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On multipliers in weighted Sobolev spaces. Part II

Let X, Y be Banach spaces whose elements are functions $y: \Omega \rightarrow \mathbb{R}$. We say that a function $z: \Omega \rightarrow \mathbb{R}$ is a pointwise multiplier on the pair (X, Y) , if $Tx = zx \in Y$ and the operator $T: X \rightarrow Y$ is bounded. $M(X \rightarrow Y)$ denotes the multiplier space on the pair (X, Y) . We introduce the norm $\|z; M(X \rightarrow Y)\| = \|T; X \rightarrow Y\|$ in $M(X \rightarrow Y)$. Let $1 \leq p < \infty$. Let m be an integer. $W_{p, \omega_0, \omega_1}^m$ denotes the weighted Sobolev space with the finite norm $\|u\|_{W_{p, \omega_0, \omega_1}^m} = \|u; W_{p, \omega_0, \omega_1}^m\| = \|\omega_0^{1/p} |\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p, v}}$. The aim of this work is to obtain descriptions of multiplier spaces for the pair of weighted Sobolev spaces $(W_{p, \rho, v}^l, W_{q, \omega_0, \omega_1}^m)$ in the case $1 \leq q < p < \infty$.

Key words: weighted Sobolev space, pointwise multiplier.

Let Ω be a domain (an open connected set) in the n -dimensional Euclidian space \mathbb{R}^n with the norm $|x| = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$. We denote by $L_p(\Omega)$, $1 \leq p < \infty$, the space of all real valued measurable functions $f: \Omega \rightarrow \mathbb{R}$ with the finite norm $\|f\|_{L_p(\Omega)} = \|f; L_p(\Omega)\| = \left(\int_{\Omega} |f(x)|^p dx\right)^{1/p}$. We denote by $L_{p, loc}(\Omega)$ the space of functions f defined a.e. in Ω such that $f \in L_p(F)$ for any compact $F \subset \Omega$. Here $L_{p, loc}^+(\Omega)$ is the space of all a.e. positive functions of $L_{p, loc}(\Omega)$, $L_{loc}(\Omega) = L_{1, loc}(\Omega)$, $L_{loc}^+(\Omega) = L_{1, loc}^+(\Omega)$. A function v of $L_{loc}^+(\Omega)$ is called weight in Ω . Let α be a measure on Ω . Below $L_{p, \alpha}(\Omega)$ is the space of all real valued functions equipped with the finite weighted Lebesgue norm $\|u\|_{L_{p, \alpha}(\Omega)} = \left(\int_{\Omega} |u|^p d\alpha(x)\right)^{1/p}$ ($1 \leq p < \infty$). If $d\alpha(x) = v(x) dx$, $v \in L_{loc}^+(\Omega)$, we write $L_{p, v}(\Omega)$. Note that $L_p(\Omega) = L_{p, v}(\Omega)$, if $v \equiv 1$. By C^∞ , C_0^∞ we denote the space of all infinitely differentiable functions in \mathbb{R}^n and the space of functions of C^∞ with compact support $\text{supp } f$ in \mathbb{R}^n , respectively. When the domain is not indicated in the notation of a space or a norm then it is assumed to be \mathbb{R}^n . Throughout the paper we assume that $0 < m < l$ are integers.

Let $1 \leq p < \infty$. Let m be an integer, $\omega_0, \omega_1 \in L_{loc}^+$. We denote by $W_{p, \omega_0, \omega_1}^m$ the completion of the set of $u \in C_0^\infty$ in the finite norm

$$\|u\|_{W_{p, \omega_0, \omega_1}^m} = \|u; W_{p, \omega_0, \omega_1}^m\| = \|\omega_0^{1/p} |\nabla_m u|\|_{L_p, \omega_0} + \|u\|_{L_{p, \omega_1}},$$

where $|\nabla_m u| = \left(\sum_{|\alpha|=m} |D^\alpha u|^2\right)^{1/2}$. Here $W_{p, \omega}^m = W_{p, \omega_0, \omega_1}^m$ with $\omega_0 = 1$, $\omega_1 = \omega$, $W_p^m = W_{p, \omega_0, \omega_1}^m$ with $\omega_0 = 1$, $\omega_1 = 1$. By $W_{p, loc}^m$ we denote the space [1] $\{u: \eta u \in W_p^m \text{ for all } \eta \in C_0^\infty\}$. Here I^n is the family of all cubes Q in the form

$$Q = Q_h = Q_h(x) = \{y \in \mathbb{R}^n: |y_i - x_i| < \frac{h}{2}, i = 1, \dots, n\}, \lambda Q = Q_{\lambda h}(x).$$

By c we denote constants depending only on the assigned numerical parameters, for example, $c = c(l, p, n)$, etc.

