

References

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About a boundary value problem for the loaded equation of mixed type

Boundary value problem for the loaded equation of mixed type in a rectangular domain is investigated. Feature of the problem is that you can not directly invert the operator of the hyperbolic and elliptic parts and reduce the initial problem to the study of the solvability of singular integral equations. Conditions of existence of a unique  $L_2$  — strong solution are found.

Key words: loaded equations, equations of mixed type,  $L_2$ -strong solution, boundary value problem for a loaded equation.

Let  $Q_1 = \{x, t | 0 < x < 2\pi, 0 < t < T\}$ ,  $Q_2 = \{x, t | 0 < x < 2\pi, -T < t < 0\}$ ,  $Q = Q_1 \cup Q_2$ . In the domain  $Q$  we consider the following boundary value problem:

$$(-1)^j \frac{\partial^2 u^j}{\partial t^2} - \frac{\partial^2 u^j}{\partial x^2} + \alpha_k u^j(x, t_k) = f^j(x, t) \quad (x, t) \in Q_j, \quad j = 1, 2, \quad k = 1, 2; \tag{1}$$

$$\frac{\partial^p u^j(0, t)}{\partial x^p} = \frac{\partial^p u^j(2\pi, t)}{\partial x^p};$$

$$\frac{\partial^p u^{(1)}(x, T)}{\partial t^p} = \mu_p \frac{\partial^p u^{(2)}(x, -T)}{\partial t^p}, \quad p = 0, 1.$$

i.e. at  $(x, t) \in Q_1$  we have:

$$\frac{\partial^2 u^{(1)}(x, t)}{\partial t^2} - \frac{\partial^2 u^{(1)}(x, t)}{\partial x^2} + \alpha_1 u^{(1)}(x, t_1) = f^{(1)}(x, t); \tag{2}$$

$$\begin{cases} u^{(1)}(0, t) = u^{(1)}(2\pi, t) \\ \frac{\partial u^{(1)}(0, t)}{\partial x} = \frac{\partial u^{(1)}(2\pi, t)}{\partial x} \end{cases} \tag{3}$$

and at  $(x, t) \in Q_2$ :

$$\frac{\partial^2 u^{(2)}(x, t)}{\partial t^2} - \frac{\partial^2 u^{(2)}(x, t)}{\partial x^2} + \alpha_2 u^{(2)}(x, t_2) = f^{(2)}(x, t); \tag{4}$$

$$\begin{cases} u^{(2)}(0, t) = u^{(2)}(2\pi, t) \\ \frac{\partial u^{(2)}(0, t)}{\partial x} = \frac{\partial u^{(2)}(2\pi, t)}{\partial x}; \end{cases} \tag{5}$$

also

$$\begin{cases} u^{(1)}(x, T) = \mu_0 u^{(2)}(x, -T); \\ \frac{\partial u^{(1)}(x, T)}{\partial t} = \mu_1 \frac{\partial u^{(2)}(x, -T)}{\partial t}. \end{cases} \tag{6}$$

Next, we put that

$$\begin{cases} T < +\infty, & f^{(1)} \in L_1(Q_1), & f^{(2)} \in L_2(Q_2), & \mu_0, \mu_1 \in C; \\ \alpha_1, \alpha_2 \in C, & t_1 \in (0, T), & t_2 \in (-T, 0) \end{cases} \quad (7)$$

are given functions and numbers.

The given equation (1) is an equation of mixed (elliptic-hyperbolic) type, and because of the loaded summand  $\alpha_k u^{(j)}(x, t_k)$  it is called the loaded, it was the object of many authors, for example [1–3].

Feature of the problem is that the domain of the hyperbolic part is not characteristic, as well as there are loaded summands, it allows to reveal some features of the problem under consideration.

The main goal of this research is to study issues of  $L_2$  — strong solvability of boundary value problems (2)–(6) under the conditions (7).

