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Comparison the readings of gravity field and it‘s gradient potential

The problem of studying the deep structure of the earth's crust is one of the strategic directions of geophysical research, ensuring the development of Earth sciences. Therewith, gravimetry is one of the main methods for studying the structure of the earth's crust. This study is associated with the concepts of gravitational field‘s potential and gradient of the underground anomaly. Solving the inverse problem of restoring density of the underground anomaly, thorough analysis of the direct problem plays an important part. This study analyzes the characteristic features of the readings of anomaly gravitational field‘s potential and gradient. Based on the results obtained by the author, it has revealed and verified the necessity of choosing a gradient as the boundary conditions of the direct problem, which will significantly improve the inverse problem calculations results – finding the density of anomaly. This research demonstrates quantitatively that anomaly gravitational field‘s gradient more accurately describes the anomaly, compared to the gravitational field‘s potential.

Keywords: inverse problem, gravitational potential, gradient method.

Introduction

Inverse problems cover a wide range of applied problems. Among hyperbolic, elliptic and parabolic inverse problems, elliptic problems are very inaccurate. In this regard, the problem itself is very complicated. In the book [1] a wide range of tasks of all types is considered. The inverse problems of hyperbolic type [1] are represented especially widely. Previous results obtained by the authors [2, 3] were published in the studies. There is a general overview of the situation on the issues of gravimetry at the field [4, 5]. On the basis of gravimetric readings on the Earth surface in the studied area, we use the mathematical apparatus for solving inverse problems of elliptic type.

When constructing a mathematical model, we simplified the objects under study as much as possible. Consider a vertical section of the ground. For simplicity, we chose it in the shape of a rectangle. It is known that inside this area in a certain place there is an anomaly, but it is unknown what type of anomaly it is (what is its density). The area of anomaly is known, and we will designate it by Ω. On the earth surface we have gravimetric readings of the field potentials $\eta_1(x)$ and its gradient $\eta_2(x)$. We denote the lower and lateral subsurface boundaries by Γ as shown in Figure 1. We artificially expand the study area so that the value of the gravitational potential of the anomaly field does not affect the external boundaries.

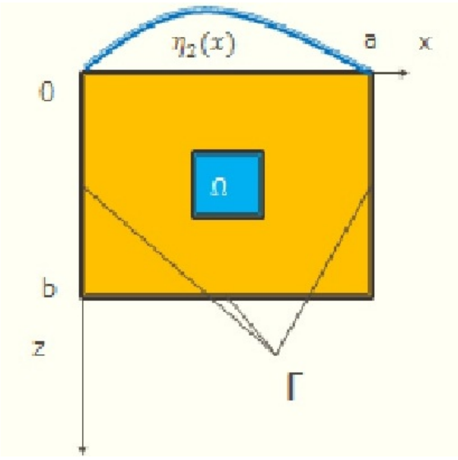

Figure 1. Interpretation of the simplified model of the problem

The equations of condition (*direct problem*) are described by the following formulas. The potential difference between perturbed and non-perturbed gravitational fields is described by the Poisson equation with boundary conditions (1)–(4). The expression (3)–(4) characterizes the results of measuring the potential and its derivative on the outer surface. Condition (5) characterizes the difference in the density of the soil with and without anomaly over the entire study area.

$$\Delta\ \eta(x,z) = -\ 4\pi G\psi(x,z); \tag{1}$$

$$\eta(x,z)|_{\Gamma} = 0; \tag{2}$$

$$\eta(x,0) = \eta_1(x); \tag{3}$$

$$\frac{\partial\eta(x,0)}{\partial z} = \eta_2(x); \tag{4}$$

$$\psi(x,z) = \begin{cases} 0, & \text{out of } \Omega; \\ \psi_0, & \text{in } \ \Omega, \end{cases} \tag{5}$$

where $\eta(x,z)$ is the potential of the gravitational field, G is the gravitational constant, $\psi(x,z)$ is the anomaly density, $\eta_1(x)$ is the measured readings of the gravitational field, $\eta_2(x)$ are the measured values of the gradient of the gravitational field, Γ is the boundary of the study area without the earth's surface, Ω is the anomaly region.

In the direct problem, we consider density as a known value. We calculate the value of the gravitational field potential and its gradient on the studied area using one of the conditions (3) or (4).

Inverse problem is a search for the anomaly density based on the results of measuring the potential and its derivative on the outer surface. The inverse problem is solved by optimization method, more precisely - a gradient method. It is necessary to introduce a functional using the standard deviation as a minimization parameter.