Let $h(\cdot)$ be a positive locally bounded function in \mathbb{R}^n . \mathfrak{B} denotes the family (basis) of cubes $Q(x) = Q_{h(x)}(x)$, $x \in \mathbb{R}^n \setminus e$, where e is a set with measure 0. We use the following notation

$$\mathfrak{B} = \{Q(x)\} \quad \text{or} \quad \mathfrak{B} = \{Q(x) = Q_h(x)\}.$$

Definition. Let $\rho \in L_{loc}^+$. We say that a weight ρ satisfies the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x)\}$, if there exist $b > 1$ such that for a.a. x

$$b^{-1}\rho(x) < \rho(y) < b\rho(x), \quad \text{for a.a. } y \in Q(x).$$

Let ρ satisfy the slow variation condition. We denote by $W_p^m(\rho^\mu, \rho^\nu)$ the space $W_{p, \omega_0, \omega_1}^m$ with $\omega_0 = (\rho^\mu)^p$, $\omega_1 = (\rho^\nu)^p$.

Theorem A [1]. Let $1 < p, q < \infty$, $pl > n$. Then

$$\|\gamma; M(W_p^l \rightarrow W_q^m)\| \leq c \sup_{\{Q_1\}} \|\gamma; W_q^m(Q_1)\|.$$

Let us denote by $\Sigma\mathfrak{B}_{(\tau)}$ ($0 < \tau \leq 1$) the set of all finite or countable subfamilies $\{\tau Q^j\} \subset \mathfrak{B}$, in which the cubes $\tau Q^j = \tau Q(x^j)$ are pairwise disjoint. Further, we take

$$T_{(\tau)} = \sup_{\{Q^j\} \subset \Sigma\mathfrak{B}_{(\tau)}} \left\{ \sum_{\{Q^j\}} \left[\int_{Q^j} \rho^{s(l-n/p)q}(x) (|\nabla_m \gamma|^q + \rho^{-smq} |\gamma|^q) dx \right]^{p/(p-q)} \right\}^{(p-q)/pq}.$$

Theorem 1. Let $1 \leq q < p < \infty$, $pl > n$, $-\infty < \mu, s < \infty$. Let $\gamma \in W_{q,loc}^m$. Assume that ρ satisfies the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x) = Q_{h(x)}(x)\}$, $h(x) = \rho(x)^s$. Then the following statements are true:

(a) If $T = T_{(1)} < \infty$, then $\gamma \in M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))$ and the norm satisfies the following inequality

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \leq cT.$$

(b) If $\gamma \in M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))$, then $\infty > \|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \geq cT_{(1/2)}$.

Proof. (a) Let $u \in C_0^\infty$, $F = \text{supp } u$. Let $\{(Q^j, \frac{2}{3}Q^j), j \in J\}$ be a Besicovitch double covering extracted from the family $\{Q(x), x \in F\}$ ($Q^j = Q(x^j)$). Let $\{\psi_j\}_{j \in J}$ be a partition of unity corresponding to the double covering, namely, $\psi_j \in C_0^\infty$, $0 \leq \psi_j \leq 1$, $\sum_{j \in J} \psi_j = 1$ and $\sup |D^\alpha \psi_j| \leq c h_j^{-|\alpha|}$ for all multiindexes $\alpha = (\alpha_1, \dots, \alpha_n)$ of order $|\alpha| = \sum_i |\alpha_i|$ ([2], Chapter 2). We have $(\gamma u)(x) = u(x) \sum_{j \in J} \psi_j(x) \gamma(x) = u(x) \sum_{j \in J} \gamma_j(x)$, $\gamma_j = \gamma \psi_j$, $j \in J$,

$$D^\alpha \gamma(x) = D^\alpha \left(\sum_{j \in J} \gamma \psi_j \right) (x) = \sum_{j \in J} D^\alpha (\gamma \psi_j) (x) = \sum_{j \in J} D^\alpha \gamma_j.$$