To solve (1) we introduce the following notation:

$$s \in S = \{\pm 1, \pm 2, \dots\}$$

$$\Delta_s = \begin{pmatrix} \frac{\sin sT + \mu_1 shsT}{s} & \cos sT - \mu_0 chsT & \frac{\cos sT - 1}{s^2} & \frac{1 - chsT}{s^2} \\ \cos sT - \mu_1 chsT & s(-\sin sT + \mu_0 shsT) & -\frac{\sin sT}{s} & \frac{shsT}{s} \\ \alpha_2 \frac{\sin s(t_2 + T)}{s} & \alpha_2 \cos s(t_2 + T) & -1 - \alpha_2 \frac{1 - \cos s(t_2 + T)}{s^2} & 0 \\ \mu_1 \alpha_1 \frac{shs(t_1 - T)}{s} & \mu_0 \alpha_1 chs(t_1 - T) & 0 & -1 - \alpha_1 \frac{1 - chs(t_1 - T)}{s^2} \end{pmatrix}.$$

*Theorem 1.* For any  $f^j, \mu_p, \alpha, t_k, T$  satisfying the requirements (7), the boundary value problems (2)–(6) have a unique  $L_2$  — strong solution if and only if following conditions are satisfied:

$$|\Delta_s| \neq 0, s \in S. \quad (8)$$

Condition (8) in terms of the data (7) gives a complete description of the correct boundary value problems of the form (2)–(6). We will obtain a number of corollaries of the theorem.

*Corollary 1.* Let at the conditions of theorem 1 in equation (1) loaded summands absent, i.e.  $\alpha_k = 0, k = 1, 2$ . Then in order to the boundary value problem (2)–(6) has a unique  $L_2$  — strong solution, if only if conditions are fulfilled:

$$\begin{vmatrix} \frac{\sin sT + \mu_1 shsT}{s} & \cos sT - \mu_0 chsT \\ \cos sT - \mu_1 chsT & s(-\sin sT + \mu_0 shsT) \end{vmatrix} \neq 0, s \in S. \quad (9)$$

*Corollary 2.* Under the conditions of Theorem 1, we assume that  $T = 2\pi, \mu_0 = \mu_1 = 1$ . Then in order to the boundary value problem (2)–(6) has a unique  $L_2$  — strong solution, if only if conditions are fulfilled:

$$\left(1 + \alpha_2 \frac{1 - \cos s(t_2 + T)}{s^2}\right) \cdot [s^2 + \alpha_1] \neq 0, \forall s \in S. \quad (10)$$

*Proof of theorem 1.* We carry out proof of the theorem by method of separation of variables, i.e. we look for a solution of problem (2)–(6) in the following form:

$$\begin{aligned} u^{(1)}(x, t) &= \sum_{s \in S} u_s^{(1)}(t) \cdot \exp\{is \cdot x\}; \\ u^{(2)}(x, t) &= \sum_{s \in S} u_s^{(2)}(t) \cdot \exp\{is \cdot x\}. \end{aligned} \quad (11)$$

We take into account the corresponding expansions for the right-hand sides of equations (2), (6):

$$\begin{aligned} f^{(1)}(x, t) &= \sum_{s \in S} f_s^{(1)}(t) \exp\{is \cdot x\}; \\ f^{(2)}(x, t) &= \sum_{s \in S} f_s^{(2)}(t) \exp\{is \cdot x\}. \end{aligned} \quad (12)$$

Boundary value problem (2)–(6) can be reduced to the study of boundary value problems for a countable system of loaded ordinary differential equations

$$-\frac{d^2u_s^{(1)}(t)}{dt^2} + s^2u_s^{(1)}(t) + \alpha_1u_s^{(1)}(t_1) = f_s^{(1)}(t) \text{ при } t \in (0;T); \tag{13}$$

$$\frac{d^2u_s^{(2)}(t)}{dt^2} + s^2u_s^{(2)}(t) + \alpha_2u_s^{(2)}(t_2) = f_s^{(2)}(t) \text{ при } t \in (-T;0); \tag{14}$$

$$\begin{cases} u_s^{(1)}(T) = \mu_0u_s^{(2)}(-T); \\ \frac{du_s^{(1)}(T)}{dt} = \mu_1\frac{du_s^{(2)}(-T)}{dt}. \end{cases} \tag{15}$$

