REDUCTION THEOREMS FOR DISCRETE HARDY OPERATOR ON THE CONES OF MONOTONE SEQUENCES

G. Karshygina[1], **A. Gogatishvili**[2], **T. Ünver**[3]

[1]Astana International University, Astana, Republic of Kazakhstan
[1]E-mail: karshygina84@mail.ru
[2]Institute of Mathematics CAS, Prague, Czechia
[2]E-mail: gogatish@math.cas.cz
[3]Kirikkale University, Kirikkale, Türkiye
[3]E-mail: tugceunver@kku.edu.tr

We will investigate the following inequalities:

Let (u_k), (a_n) and (b_n) be given non-negative sequences. Let $q \in (0, \infty)$ and $0 < p < 1$.

$$\left(\sum_{n=1}^{\infty}\left(\sup_{n \le i < \infty} u_i \sum_{k=1}^{i} x_k\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} x_n^p b_n\right)^{\frac{1}{p}} \tag{1}$$

$$\left(\sum_{n=1}^{\infty}\left(\sup_{n \le i < \infty} u_i \sum_{k=i}^{\infty} x_k\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} x_n^p b_n\right)^{\frac{1}{p}} \tag{2}$$

for non-negative, non-increasing sequences $x = (x_n)$ and the constant $C > 0$ is independent of x.

The corresponding problem for unrestricted, non-negative sequences x was solved by the authors in [1] and [3].

Our main result is the discrete analog of the theorem of Gogatishvili and Pick [2]].

This problem for continuous cases has been extensively studied over the last twenty years; see papers [4] and [5] and see reference there. As the supremum operator is not a linear operator, Sawyer's duality principle [6], doesn't work.

There is a difference between discrete and continuous cases. The proof in the discrete case does not work directly, as the power rule theorem, which is true in the continuous case, is not true in the discrete case. We also show that the precise formulation of the result from [4] is not true in discrete case.

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ON COMPACTNESS OF COMMUTATORS FOR SINGULAR INTEGRAL OPERATOR ON GENERALIZED MORRY SPACES

Dauren Matin[1], **Talgat Akhazhanov**[2]

[1,2]L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
[1]E-mail: d.matin@mail.kz
[2]E-mail: talgat-a2008@mail.ru

In this paper, we give the sufficient conditions for the compactness of the commutators of the singular integral operator Calderón-Zygmund in generalized Morrey spaces.

The compactness of the commutator for the singular integral operator Calderón-Zygmund on the Morrey spaces M_p^λ was considered in (4). The boundedness of the singular integral operator Calderón-Zygmund in generalized Morrey spaces was discussed in (1). The pre-compactness of sets on the generalized Morrey spaces $M_p^{w(\cdot)}$ was examined in (2).

Definition 1. Let $1 \leq p \leq \infty$ and let w be a measurable non-negative function on $(0, \infty)$ that is not equivalent to zero. The generalized Morrey space $M_p^{w(\cdot)} \equiv M_p^{w(\cdot)}(R^n)$ is defined as the set of all functions $f \in L_p^{loc}(R^n)$ with $\|f\|_{M_p^{w(\cdot)}} < \infty$, where

$$\|f\|_{M_p^{w(\cdot)}} = \sup_{x\in R^n, \quad r>0} \left(w(r)\, \|f\|_{L_p(B(x,r))} \right).$$

The space $M_p^{w(\cdot)}$ coincides with the Morrey space M_p^λ if $w(r) = r^{-\lambda}$, where $0 \leq \lambda \leq \frac{n}{p}$.

By $\Omega_{p\infty}$ we denote the set of all non-negative, measurable on $(0, \infty)$ functions, not equivalent to 0 and such that for some $t > 0$,

$$\|w(r) r^{\frac{n}{p}}\|_{L_\infty(0,t)} < \infty, \quad \|w(r)\|_{L_\infty(t,\infty)} < \infty.$$

The space $M_p^{w(\cdot)}$ is non-trivial if and only if $w \in \Omega_{p\infty}$ (3).

Next, we will provide the definition of the singular integral operator for the Calderón-Zygmund T, which plays a crucial role in harmonic analysis and potential theory.

$$Tf(x) = p.v. \int\limits_{R^n} \frac{f(y)K(x-y)}{|x-y|^n} dy.$$