Moreover, there exist finite constants $\tilde{\varkappa}_1, \tilde{\varkappa}_2 > 0$, such that

$$\begin{aligned} & \int_F \left(\rho^{\mu q}(x) |\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x) |\gamma u|^q \right) dx \leq \\ & \leq \tilde{\varkappa}_1^q \sum_{k \in J} \int_{Q^k} \sum_{j \in J} \rho^{\mu q}(x) \left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x) |(\gamma_j u)(x)|^q \right) dx \leq \\ & \leq c \tilde{\varkappa}_1^q \sum_{j \in J} \sum_{k \in J} \int_{Q^k \cap Q_j} \rho^{\mu q}(x) \left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x) |(\gamma_j u)(x)|^q \right) dx \leq \\ & \leq c \tilde{\varkappa}_1^q \sum_{j \in J} \int_{Q_j} \rho^{\mu q}(x) \left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x) |(\gamma_j u)(x)|^q \right) dx \leq \\ & \leq c \tilde{\varkappa}_1^q \tilde{\varkappa}_2 \max_{1 \leq i \leq \tilde{\varkappa}_2} \sum_{j \in J_i} \int_{Q_j} \rho^{\mu q}(x) \left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x) |(\gamma_j u)(x)|^q \right) dx \leq \\ & \leq c \tilde{\varkappa}_1^q \tilde{\varkappa}_2 \sum_{j \in J_{i_0}} \int_{Q_j} \rho^{\mu q}(x) \left(|\nabla_m(\gamma_j u)(x)|^q + \rho^{-smq}(x) |(\gamma_j u)(x)|^q \right) dx, \end{aligned}$$

where the maximum coincides with the sum by the subfamily J_{i_0} .

Let $U \in C_0^\infty$ be the continuation of $\tilde{u}(\xi) = u(x^j + h_j \xi)$ from $Q_1 = Q_1(0)$ to the cube $\frac{3}{2}Q_1$ such that $\text{supp } U \subset \frac{3}{2}Q_1$ and

$$\|U; W_p^l\| \leq c_5 \|\tilde{u}; W_p^l(Q_1)\| \tag{1}$$

(see [3]). Let $\tilde{\gamma}_j(\xi) = \gamma(x^j + h_j \xi) \psi_j(x^j + h_j \xi)$. Note that $\text{supp } \tilde{\gamma}_j(\xi) \subset \bar{Q}_1$. Thus, by using (1), Theorem A, we obtain

$$\begin{aligned} & \int_{Q_j} \left(|\nabla_m(u \gamma_j)|^q \rho^{\mu q}(x) + |u \gamma_j|^q \rho^{(\mu-sm)q}(x) \right) dx \leq \rho_j^{\mu q} h_j^{n-mq} \int_{Q_1} \left(|\nabla_m(\tilde{u} \tilde{\gamma}_j)|^q + |\tilde{u} \tilde{\gamma}_j|^q \right) d\xi \leq \\ & \leq \rho_j^{\mu q} h_j^{n-mq} \|\tilde{\gamma}_j; M(W_p^l \rightarrow W_q^m)\|^q \|\tilde{u}; W_p^l(Q_1)\|^{q/p} \leq \\ & \leq c \rho_j^{\mu q} h_j^{n-mq} \sup_x \int_{Q_1(y) \cap Q_1(0)} \left(|\nabla_m \tilde{\gamma}_j|^q + |\tilde{\gamma}_j|^q \right) d\xi \times \left(h_j^{lp-n} \int_{Q_j} |\nabla_l u|^p + h_j^{-n} \int_{Q_j} |u|^p \right)^{q/p} \leq \\ & \leq c \int_{Q_1(0)} \left(|\nabla_m \tilde{\gamma}_j|^q + |\tilde{\gamma}_j|^q \right) d\xi \times h_j^{n-mq} \left(h_j^{lp-n} \int_{Q_j} \rho^{\mu p} |\nabla_l u|^p + h_j^{-n} \rho^{\mu p} \int_{Q_j} |u|^p \right)^{q/p} = \\ & = c h_j^{(l-n/p)q} \left(\int_{Q_j} \left(|\nabla_m \tilde{\gamma}_j|^q + h^{-mq} |\tilde{\gamma}_j|^q \right) dx \right) \times \left(\int_{Q_j} \left(\rho^{\mu p}(x) |\nabla_l u|^p + \rho^{(\mu-sl)p}(x) |u|^p \right) dx \right)^{q/p}. \end{aligned}$$