We introduce a systems of numbers  $\{v_s, s \in S\}$ ,  $\{\varphi_s, s \in S\}$ , as long as temporarily unknown, by which instead of the problems (13)–(15) we consider the following boundary value problems

$$\begin{cases} -\frac{d^2u_s^{(1)}(t)}{dt^2} + s^2u_s^{(1)}(t) + \alpha_1u_s^{(1)}(t_1) = f_s^{(1)}(t) & t \in (0;T); \\ u_s^{(1)}(T) = \mu_0v_s & \frac{du_s^{(1)}(T)}{dt} = \mu_1\varphi_s & s \in S; \end{cases} \tag{16}$$

$$\begin{cases} \frac{d^2u_s^{(2)}(t)}{dt^2} + s^2u_s^{(2)}(t) + \alpha_2u_s^{(2)}(t_2) = f_s^{(2)}(t) & t \in (-T;0); \\ u_s^{(2)}(-T) = v_s & \frac{du_s^{(2)}(-T)}{dt} = \varphi_s & s \in S. \end{cases} \tag{17}$$

The general solution of equation (16) takes the form:

$$\begin{aligned} u_{o.p}^{(1)} &= C_1chst + C_2shst + \int_t^T \frac{shs(t-\tau)}{s} \cdot F_s^{(1)}(\tau)d\tau = \\ &= C_1chst + C_2shst + \int_t^T \frac{shs(t-\tau)}{s} \cdot f_s^{(1)}(\tau)d\tau - \int_t^T \frac{shs(t-\tau)}{s} \alpha_1u_s^{(1)}(t_1)d\tau. \end{aligned} \tag{18}$$

The general solution of equation (17):

$$\begin{aligned} u_{o.p}^{(2)} &= \tilde{C}_1 \cos st + \tilde{C}_2 \sin st + \int_{-T}^t \frac{\sin s(t-\tau)}{s} \cdot F_s^{(2)}(\tau)d\tau = \tilde{C}_1 \cos st + \\ &+ \tilde{C}_2 \sin st + \int_{-T}^t \frac{\sin s(t-\tau)}{s} \cdot f_s^{(2)}(\tau)d\tau - \int_{-T}^t \frac{\sin s(t-\tau)}{s} \alpha_2u_s^{(2)}(t_2)d\tau. \end{aligned} \tag{19}$$

Now we find  $C_1, C_2$  and  $\tilde{C}_1, \tilde{C}_2$  from the conditions  $u_s^{(1)}(T) = \mu_0v_s, \frac{du_s^{(1)}(T)}{dt} = \mu_1\varphi_s, s \in S$  and  $u_s^{(2)}(-T) = v_s, \frac{du_s^{(2)}(-T)}{dt} = \varphi_s, s \in S$  ((16), (17)):

$$\begin{aligned} C_1 &= \mu_0v_schsT - \frac{\mu_1\varphi_s}{s} \cdot shsT; \\ C_2 &= \frac{\mu_1\varphi_s}{s} \cdot chsT - \mu_0v_s \cdot shsT; \\ \tilde{C}_1 &= v_s \cos s(-T) - \frac{\varphi_s}{s} \sin s(-T); \\ \tilde{C}_2 &= \frac{\varphi_s}{s} \cos s(-T) + v_s \sin s(-T). \end{aligned}$$

Found  $C_1, C_2$  and  $\tilde{C}_1, \tilde{C}_2$  we substitute in (18) and (19), respectively, we obtain the following representation:

$$u_s^{(1)}(t) = \mu_0 v_s \operatorname{ch} s(t-T) + \frac{\mu_1 \varphi_s}{s} \operatorname{sh} s(t-T) + \int_t^T \frac{\operatorname{sh} s(t-\tau)}{s} \cdot f_s^{(1)}(\tau) d\tau - \alpha_1 u_s^{(1)}(t_1) \frac{1 - \operatorname{ch} s(t-T)}{s^2}$$
(20)

and

$$u_s^{(2)}(t) = v_s \cos s(t+T) + \frac{\varphi_s}{s} \sin s(t+T) + \int_{-T}^t \frac{\sin s(t-\tau)}{s} \cdot f_s^{(2)}(\tau) d\tau - \alpha_2 u_s^{(2)}(t_2) \cdot \frac{1 - \cos s(t+T)}{s^2}$$
(21)