First statement of optimization problem looks as follows:

$$\Delta\ \eta(x,z) = -\ 4\pi G\psi(x,z);$$
$$\eta(x,z)|_{\Gamma} = 0;$$
$$\eta(x,0) = \eta_1(x);$$
$$\frac{\partial\eta(x,0)}{\partial z} = \eta_2(x);$$
$$\psi(x,z) = \begin{cases} 0, & \text{out of } \Omega; \\ \psi_0, & \text{in } \ \Omega; \end{cases}$$
$$\mathrm{I}\,(\psi_0) = \int_0^{\mathrm{L}} \left(\frac{\partial\eta(\mathrm{x},0)}{\partial \mathrm{z}} - \eta_2(\mathrm{x})\right)^2 \mathrm{dx} \to \min.$$

Second statement of optimization problem looks like this:

$$\Delta\ \eta(x,z) = -\ 4\pi G\psi(x,z);$$
$$\eta(x,z)|_{\Gamma} = 0;$$
$$\eta(x,0) = \eta_1(x);$$
$$\frac{\partial\eta(x,0)}{\partial z} = \eta_2(x);$$
$$\psi(x,z) = \begin{cases} 0, & \text{out of } \Omega; \\ \psi_0, & \text{in } \ \Omega; \end{cases}$$
$$\mathrm{I}\,(\psi_0) = \int_0^{\mathrm{L}} (\eta(\mathrm{x},0) - \eta_1(\mathrm{x}))^2 \mathrm{dx} \to \min.$$

We do not know yet, which of these forms is better. However, some information about the properties of these optimization problems can be obtained on the basis of a quantitative analysis of the direct problem. We want to find out what happens at the upper boundary (earth surface) at different locations of the anomaly inside the area.

Suppose we need to examine a region with a size of 100 horizontally and 50 vertically. We artificially expand the region up by 100 in order to analyze what will happen at the upper boundary z = 50. All calculations were performed on COMSOL Multiphysics 5.2 (Fig. 2). The anomaly has a dimension of 2 to 2. We will change the location of the position of the anomaly horizontally along j = 0, 10, 20, 30, 35, 40, 45. It will be sufficient to change the anomaly location to the middle of the study area, since earlier in the studies we found that the results are symmetrical. The vertical shifts along i = 5, 10, 15, 20, 30, 40. Figure 3 shows the surge of the gravitational potential on the surface z = 50, that is, theoretically, the readings of a gravimeter on the surface of the earth. Figure 4 shows a graph of the value of the anomaly gravitational field gradient, located as in Figure 2. Later in the tables, we analyzed the indication of the potential and its gradient for different variations of the anomaly location.

Figure 2. Anomaly location at the extended upper boundary.
The angle of the lower left edge of the anomaly is located at (20;10)

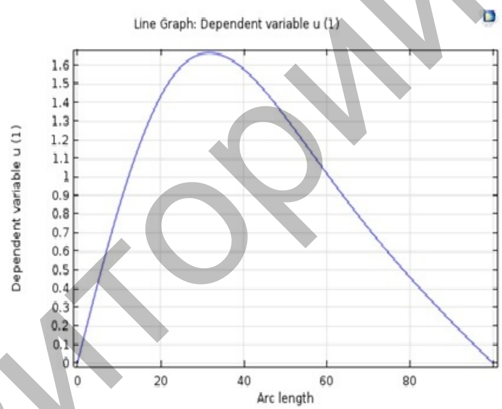

Figure 3. The value of the gravitational field potential on the surface z = 50
at the anomaly location in the coordinate (20;10)

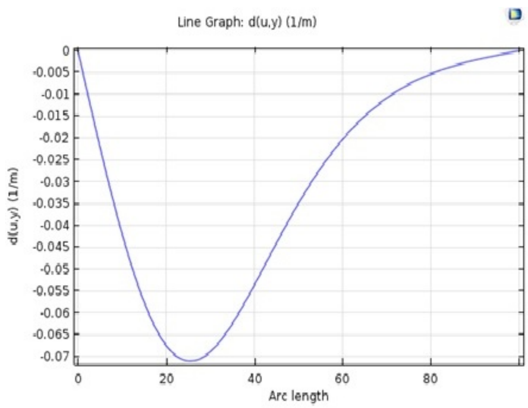

Figure 4. Gradient value of the gravitational field on the surface z = 50
at the anomaly location in the coordinate (20;10)

Below there are table of symbols describing the parameters of the gravitational field potential and gradient.

- i is the line – horizontal position, bottom up;
- j is the column – vertical position, left to right;
- a is the horizontal coordinate of the anomaly;
- ap is the coordinate of the potential peak on the outer surface;
- ag is the coordinate of the potential gradient peak on the outer surface;
- bp is the value of the potential peak on the outer surface;
- bg is the value of the gradient peak on the outer surface;
- cpl is the coordinate of the point to the left of the peak with a drop in the potential value by an order of magnitude compared with the peak value;
- cgl is the coordinate of the point to the left of the peak with the gradient value falling by an order of magnitude compared with the peak value;
- cpr is the coordinate of the point to the right of the peak with a drop in the potential value by an order of magnitude compared with the peak value;
- cgr is the coordinate of the point to the right of the peak with the gradient value falling by an order of magnitude compared with the peak value.

Consider the most extreme point of the anomaly both vertically and horizontally (Tables 1, 2).

