is a scheme of a fractional detector with an autoregressive estimate of a power spectrum of the ground clutter. The autoregressive model is a linear prediction model that estimates the power spectrum of the clutter and forms its autocorrelation matrix.

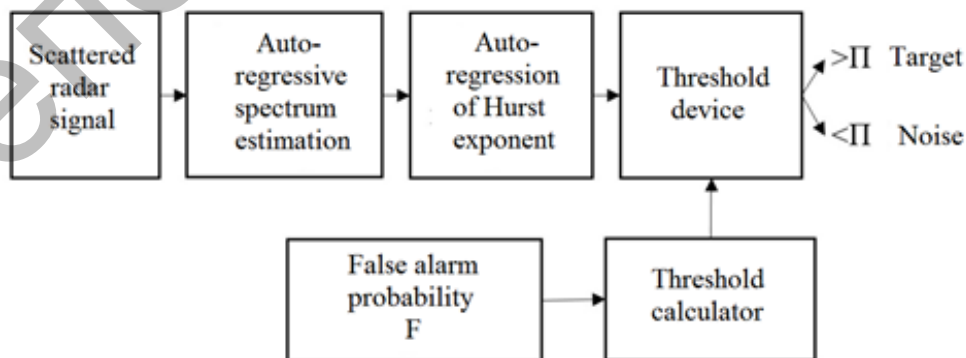


Fig.7. A fractal detector with an autoregression estimate of interference spectrum and the Hurst exponent.

An autoregression equation governs the relationship between the current and previous samples of a sampled stochastic process. Earlier in the 1980-ies the problem of autoregression on the basis of canonical system of Yule-Walker equations with transformation of luminance histogram was solved by the investigators. Thus, the detector in Fig. 7 uses real fractal properties of the power spectrum based on the autoregressive spectral estimation, used for the detection of low-contrast targets. Note that schemes like in Fig. 6 and 7 are often studied in China at present [2]. Similar detectors were used by the author in the texture processing of APS and radar images as early as in the 80-ies of the XX century. The author emphasizes that the correlation dimension (Fig. 4) that requires a large sample collection size, which is unrealistic in radiolocation cannot be considered as detection statistics.

4. Innovative fractal-scaling technologies: creation, development and application of fractal methods for radiolocation tasks and developing the foundations of fractal element base

During 35 years of research, the developed global fractal-scaling method completely lived up to expectations having found numerous applications (Fig. 8). This is a challenge of time. The facts themselves are in the author's favour!

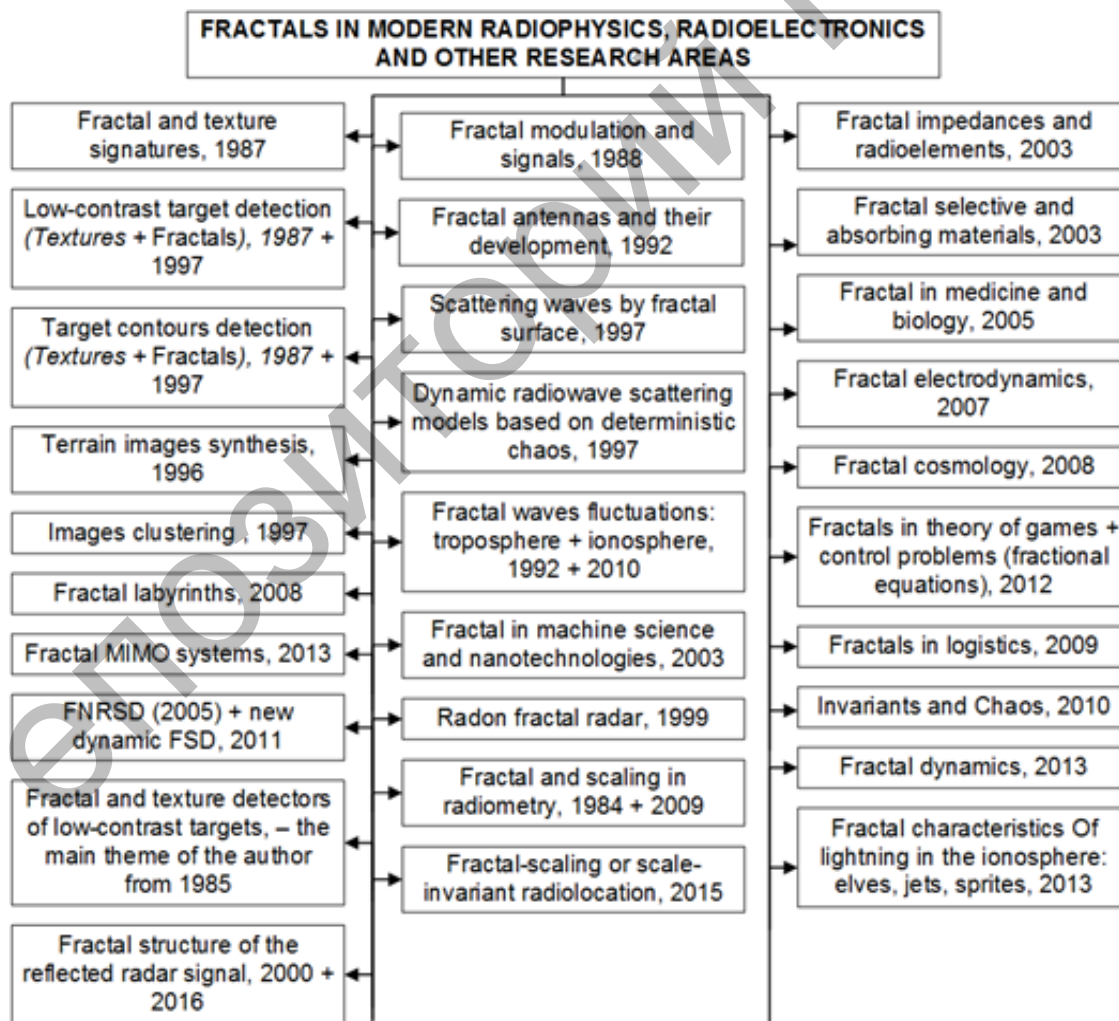


Fig.8. The layout of the author's development of new information technologies based on fractals, fractional operators and scaling effects for nonlinear physics and electronics.

