PERIODIC BOUNDARY VALUE PROBLEM FOR A SYSTEM OF THE HYPERBOLIC EQUATIONS WITH DELAY ARGUMENT

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Numerous problems of application such as problems of population dynamics, management of technical systems, the problem of physics, mathematical economics, ecology and etc., variational problems related to the regulatory process, the optimal control problem with delay systems leads to boundary value problems for differential equations with deviating argument [1]. One of the rapidly growing field of the theory of differential equations with deviating argument is the theory of boundary value problems for differential equations with delay argument. Periodic boundary value problems for hyperbolic equations with delay argument are widely used in various applications. Conditions of a solvability of the periodic boundary value problem for hyperbolic equations with delay argument connected with the solvability of a family of periodic boundary value problems for ordinary differential equations with delay argument.

We consider the periodic boundary value problem for the system of the hyperbolic equations second order with delay argument on the domain $\Omega^{\tau} = [-\tau, T] \times [0, \omega]$

$$\frac{\partial^2 u(t,x)}{\partial t \partial x} = A(t,x)\frac{\partial u(t,x)}{\partial x} + A_0(t,x)\frac{\partial u(t-\tau,x)}{\partial x} + B(t,x)\frac{\partial u(t,x)}{\partial t} + C(t,x)u(t,x) + f(t,x), \quad (1)$$

$$(t,x) \in [0,T] \times [0,\omega],$$

$$\frac{\partial u(z,x)}{\partial x} = diag\left[\frac{\partial u(0,x)}{\partial x}\right] \cdot \varphi(z), \qquad z \in [-\tau, 0], \qquad u(0,x) = u(T,x), \qquad x \in [0,\omega], \quad (2)$$

$$u(t,0) = \psi(t), \qquad t \in [0,T], \quad (3)$$

where $u(t,x) = col(u_1(t,x), u_2(t,x), ..., u_n(t,x))$ is unknown function, the $(n \times n)$ matrices $A(t,x)$, $A_0(t,x)$, $B(t,x)$, $C(t,x)$, and n vector-function $f(t,x)$ are continuous on $\Omega = [0,T] \times [0,\omega]$, the n vector-function $\varphi(t)$ is continuously differentiable and given on the initial set $[-\tau, 0]$ such that $\varphi_i(0) = 1, i = 1,2,..., n$, $\tau > 0$ is constant delay, the n vector-function $\psi(t)$ is continuously differentiable on $[0,T]$, and the compatibility condition is valid: $\psi(0) = \psi(T)$.

We introduce a new unknown functions $v(t,x) = \frac{\partial u(t,x)}{\partial x}$, $w(t,x) = \frac{\partial u(t,x)}{\partial t}$ [2] and the problem (1)—(3) reduce to an equavalent problem, consisting the family of periodic boundary value problem for system of differential equations with delay argument and integral relations. For constructing of algorithms of finding approximate solutions to the equavalent problem are used results of paper [3]. For solve of the family to the periodic boundary value problems for system of differential equations with delay argument are used results of articles [4]. Algorithms of finding solutions to the families of periodic boundary value problems for differential equations with delay argument are constructed and their convergence proved. The conditions of the solvability to the periodic boundary value problems for hyperbolic equations with delay argument are established.

Репозиторий КарГУ

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