In these representations the unknowns are

$$v_s, \varphi_s, \alpha_1 u_s^{(1)}(t_1), \alpha_2 u_s^{(2)}(t_2).$$

For finding them we use the representations (20) and (21). At first, we find the representations for the derivatives of solutions of problems (16) and (17):

$$\frac{du_s^{(1)}(t)}{dt} = \int_t^T f_s^{(1)}(\tau) \cdot \operatorname{ch} s(t-\tau) d\tau + \mu_1 \varphi_s \cdot \operatorname{ch} s(t-T) + \mu_0 v_s \cdot s \cdot \operatorname{sh} s(t-T) + \alpha_1 u_s^{(1)}(t_1) \frac{\operatorname{sh} s(t-T)}{s};$$
(22)

$$\frac{du_s^{(2)}(t)}{dt} = \int_{-T}^t f_s^{(2)}(\tau) \cdot \cos s(t-\tau) d\tau + \varphi_s \cdot \cos s(t+T) - v_s \cdot s \cdot \sin s(t+T) - \alpha_1 u_s^{(1)}(t_1) \frac{\sin s(t+T)}{s}$$
(23)

Further, using on the line  $t=0$  in the domain  $Q$ , the conjugation conditions for the solutions (20)–(21) and their derivatives (22)–(23):

$$\begin{cases} u_s^{(1)}(0+) = u_s^{(2)}(0-); \\ \frac{du_s^{(1)}(0+)}{dt} = \frac{du_s^{(2)}(0-)}{dt}, \end{cases}$$

we finally obtain:

$$\begin{aligned} \left( \frac{\sin sT}{s} + \mu_1 \frac{\operatorname{sh} sT}{s} \right) \cdot \varphi_s + (\cos sT - \mu_0 \operatorname{ch} sT) \cdot v_s - \frac{1 - \cos sT}{s^2} \cdot \alpha_2 u_s^{(2)} + \frac{1 - \operatorname{ch} sT}{s^2} \cdot \alpha_1 u_s^{(1)} &= F_s^1; \\ (\cos sT - \mu_1 \operatorname{ch} sT) \cdot \varphi_s + s(-\sin sT + \mu_0 \operatorname{sh} sT) \cdot v_s - \frac{\sin sT}{s} \cdot \alpha_2 u_s^{(2)} + \frac{\operatorname{sh} sT}{s} \cdot \alpha_1 u_s^{(1)} &= F_s^2, \end{aligned}$$
(24)

where

$$\begin{aligned} F_s^1 &= \int_{-T}^0 \frac{\sin s\tau}{s} \cdot f_s^{(2)}(\tau) d\tau - \int_0^T \frac{\operatorname{sh} s\tau}{s} \cdot f_s^{(1)}(\tau) d\tau; \\ F_s^2 &= - \int_{-T}^0 \cos s\tau \cdot f_s^{(2)}(\tau) d\tau + \int_0^T \operatorname{ch} s\tau \cdot f_s^{(1)}(\tau) d\tau. \end{aligned}$$
(25)

By submissions (20) and (21) we assume  $t=t_1$  and  $t=t_2$ , and multiply the obtained expressions for the corresponding  $\alpha_1, \alpha_2$ . As a result, we will have:

$$\begin{aligned} \alpha_1 \mu_0 \operatorname{ch} s(t_1-T) \cdot v_s + \frac{\alpha_1 \mu_1}{s} \operatorname{sh} s(t_1-T) \cdot \varphi_s - \alpha_1 u_s^{(1)}(t_1) \left( 1 + \alpha_1 u_s^{(1)}(t_1) \frac{1 - \operatorname{ch} s(t_1-T)}{s^2} \right) &= F_s^3; \\ \alpha_2 \cos s(t_2+T) \cdot v_s + \frac{\alpha_2}{s} \sin s(t_2+T) \cdot \varphi_s - \alpha_2 u_s^{(2)}(t_2) \left( 1 + \alpha_2 u_s^{(2)}(t_2) \cdot \frac{1 - \cos s(t_2+T)}{s^2} \right) &= F_s^4, \end{aligned}$$
(26)

where

$$\begin{aligned}
 F_s^3 &= -\alpha_1 \int_{t_1}^T \frac{shs(t_1 - \tau)}{s} \cdot f_s^{(1)}(\tau) d\tau; \\
 F_s^4 &= -\alpha_2 \int_{-T}^{t_2} \frac{\sin s(t_2 - \tau)}{s} \cdot f_s^{(2)}(\tau) d\tau.
 \end{aligned}
 \tag{27}$$