The fundamental principle of "maximum topology with a minimum of energy" for the received signal that makes for more efficient use of the advantages of fractal-scaling processing of information has been developed and introduced; and this principle is consistently being put into practice [1-12, 17-26].

5. Postulates of Fractal Radar

Fractal radar defined in [1-12, 17-26] is based on four main postulates:

- 1) Intelligent signal processing based on the theory of fractional measure, scaling effects and fractional operator's theory;
- 2) Hausdorff dimension or fractal dimension D of a signal or a radar image is directly connected with the topological dimension;
- 3) robust non-Gaussian probability distributions of the fractal dimension of the processed signal;
- 4) "Maximum topology with a minimum of energy" for the received signal. It allows take advantages of fractal scaling information processing more effectively.

The key point of fractal approach is to focus on describing and processing of radar signal (fields) exclusively in the space of fractional measure with the use of the scaling hypothesis and distributions with heavy-tailed or stable distributions (non-Gaussian). Fractal-scaling processing methods of signals, wave fields and images are in a broad sense based on the pieces of information, which isn't usually taken into account and irretrievably lost if classical methods of processing are applied.

This work is concerned with the main radio physical area – radiolocation and it aims to ascertain what's done and things to do in this field on the basis of the fractal theory. Investigations carried out showed the correctness of the path chosen by the author (since 1980) to improve the radiolocation technique.

Is necessary to think about the processing of the input signals with a low threshold at high levels of false alarm and then a transition to a low level of false alarms. Moreover, the false alarm probability is never measured in real time. In principle, we need a new metric, and the new parameters of radar detection.

6. A New Direction in the Theory of Statistical Solutions

Fast development of the fractal theory in radar and radio physics led to establishing of the new theoretical direction in modern radar. It can be described as «*Statistical theory of fractal radar*». This direction includes (at least at the initial stage) the following fundamental questions:

- 1). The theory of the integer and fractional measure.
- 2). Caratheodory construction in the measure theory.
- 3). Hausdorff measure and Hausdorff-Besicovitch dimension.
- 4). The theory of topological spaces.
- 5). The dimension theory.
- 6). The line from the point of view of mathematician.
- 7). Non-differentiable functions and sets.
- 8). Fundamentals of the theory of probability.
- 9). Stable probability distributions.
- 10). The theory of fractional calculus.
- 11). The classical Brownian motion.
- 12). Generalized Brownian motion.
- 13). Fractal sets.
- 14). Anomalous diffusion.
- 15). The main criteria for statistical decision theory in radar.

- 16). Wave propagation in fractal random media.
- 17). Wave scattering generalized Brownian surface.
- 18). Wave scattering surface on the basis of non-differentiable functions.
- 19). Diffractals.
- 20). Cluster analysis.
- 21). Theory and circuitry of fractal detectors.
- 22). Fractal-scaling or scale-invariant radar.
- 23). The multi-radar.
- 24). MIMO radar.
- 25). Cognitive radar.

This list of studied questions, of course, is supposed to be expanded and refined in the future. The author has been dealing with it for nearly 40 years of his scientific career.

Conclusion

The author created, developed and applied fractal-scaling methods for radiolocation problems and forming the foundations of fractal element base [1-12, 17-26]. For the first time ever approaches to development of a fractal radar and a fractal MIMO- radar were considered. The author emphasizes that the synthesis of topological (fractal, textural, chaotic, etc.) detectors makes for a fresh look at the problem of detecting super weak actual signals. As a result of that, the author's discovery in the away-back 1980-ies takes the meaning of *generalized detection*. Thus, pure energy and pure topological detectors are not contrary to each other and they do not duplicate, but complement one another.

Due to topological detectors it is possible to see the process of energy detection in a new light and to find some essential faults in it. Consequently, topological detection becomes not less, if not more, valuable for theory and practice than energy detection. The theory of topological detection is formulated in [1-12, 16-25]. It is especially necessary for the purpose of reexamining the former theory and in that way producing new results that are not available to traditional concepts of radiolocation.

Thus, topological detection opens the door to a radically new field of statistical decision theory and provides an opportunity to correct ideas in this field, and even to create new ones, which is of great theoretical and practical importance. The sufficiently detailed reasoning reported here should contribute to a better understanding of proposed by the author fundamentally new interpretation of the problem of radar (and other kinds of) detection. The proposed theory has much in common with cognitive radar.

Thus, during more than 35 years, almost from scratch, fundamental bases of the theory that will be applied in the following decades were formed. Not results, not specific solutions are the most valuable, but namely the solution method, the approach to it. The created method is presented in [1-12, 17-26].

The author raised the foregoing problems as early as in 1980 and for more than 35 years he has been successfully working on their solution and development. Careful bibliographic studies show complete and absolute world priority of the author in all "fractal" fields of radiolocation and radio-physics (the list of the author's works in cooperation with students has about 800 publications, including 23 monographs).

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