For each $x \in Q_j$, we have

$$\begin{aligned} |\nabla_m \tilde{\gamma}_j(x)| & \leq c \sum_{|\alpha|=m} |D^\alpha(\gamma \psi_j)(x)| \leq \sum_{|\alpha|=m} \sum_{0 \leq \beta \leq \alpha} |D^\beta \gamma(x)| |D^{\alpha-\beta} \psi_j(x)| \leq \\ & \leq c \sum_{|\alpha|=m} \sum_{0 \leq \beta \leq \alpha} |D^\beta \gamma(x)| h_j^{-|\alpha-\beta|} \leq c \sum_{k=0}^m |\nabla_k \gamma(x)| h_j^{k-m}. \end{aligned} \tag{2}$$

By (2) and embedding theorems of Sobolev spaces $W_p^l(Q_1)$ [4], we have

$$\int_{Q_j} |\nabla_m \tilde{\gamma}_j|^q \leq c \left(\sum_{k=0}^m h_j^{k-m} \|\nabla_m \gamma; L_q(Q^j)\| \right)^q \leq c \int_{Q_j} \left(|\nabla_m \gamma|^q + h^{-mq} |\gamma|^q \right) dx. \tag{3}$$

Inequality (3) implies that

$$\begin{aligned} & \int_{Q_j} \left(|\nabla_m(u\gamma_j)|^q \rho^{\mu q}(x) + |u\gamma_j|^q \rho^{(\mu-sm)q}(x) \right) dx \leq \\ & \leq c h_j^{(l-n/p)q} \left(\int_{Q_j} (|\nabla_m \gamma|^q + h^{-mq} |\gamma|^q) dx \right) \times \left(\int_{Q_j} \left(\rho^{\mu p}(x) |\nabla_l u|^p + \rho^{(\mu-sl)p}(x) |u|^p \right) dx \right)^{q/p}. \end{aligned} \quad (4)$$

By (4) and the Holder inequality, we have

$$\begin{aligned} & \int_F \left(\rho^{\mu q}(x) |\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x) |\gamma u|^q \right) dx \leq \\ & \leq c \tilde{\varkappa}_1^q \tilde{\varkappa}_2 \sum_{j \in J_{i_0}} \left(\int_{Q_j} \rho^{s(l-n/p)q}(x) (|\nabla_m \gamma|^q + \rho^{-smq} |\gamma|^q) dx \right) \times \\ & \quad \times \left(\int_{Q_j} \left(|\rho^\mu \nabla_l u|^p + |\rho^{(\mu-sl)} u|^p \right) dx \right)^{q/p} \leq \\ & \leq c \left\{ \sum_{j \in J_{i_0}} \left[\int_{Q_j} \rho^{s(l-n/p)q}(x) (|\nabla_m \gamma|^q + \rho^{-smq} |\gamma|^q) dx \right]^{p/(p-q)} \right\}^{(p-q)/p} \times \\ & \quad \times \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \leq c T^q \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q. \end{aligned} \quad (5)$$

Hence it follows the upper estimate of $\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\|$.

(b) We take $\eta \in C_0^\infty(Q_1)$, $0 \leq \eta \leq 1$, $\eta = 1$ in $\frac{1}{2}Q_1$. Assume that there exist $\varphi_j \in C_0^\infty(Q^j)$, $\varphi_j(x) = \eta\left(\frac{x-x^j}{h_j}\right)$, such that $\varphi_j = 1$ in $\frac{1}{2}Q^j$. Here $\{Q^j, j \in \Lambda\} \subset \Sigma \mathfrak{B}$, $Q^j = Q(x^j)$. Then

