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FORCING COMPANIONS OF MUTUALLY CONSISTENT THEORIES IN PERMISSIBLE ENRICHMENTS

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This study is devoted to the study of the forcing companions of the Jonsson AP-theories in the enriched signature. It is proved that the forcing companion of the theory does not change when expanding the theories under consideration, which have some properties, by adding new predicate and constant symbols to the language. The model-theoretic results obtained in the general form are supported by examples from differential algebra.

Definition 1 [3]. Let T be a theory of the language L. A forcing companion of the theory T is a theory T^f that satisfies the following equation:

$$T^f = \{\phi \mid T \Vdash \neg\neg\phi\}.$$

The following results were proved by J. Barwise and A. Robinson:

Theorem 1 [3]. Let T_1 and T_2 be the theories of the language L. Then T_1 and T_2 are mutually model consistent if and only if $T_1^f = T_2^f$.

Theorem 2 [3]. Let T be mutually model consistent with some inductive theory T'. Then $T' \subseteq E^a$. Therefore, if T is an inductive theory then $T \subseteq E^a$.

We are working within the framework of the following definition of Jonsson theory published in the Russian edition of [1].

Definition 2 [1, p. 80]. A theory T is called Jonsson if the theory T has at least one infinite model; T is an inductive theory;T has the amalgam property (AP) and the joint embedding property (JEP).

Definition 3 [2]. A theory T is called an AP-theory if, from the fact that it has the amalgam property, it follows that T also has the joint embedding property, i.e. $AP \rightarrow JEP$.

We consider the theories Δ_1, Δ_2, $\Delta_3$to satisfy the following conditions:

1) Δ_1 is an inductive theory that is not a Jonsson theory, but has a model companion which is the theory Δ_3,

2) Δ_2 is a hereditary Jonsson AP-theory that has a model companion, which is also Δ_3.

All three theories are mutually model consistent because Δ_3 is mutually model consistent with both Δ_1 and Δ_2, for which Δ_3 is the model companion, which means that Δ_1 and Δ_2 are mutually model consistent with each other. At the same time, according to Theorem 1, the forcing companions of mutually model consistent theories must coincide, which means that $\Delta_1^f = \Delta_2^f$. Δ_2 is a perfect Jonsson theory, while $\Delta_2^* = Th(C) = \Delta_3$, C is a semantic model of Δ_2. In addition, Δ_3 is also a forcing companion of Δ_2, i.e. $\Delta_3 = \Delta_2^f$. So we get $\Delta_1^f = \Delta_2^f = \Delta_3$.

We consider the following extensions of the theories Δ_1, Δ_2, Δ_3 in various language enrichment L by adding new constant and predicate symbols c and P. Let $\overline{\Delta_1}$ be a theory extending Δ_1 by enriching the language L with the predicate symbol P as follows:

$$\overline{\Delta_1} = \Delta_1 \cup \Delta_1^f \cup \{P, \subseteq\},$$

where $\{P, \subseteq\}$ is an infinite list of $\exists$-sentences and interpretation of P is an existentially closed submodel in amodel of Δ_1.

Let $\overline{\Delta_2}$ be a theory that extends Δ_2 when a new constant symbol c is added to the language L and defined as follows:

$$\overline{\Delta_2} = \Delta_2 \cup \Delta_2^f \cup Th_{\forall\exists}(C, c),$$

where C is a semantic model of Jonsson theory Δ_2. Since Δ_2 is a hereditary Jonsson theory, $\overline{\Delta_2}$ is also a Jonsson theory.

Based on these assumptions, we obtain the following results.

Theorem 3.$\overline{\Delta_1}^f = \Delta_1^f$.

Thus, we can conclude that the forcing companion of the inductive theory Δ_1 does not change when enriching the language of this theory with a new predicate symbol P.

Theorem 4.$\overline{\Delta_2}^f = \Delta_2^f$.

This means that the addition of the new constant c to language L did not affect the forcing companion when expanding theory Δ_2 to $\overline{\Delta_2}$.

Theorem 5 [1, p. 77]. Let T be a complete theory of language L, languages L_1 and L_2 are extensions of language L such that $L_1 \cap L_2 = L$, and theories T_1 and T_2 are consistent extensions of theory T in languages L_1 and L_2 respectively. Then $T_3 = T_1 \cup T_2$ is a consistent theory.

Now we can formulate and prove the following result.

Theorem 6 i) The theory $\overline{\Delta_1} \cup \overline{\Delta_2}$ is consistent.ii) $(\overline{\Delta_1} \cup \overline{\Delta_2})^f = \Delta_1^f = \Delta_2^f$

The results formulated above, described for the general situation in model theory, can be interpreted using examples of differential algebra, namely, when considering the theory DF_0 of differential fields of characteristic 0, the theory DCF_0 of differentially closed fields of characteristic 0, the theory DF_p of differential fields of characteristic p, the theory DPF_p of differentially perfect fields of characteristic p, the theory DCF_p of differentially closed fields of characteristic p. In [2], it was proved that DF_0 and DPF_p are perfect Jonsson theories, DCF_0 and DCF_p are their centers correspondingly, while DF_p is not Jonsson. In addition, DF_0 and DPF_p are strongly convex theories in the classical Robinson sense, which allows us to state that DF_0 and DPF_p are Jonsson AP-theories.

Due to the above facts, we can project the results described in the previous paragraph to the case of differentially closed fields of zero and positive characteristic. Here we assume that DF_0 and DPF_p are hereditary Jonsson theories.

Theorem 7.$\overline{DF_p}^f = DF_p^f$.

Theorem 8.$\overline{DPF_p}^f = DPF_p^f$.

Theorem 9. i) $\overline{DF_p} \cup \overline{DPF_p}$ is consistent.

ii) $(\overline{DF_p} \cup \overline{DPF_p})^f = DF_p^f = DPF_p^f$

All concepts whose definitions are not given here can be found in [4] and [2].

This work was supported by the Science Committee of the Ministry of Education and Science of theRepublic of Kazakhstan (grant AP09260237).

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