EURASIAN PHYSICAL TECHNICAL JOURNAL

2025, Volume 22, No.2 (52)

https://doi.org/10.31489/2025N2/147-154

Received: 30/11/20254 Revised: 21/04/2025 Accepted: 25/06/2025 Published online: 30/06/2025
Original Research Article

UDC 539.124; 539.12...162

DEMONSTRATING NONLINEAR OSCILLATIONS OF CHARGED PARTICLE BY THE HOMOTOPY PERTURBATION METHOD

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Abstract. *This document describes explicitly the nonlinear equation handling the oscillation of charged particles affected by electrical field using the Homotopy Perturbation Method. This method success to construct a significant approximate solution. Our analysis has resulted in the development of an initial set of four equations that govern the oscillatory of charged particles. Following this, we have visually depicted these equations and satisfied a deep clear interpretations vision of the investigated outcomes. Our analysis has resulted in the development of an initial set of four equations that govern the oscillatory of charged particles. Following this, we have visually depicted these equations and satisfied a deep clear interpretations vision of the investigated outcomes.*

Keywords: Charged particle oscillation; Nonlinear harmonic equation, Homotopy Perturbation Method.

1. Introduction

The aim of this study is to provide an explicit interpretation and describe the behavior of a charged body moved under influence of electrical forces that expressed as in real natural physical system by equation(1) [1]. This actual serious harmonic equation is commonly used in physics to describe the motion of a charged body under the influence of electrical forces between it and its neighbored charges. Such equations are used in physics to understand the motion and dynamics of systems affected by electrical forces, aiding in the comprehension of real natural phenomena, and predicting the behavior of objects in such contexts:

$$\ddot{x}(t) + \frac{kqQ}{m}\left(\frac{1}{2R^3}x(t) + \frac{9}{16R^5}x(t)^3 + \cdots\right) = 0 \tag{1}$$

where $x(t), \ddot{x}(t)$the displacement and time dependent acceleration of a particular body from its equilibrium position over time, the kqQ/R^2term represents the electrical force between two charged bodies, where: k is the Coulomb's constant. And the last two terms introduced as a primary and secondary terms components of the electrical forces between the two bodies respectively.

The second term$\frac{kqQ}{m}\frac{1}{2R^3}x(t)$: the linear restoring force (attractive force), similar as Hooke's law.

The third term $\frac{kqQ}{m}\frac{9}{16R^5}x(t)^3$: the nonlinear correction to the restoring force

.

The Homotopy perturbation (HP) method is one of the most significant methods that considered as a universal method for solving non-linear partial differential equations (DF) of various kinds. It investigates the analytic solution by an infinite series which using a phony parameter p. Furthermore, HPM does not require any small parameter in the equation as that of other iteration methods but it has a parameter p∈[0,1] (a small parameter) so it catches a significant advantage. Otherwise, it is the first time used to solve our interested problem in our known, that provide a new look on this problem and test other method on competing to provide accurate solution. This method shows a great efficiency in providing accurate solutions to partial DF in some physical systems [2]. Biazar and Ghazvini [3] studied the efficient valid condition for convergence of the HP method, and provide a test of this condition for the three well-known problems of Burgers' equation [2], the 4th variable coefficients parabolic, and Schrödinger partial DF [3]. Researchers found that the analytical approximation to the solutions is reliable and confirms the power and ability of the HP method as an easy device for computing the solution of partial DF. The examination of convergence conditions for DF, integral equations, integrodifferential equations, and their systems operating the HP method, is also developed [4]. The (HP) Transform Method is used to solve the nonlinear RLC circuit equation [5]. This paper proposes three third-order, sixth-order, and seventh-order iterative methods, respectively, to solve nonlinear equations using the modified HP technique attached with the equations of the system [6]. It also provides approximate and exact solutions to the nonlinear Burger's equation by means of (HP) method [7]. The (HP) method predicts an approximate analytical description for the nonlinear Schrödinger equation with a harmonic oscillator. Accordingly, the nonlinear Schrödinger equation was presented in one and two dimensions to demonstrate the effects of the harmonic oscillator on the behavior of the wave function [8]. Laplace–Adomian decomposition method (LDM) is hold for describing charge oscillation and it provides a good sense.

This study is unique since it's the first for providing a particular description and solution for nonlinear serious harmonic charged oscillation using the significant HP method. Also, it instructs typical model procedure to demonstrate a real serious nonlinear harmonic oscillation of charges. Otherwise it provides us with regular mathematical variables as control switch buttons to recognize and understand the change in physical properties of the wave motion such amplitude and frequency variance. Finally this method significantly shortens a lot of mathematical computation, calculation and provides a high degree of accuracy and efficiency. This paper is settled as follows: section 2 provides the Basic idea for HP method, section 3 demonstrates result and analysis, section 4 declares the conclusion.