$$\begin{aligned} & \frac{\|\gamma \varphi_j; W_q^m(\rho^\mu, \rho^{\mu-sm})\|^q}{\|\varphi_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q} \geq c \frac{\rho_j^{\mu q} \int_{\frac{1}{2}Q^j} \left(|\nabla_m \gamma|^q + \rho_j^{-smq} |\gamma|^q \right) dx}{\rho_j^{\mu q - s(l-n/p)q}} = \\ & = c \left(\rho_j^{s(l-n/p)q} \int_{\frac{1}{2}Q^j} |\nabla_m \gamma|^q dx + \rho_j^{s(l-m-n/p)q} \int_{\frac{1}{2}Q^j} |\gamma|^q dx \right) = c a_j^q, \end{aligned}$$

where $a_j^q = \rho_j^{s(l-n/p)q} \int_{\frac{1}{2}Q^j} |\nabla_m \gamma|^q dx + \rho_j^{s(l-m-n/p)q} \int_{\frac{1}{2}Q^j} |\gamma|^q dx$.

Let $u_j = a_j^{q/(p-q)} \frac{\varphi_j}{\|\varphi_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|}$. We take $u = \sum_{j \in \Lambda} u_j$. Then

$$\|\gamma u_j; W_q^m(\rho^\mu, \rho^{\mu-sm})\|^q \geq a_j^{q^2/(p-q)} \left(\frac{\|\gamma \varphi_j; W_q^m(\rho^\mu, \rho^{\mu-sm})\|}{\|\varphi_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|} \right)^q \geq c a_j^{pq/(p-q)}.$$

In addition, $\|u_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\| = a_j^{q/(p-q)}$. So that,

$$\begin{aligned} & \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p = \sum_{j \in \Lambda} \|u_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p = \sum_{j \in \Lambda} a_j^{pq/(p-q)} \leq \\ & \leq c \sum_{j \in \Lambda} \|\gamma u_j; W_q^m(\rho^\mu, \rho^{\mu-sm})\|^q = c \|\gamma u; W_q^m(\rho^\mu, \rho^{\mu-sm})\|^q \leq \\ & \leq c \|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\|^q \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \end{aligned}$$

for $u = \sum_{j \in \Lambda} u_j$, which implies

$$\|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^{p-q} \leq c \|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\|^q < \infty$$

for $u \in C^\infty \cap W_p^l(\rho^\mu, \rho^{\mu-sl})$. Next, we show

$$\begin{aligned} \|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\|^{pq/(p-q)} &\geq c \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p = \\ &= c \sum_{j \in \Lambda} \|u_j; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^p = c \sum_{j \in \Lambda} a_j^{pq/(p-q)} = \\ &= c \sum_{j \in \Lambda} \left[\rho_j^{s(l-n/p)q} \int_{\frac{1}{2}Q_j} |\nabla_m \gamma|^q dx + \rho_j^{s(l-m-n/p)q} \int_{\frac{1}{2}Q_j} |\gamma|^q dx \right]^{p/(p-q)}. \end{aligned}$$

Thus, we have the following final estimate

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}) \rightarrow W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \geq c T_{(1/2)}.$$

The proof of Theorem 1 is complete.

Remark. Since a lattice of unit cubes is contained in $\{Q_1(x), x \in \mathbb{R}^n\}$, Theorem 1 implies the two-sided estimate of $\|\gamma; M(W_p^l \rightarrow W_q^m)\|$ ($1 \leq q < p < \infty$), obtained in [1].

Theorem 2. Let $1 \leq q < p < \infty$, $pl > n$, $-\infty < \mu, s < \infty$. Let $\gamma \in W_{q,loc}^m$. Let ρ satisfy the slow variation condition with respect to the basis of cubes $\mathfrak{B} = \{Q(x) = Q_{h(x)}(x)\}$, where $h(x) = \rho(x)^s$. Then

$$W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)}) \cap W_{q,loc}^m \subset M(W_p^l(\rho^\mu, \rho^{\mu-sl}), W_q^m(\rho^\mu, \rho^{\mu-sm})).$$

And the following inequality holds:

$$\|\gamma; M(W_p^l(\rho^\mu, \rho^{\mu-sl}), W_q^m(\rho^\mu, \rho^{\mu-sm}))\| \leq c \|\gamma; W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)})\|.$$