Now from (24) and (26) we define the unknown quantities  $\varphi_s, v_s, \alpha_1 u_s^{(1)}(t_1), \alpha_2 u_s^{(2)}(t_2)$ , by the formulas  $\forall s \in S$ :

$$\varphi_s = \frac{|\Delta_{\varphi_s}|}{|\Delta_s|}, \quad v_s = \frac{|\Delta_{v_s}|}{|\Delta_s|}, \quad \alpha_1 u_s^{(1)}(t_1) = \frac{|\Delta_{\alpha_1 u_s^{(1)}(t_1)}|}{|\Delta_s|}, \quad \alpha_2 u_s^{(2)}(t_2) = \frac{|\Delta_{\alpha_2 u_s^{(2)}(t_2)}|}{|\Delta_s|}.
 \tag{28}$$

where the matrices  $\Delta_{\varphi_s}, \Delta_{v_s}, \Delta_{\alpha_1 u_s^{(1)}(t_1)}, \Delta_{\alpha_2 u_s^{(2)}(t_2)}$  are obtained from the matrix  $\Delta_s$  by replacing the corresponding columns to elements  $F_s^1, F_s^2, F_s^3, F_s^4$ .

Next, substituting (28) into (20) and (21), we obtain the final representation of the solutions of boundary problems (16), (17):

$$\begin{aligned}
 u_s^{(1)}(t) &= \frac{|\Delta_{v_s}|}{|\Delta_s|} \cdot \mu_0 \cdot chs(t-T) + \frac{|\Delta_{\varphi_s}|}{|\Delta_s|} \cdot \mu_1 \cdot \frac{shs(t-T)}{s} + \int_t^T \frac{shs(t-\tau)}{s} \cdot f_s^{(1)}(\tau) d\tau - \\
 &\quad - \frac{|\Delta_{\alpha_1 u_s^{(1)}(t_1)}|}{|\Delta_s|} \cdot \frac{1 - chs(t-T)}{s^2}, \quad s \in S;
 \end{aligned}
 \tag{29}$$

$$\begin{aligned}
 u_s^{(2)}(t) &= \frac{|\Delta_{v_s}|}{|\Delta_s|} \cdot \cos s(t+T) + \frac{|\Delta_{\varphi_s}|}{|\Delta_s|} \cdot \frac{\sin s(t+T)}{s} + \int_{-T}^t \frac{\sin s(t-\tau)}{s} \cdot f_s^{(2)}(\tau) d\tau - \\
 &\quad - \frac{|\Delta_{\alpha_2 u_s^{(2)}(t_2)}|}{|\Delta_s|} \cdot \frac{1 - \cos s(t+T)}{s^2}, \quad s \in S.
 \end{aligned}
 \tag{30}$$

Now let us discuss the issue of establishing  $L_2$  — estimates for solutions of (29)–(30), uniform in  $s \in S$ , i.e. estimates of the form:

$$\|u_s^{(1)}(t)\|_{L_2(0,T)} \leq C_1 \|f_s^{(1)}\|_{L_2(0,T)}, \quad s \in S
 \tag{31}$$

$$\|u_s^{(2)}(t)\|_{L_2(-T,0)} \leq C_2 \|f_s^{(2)}\|_{L_2(-T,0)}, \quad s \in S
 \tag{32}$$

where the constants  $C_1, C_2$  do not depend on  $s$ .

For this we first consider the case of absence of loaded summands. From (29), (30) we obtain the following representation for the desired solutions ( $s \in S$ ):

$$u_s^{(1)}(t) = \int_t^T \frac{shs(t-\tau)}{s} \cdot f_s^{(1)}(\tau) d\tau + \frac{|\tilde{\Delta}_{\varphi_s}|}{|\tilde{\Delta}_s|} \cdot \mu_1 \cdot \frac{shs(t-T)}{s} + \frac{|\tilde{\Delta}_{v_s}|}{|\tilde{\Delta}_s|} \cdot \mu_0 \cdot chs(t-T);
 \tag{33}$$