Table 1

Horizontally anomaly location at x = 5 (i = 5 line). The anomaly «runs through» from left to right to the middle of the area (depending on the j column)

j	a	ap	ag	bp	bg	cpl	cgl	cpr	cgr
0	5	19	20	0.04	-0.01	18.54	13.91	38.82	28.26
10	5	29	21	0.51	-0.02	19.43	15.18	40.67	29.96
20	5	32	26	0.88	-0.04	22.26	18.22	44.27	34.82
30	5	37	32	1.14	-0.04	26.72	23.88	49.78	42.13
35	5	40	37	1.22	-0.04	29.15	27.53	53.13	46.67
40	5	43	41	1.28	-0.05	31.97	31.97	56.27	51.35
45	5	47	46	1.31	-0.05	35.21	36.42	59.51	56.06

Table 2

Vertical position of the anomaly at y = 0 (j = 0 column). The anomaly «runs» from bottom to top to the middle (from the depth to the surface) of the region z = 50 (depending on the i line)

i	a	ap	ag	bp	bg	cpl	cgl	cpr	cgr
5	5	19	21	0.04	-0.002	18.54	13.91	38.82	28.26
10	10	26	19	0.09	-0.004	17.85	13	37.99	26.69
15	15	25	18	0.13	-0.007	16.95	12.11	36.25	24.83
20	20	23	15	0.19	-0.011	15.11	10.67	33.49	22.43
30	30	17	10	0.37	-0.028	10.89	7.26	25.83	15.28
40	40	9	5	0.86	-0.128	5.53	3.65	13.9	7.31

Now consider the central location of the anomaly (Table 3).

Table 3

Vertical location of the anomaly at y = 45 (j = 45 column). The anomaly «runs» from bottom to top to the middle (from the depth to the surface) of the region z = 50 (depending on the i line)

i	a	ap	ag	bp	bg	cpl	cgl	cpr	cgr
5	5	47	46	1.31	-0.04	35.21	36.42	59.51	56.06
10	10	47	46	2.45	-0.08	35.17	36.72	59.35	55.64
15	15	46	46	3.68	-0.13	35.56	37.31	59.03	54.99
20	20	47	45	5.05	-0.18	36.01	37.96	58.2	54.12
30	30	46	46	8.59	-0.35	37.32	40.01	55.96	51.82
40	40	46	45	14.71	-0.84	40.34	43.05	52.17	49.01

Studying the obtained results, it was found that the value of the gravitational field gradient more accurately describes the anomaly location and provides the most accurate anomaly center readings and boundaries. All calculation tables (all possible combinations of location) were in favor of the gravitational field gradient. Thus, it is better to be guided by the readings of the gravitational field gradient in the search for the vertical anomaly location.

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М.О. Кенжебаева

Аномалиядағы гравитациялық өрістің потенциалын және градиенттін талдау

Жер қыртысының терең құрылымын зерттеу мәселесі Жер туралы ғылымды дамытуды қамтамасыз ететін геофизикалық зерттеулердің стратегиялық бағыттарының бірі болып табылады. Сонымен қатар гравитациялық барлау жер қыртысының құрылымын зерттеудің негізгі әдістерінің бірі болып есептеледі. Бұл жұмыс жер асты аномалиясының гравитациялық өрісінің потенциалы мен градиенті туралы түсініктермен байланысты. Жер асты аномалиясының тығыздығын қалпына келтірудің кері есебін шешу тікелей есепті мұқият талдауда маңызды рөл атқарады. Мақалада аномалияның гравитациялық өрісінің потенциалы мен градиентінің белгілеріне тән ерекшеліктері талданды. Авторлармен алынған нәтижелер негізінде тікелей есептің шекаралық шарттары ретінде градиентті таңдау қажеттілігі анықталды және дәлелденді. Бұл кері есепті шығару нәтижелерін – аномалияның тығыздығын іздеуді жақсартады. Аталған жұмыста аномалияның гравитациялық өрісінің градиенті, гравитациялық өрістің потенциалына салыстырмалы түрде қарағанда, аномалияны нақтырақ анықтайтындығы сандық көрсеткіштермен сипатталды.

Кілт сөздер: кері есеп, гравитациялық потенциал, градиент әдісі, тікелей есептің шекаралық шарттары.

М.О. Кенжебаева

Анализ градиента и потенциала гравитационного поля аномалии

Проблема изучения строения земной коры является одним из стратегических направлений геофизических исследований, обеспечивающих развитие науки о Земле. При этом гравиразведка является одним из основных методов изучения строения земной коры. Данная работа связана с понятиями потенциала и градиента гравитационного поля подземной аномалии. При решении обратной задачи восстановления плотности подземной аномалии важную роль играет тщательный анализ прямой задачи. В статье проанализированы характерные особенности показания потенциала и градиента гравитационного поля аномалии. На основе результатов, полученных автором, выявлена и обоснована необходимость выбора градиента в качестве граничных условий при решении прямой задачи. Кроме того, количественно показано, что градиент гравитационного поля аномалии точнее описывает аномалию, по сравнению с потенциалом гравитационного поля.

Ключевые слова: обратная задача, гравитационный потенциал, градиентный метод, решение прямой задачи.

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