Proof. Let $u \in C_0^\infty$, $F = \text{supp } u$. By using (), we have the estimate

$$\begin{aligned} &\int_F (\rho^{\mu q}(x) |\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x) |\gamma u|^q) dx \leq \\ &\leq c \left\{ \sum_{j \in J_{i_0}} \left[\int_{Q_j} \rho^{s(l-n/p)q}(x) (|\nabla_m \gamma|^q + \rho^{-smq} |\gamma|^q) dx \right]^{p/(p-q)} \right\}^{(p-q)/p} \times \\ &\quad \times \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \leq c T^q \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q. \end{aligned}$$

Let us continue to estimate ()

$$\begin{aligned} &\int_F (\rho^{\mu q}(x) |\nabla_m(\gamma u)|^q + \rho^{(\mu-sm)q}(x) |\gamma u|^q) dx \leq \\ &\leq c \left\{ \int_{Q_i} \rho^{s(l-n/p)q}(x) (|\nabla_m \gamma|^q + \rho^{-smq} |\gamma|^q) dx \right\} \times \\ &\quad \times \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q \leq c \|\gamma; W_q^m(\rho^{s(l-n/p)}, \rho^{s(l-m-n/p)})\|^q \|u; W_p^l(\rho^\mu, \rho^{\mu-sl})\|^q, \end{aligned}$$

which implies the statement of Theorem 2. The proof of the theorem is complete.

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А.Мырзағалиева

Салмақты Соболев кеңістіктеріндегі мультипликаторлар жайлы. II-бөлім

$X, Y - y: \Omega \rightarrow \mathbb{R}$ функцияларынан тұратын банах кеңістіктері болсын. Егер $Tx = zx \in Y$ және $T: X \rightarrow Y$ операторы шенелген болса, онда $z: \Omega \rightarrow \mathbb{R}$ функциясы (X, Y) жұбындағы нүктелік мультипликатор деп аталады. $M(X \rightarrow Y)$ арқылы (X, Y) жұбындағы мультипликаторлар кеңістігін белгілейміз. $M(X \rightarrow Y)$ мультипликаторлар кеңістігінде норманы келесідей анықтаймыз: $\|z; M(X \rightarrow Y)\| = \|T; X \rightarrow Y\|$. $1 \leq p < \infty$, m – бүтін сан болсын. $W_{p, \omega_0, \omega_1}^m$ арқылы салмақты Соболев кеңістігін белгілеп, норманы келесідей анықтаймыз: $\|u\|_{W_{p, \omega_0, \omega_1}^m} = \|u; W_{p, \omega_0, \omega_1}^m\| = \|\omega_0^{1/p} |\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p, v}}$. Аталмыш жұмыстың мақсаты — салмақты Соболев кеңістіктерінің $(W_{p, \rho, v}^l, W_{q, \omega_0, \omega_1}^m)$ жұбы үшін мультипликаторлар кеңістіктерін сипаттау.

А.Мырзағалиева

О мультипликаторах в весовых пространствах Соболева. Часть II

Пусть $X, Y -$ банаховы пространства функций $y: \Omega \rightarrow \mathbb{R}$. Функция $z: \Omega \rightarrow \mathbb{R}$ называется точечным мультипликатором в паре (X, Y) , если $Tx = zx \in Y$ и оператор $T: X \rightarrow Y$ ограничен. Через $M(X \rightarrow Y)$ обозначается пространство мультипликаторов в паре (X, Y) . В $M(X \rightarrow Y)$ вводится норма $\|z; M(X \rightarrow Y)\| = \|T; X \rightarrow Y\|$. Пусть $1 \leq p < \infty$, m – целое. Через $W_{p, \omega_0, \omega_1}^m$ обозначается весовое пространство Соболева с конечной нормой вида $\|u\|_{W_{p, \omega_0, \omega_1}^m} = \|u; W_{p, \omega_0, \omega_1}^m\| = \|\omega_0^{1/p} |\nabla_m u|\|_{L_p} + \|\omega_1^{1/p} u\|_{L_{p, v}}$. Цель данной работы заключается в описании пространств мультипликаторов для пары весовых пространств Соболева $(W_{p, \rho, v}^l, W_{q, \omega_0, \omega_1}^m)$.

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