$$u_s^{(2)}(t) = \int_{-T}^t \frac{\sin s(t-\tau)}{s} \cdot f_s^{(2)}(\tau) d\tau + \frac{|\tilde{\Delta}_{\varphi_s}|}{|\tilde{\Delta}_s|} \cdot \frac{\sin s(t+T)}{s} + \frac{|\tilde{\Delta}_{v_s}|}{|\tilde{\Delta}_s|} \cdot \cos s(t+T),
 \tag{34}$$

where:

$$\begin{aligned}
 |\tilde{\Delta}_s| &= \begin{vmatrix} \frac{\sin sT}{s} + \mu_1 \frac{shsT}{s} & \cos sT - \mu_0 chsT \\ \cos sT - \mu_1 chsT & s(-\sin sT + \mu_0 shsT) \end{vmatrix} = \\
 &= -1 - \mu_0 \mu_1 + (\mu_0 - \mu_1) \sin sT \cdot shsT + (\mu_0 + \mu_1) \cos sT \cdot chsT;
 \end{aligned}
 \tag{35}$$

$$\begin{aligned}
 |\tilde{\Delta}_{\varphi_s}| &= \left| \begin{array}{cc} \int_{-T}^0 \frac{\sin s\tau}{s} \cdot f_s^{(2)}(\tau) d\tau - \int_0^T \frac{sh s\tau}{s} \cdot f_s^{(1)}(\tau) d\tau & \cos sT - \mu_0 chsT \\ - \int_{-T}^0 \cos s\tau \cdot f_s^{(2)}(\tau) d\tau + \int_0^T chs\tau \cdot f_s^{(1)}(\tau) d\tau & s(-\sin sT + \mu_0 shsT) \end{array} \right| = \\
 &= \int_{-T}^0 f_s^{(2)}(\tau) [\cos s(\tau+T) - \mu_0(\cos s\tau \cdot chsT - \sin s\tau \cdot shsT)] d\tau + \\
 &\quad + \int_0^T f_s^{(1)}(\tau) [\mu_0 chs(\tau-T) - chs\tau \cdot \cos sT + shs\tau \sin sT] d\tau; \tag{36}
 \end{aligned}$$

$$\begin{aligned}
 |\tilde{\Delta}_{\psi_s}| &= \left| \begin{array}{cc} \frac{\sin sT}{s} + \mu_1 \frac{shsT}{s} & \int_{-T}^0 \frac{\sin s\tau}{s} \cdot f_s^{(2)}(\tau) d\tau - \int_0^T \frac{sh s\tau}{s} \cdot f_s^{(1)}(\tau) d\tau \\ \cos sT - \mu_1 chsT & - \int_{-T}^0 \cos s\tau \cdot f_s^{(2)}(\tau) d\tau + \int_0^T chs\tau \cdot f_s^{(1)}(\tau) d\tau \end{array} \right| = \\
 &= \frac{1}{s} \int_{-T}^0 f_s^{(2)}(\tau) [-\sin s(\tau+T) + \mu_1(\sin s\tau \cdot chsT - shsT \cdot \cos s\tau)] d\tau + \\
 &\quad + \frac{1}{s} \int_0^T f_s^{(1)}(\tau) [\mu_1 shs(T-\tau) + shs\tau \cdot \cos sT + \sin sT \cdot chs\tau] d\tau. \tag{37}
 \end{aligned}$$

Substituting (35)–(37) into (33), we obtain:

$$\begin{aligned}
 u_s^{(1)}(t) &= \frac{1}{s \cdot |\tilde{\Delta}_s|} \cdot \int_{-T}^0 f_s^{(2)}(\tau) \{ \mu_1 shs(t-T) \cdot \cos s(\tau+T) - \mu_0 \sin s(\tau+T) \cdot chs(t-T) + \\
 &\quad + \mu_0 \mu_1 [\cos s\tau \cdot chs(2T-t) - shs\tau \cdot \cos s\tau] \} d\tau + \\
 &+ \frac{1}{s \cdot |\tilde{\Delta}_s|} \cdot \int_0^T f_s^{(1)}(\tau) \{ \mu_0 \mu_1 shs(t-\tau) + \mu_0 \sin sT \cdot chs(t-T) chs\tau - \mu_1 \cos sT \cdot shs(t-T) chs\tau \} d\tau + \\
 &+ \frac{1}{s \cdot |\tilde{\Delta}_s|} \int_t^T f_s^{(1)}(\tau) \{ -(1 + \mu_0 \mu_1) shs(t-\tau) + (\mu_0 - \mu_1) \sin sT \cdot shsT \cdot sh(t-\tau) + \\
 &\quad + (\mu_0 + \mu_1) \cos sT \cdot chsT \cdot shs(t-\tau) \} d\tau. \tag{38}
 \end{aligned}$$

Formulas for solutions of (36) and (40) provide the required  $L_2$ -estimates for the functions themselves

$u_s^{(1)}(t)$ ,  $u_s^{(2)}(t)$  and their derivatives  $\frac{du_s^{(1)}(t)}{dt}$ ,  $\frac{du_s^{(2)}(t)}{dt}$ ,  $\frac{d^2u_s^{(1)}(t)}{dt^2}$ ,  $\frac{d^2u_s^{(2)}(t)}{dt^2}$ :

$$\left\| u_s^{(1)}(t) \right\|_{L_2(0,T)} \leq C_1 \left[ \left\| f_s^{(1)}(t) \right\|_{L_2(0,T)} + \left\| f_s^{(2)}(t) \right\|_{L_2(-T,0)} \right]; \tag{39}$$

$$\left\| u_s^{(2)}(t) \right\|_{L_2(0,T)} \leq C_2 \left[ \left\| f_s^{(1)}(t) \right\|_{L_2(0,T)} + \left\| f_s^{(2)}(t) \right\|_{L_2(-T,0)} \right]; \tag{40}$$

$$\left\| \frac{du_s^{(1)}(t)}{dt} \right\|_{L_2(0,T)} \leq C_3 \left[ \left\| f_s^{(1)}(t) \right\|_{L_2(0,T)} + \left\| f_s^{(2)}(t) \right\|_{L_2(-T,0)} \right]; \tag{41}$$

$$\left\| \frac{d^2u_s^{(1)}(t)}{dt^2} \right\|_{L_2(0,T)} \leq C_4 \left[ \left\| s \cdot f_s^{(1)}(t) \right\|_{L_2(0,T)} + \left\| s \cdot f_s^{(2)}(t) \right\|_{L_2(-T,0)} \right]; \tag{42}$$

$$\left\| \frac{du_s^{(2)}(t)}{dt} \right\|_{L_2(0,T)} \leq C_5 \left[ \left\| f_s^{(1)}(t) \right\|_{L_2(0,T)} + \left\| f_s^{(2)}(t) \right\|_{L_2(-T,0)} \right]; \tag{43}$$

$$\left\| \frac{d^2 u_s^{(2)}(t)}{dt^2} \right\|_{L_2(0,T)} \leq C_6 \left[ \|s \cdot f_s^{(1)}(t)\|_{L_2(0,T)} + \|s \cdot f_s^{(2)}(t)\|_{L_2(-T,0)} \right]. \quad (44)$$

*Lemma 1.* Problem (2)–(6) under the conditions (7) has a unique  $L_2$ -strong solution if and only if all the boundary problems from (10)–(11) and (12) are uniquely solvable, and there are not dependent on  $s$  constants  $C_1, C_2$  such that the estimates (39) and (40) are valid.

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#### Аралас типті жүктеулі тендеу үшін берілген бір шеттік есеп туралы

Тік төртбұрышты облыста берілген аралас типті жүктеулі тендеу үшін шекаралық есеп зерттелген. Есептің ерекшелігі гиперболалық және эллипстік бөліктерінің операторын тікелей айналдыруға және берілген есепті сингулярлы интегралдық тендеулерді шешуге әкелуге болмайтындығында. Жалғыз  $L_2$ -әлді шешімінің бар болу шарттары табылған.

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#### Об одной краевой задаче для нагруженного уравнения смешанного типа

Исследована граничная задача для нагруженного уравнения смешанного типа в прямоугольной области. Особенностью задачи является то, что не удается непосредственно обратиться оператор гиперболической и эллиптической частей и свести исходную задачу к исследованию разрешимости сингулярных интегральных уравнений. Найдены условия существования единственного  $L_2$ -сильного решения.

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