2. Basic Idea of HP Method

Here we work to clarify the basic idea of HP Method. So let us suppose the following differential equation.

$$A(u) - f(r) = 0, \qquad r \in \Omega \tag{2}$$

The boundary conditions.

$$B\left(u, \frac{\partial u}{\partial r}\right) = 0, \qquad r \in \Gamma \tag{3}$$

(A) is a general calculus operator, *(B)* is a parametric operator, (Ω) is a domain bound, and *(f(r))* is a well-known analytic function. The linear part and the non-linear part can be obtained by decomposing the operator (A) into two parts, become eq (2):

$$L(u) + N(u) - f(r) = 0 \tag{4}$$

where *L(u)* linear and *N(u)* non-linear. If a nonlinear equation does not contain (slight parameter), we can assume an artificial parameter in eq (2) [4].

$$L(u) + p\big(N(u)\big) - f(r) = 0 \tag{5}$$

where p∈[0,1] is an artificial parameter, Equation (4) can be indicated as a series of power p [9].

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \cdots \tag{6}$$

When p→1 in eq (5) then the approximate solution is obtained as eq (4). The adjustment parameter is quiet.

3. Result and Analysis

The equation

$$\ddot{x}(t) + \frac{kqQ}{m}\left(\frac{1}{2R^3}x(t) + \frac{9}{16R^5}x(t)^3 + \cdots\right) = 0$$

The initial conditions

$x(0) = 1$, $x(0)' = 0$

$$\ddot{x}(t) + \left(\frac{kqQ}{2mR^3}x(t) + \frac{9kqQ}{16mR^5}x(t)^3\right) = 0$$

But $\frac{\mathrm{kqQ}}{2\mathrm{mR}^2} =$ constant and $\frac{9\mathrm{kqQ}}{16\mathrm{mR}^2} =$ constant, so we can assume.

$$\frac{kqQ}{2mR^3} = C$$

And

$$\frac{9kqQ}{16mR^5} = D$$

To become the equation

$$\ddot{x}(t) + C\,x(t) + D\,x(t)^3 = 0 \quad (7)$$

But

$f(x) = 0$

And

$L(u) = \ddot{x}(t) + Cx(t)$ $\quad N(u) = D\,x(t)^3$

We are now using the Homotopy perturbation method, with initial condition. Using eq (5) we construct (HPM) shown below.

$$H(x,p) = (1-p)(\ddot{x} + Cx) + p(\ddot{x} + Cx + Dx^3) = 0$$

Become

$$\ddot{x} + Cx = pDx^3 \quad (8)$$

But

$$x = x_0 + px_1 + p^2x_2 + \cdots \quad (9)$$

where

$$x(t) = \lim_{p\to 1} x = x_0 + x_1 + x_2 + \cdots \quad (10)$$

Substituting eq (8) in eq (9), we get

$$\frac{\partial^2}{\partial t^2}(x_0 + px_1 + p^2x_2 + \cdots) + C(x_0 + px_1 + p^2x_2 = p(D(v_0 + pv_1 + p^2v_2 + \cdots)^3) \quad (11)$$

But

$$\begin{aligned}(v_0 + pv_1 + p^2v_2 + \cdots)^3 &= p^{12}\,v_4^3 + 3\,p^{11}\,v_3\,v_4^2 + 3\,p^{10}\,v_2\,v_4^2 + 3\,p^{10}\,v_3^2\,v_4 + p^9\,v_3^3 \\ &+ 3\,p^9\,v_1\,v_4^2 + 6\,p^9\,v_2\,v_3\,v_4 + 3\,p^8\,v_2\,v_3^2 + 3\,p^8\,v_0\,v_4^2 \\ &+ 3\,p^8\,v_2^2\,v_4 + 6\,p^8\,v_1\,v_3\,v_4 + 3\,p^7\,v_1\,v_3^2 + 3\,p^7\,v_2^2\,v_3 \\ &+ 6\,p^7\,v_1\,v_2\,v_4 + 6\,p^7\,v_0\,v_3\,v_4 + p^6\,v_2^3 + 3\,p^6\,v_0\,v_3^2 \\ &+ 6\,p^6\,v_1\,v_2\,v_3 + 3\,p^6\,v_1^2\,v_4 + 6\,p^6\,v_0\,v_2\,v_4 + 3\,p^5\,v_1\,v_2^2 \\ &+ 3\,p^5\,v_1^2\,v_3 + 6\,p^5\,v_0\,v_2\,v_3 + 6\,p^5\,v_0\,v_1\,v_4 + 3\,p^4\,v_0\,v_2^2 \\ &+ 3\,p^4\,v_1^2\,v_2 + 6\,p^4\,v_0\,v_1\,v_3 + 3\,p^4\,v_0^2\,v_4 + p^3\,v_1^3 \\ &+ 6\,p^3\,v_0\,v_1\,v_2 + 3\,p^3\,v_0^2\,v_3 + 3\,p^2\,v_0\,v_1^2 + 3\,p^2\,v_0^2\,v_2 \\ &+ 3\,p\,v_0^2\,v_1 + v_0^3\end{aligned}$$

We compare both sides of the equation with equal the power of p, we have.

.

$$p^0: \frac{\partial^2 x_0}{\partial t^2} + C\, x_0 = 0 \quad (12)$$

$$p^1: \frac{\partial^2 x_1}{\partial t^2} + C\, x_1 = -D\, x_0^3 \quad (13)$$

$$p^2: \frac{\partial^2 x_2}{\partial t^2} + C\, x_2 = 3D\, x_0^2\, x_1 \quad (14)$$

$$p^3: \frac{\partial^2 x_3}{\partial t^2} + C x_3 = -D(3\, x_0^2\, x_2 + 3\, x_0\, x_1^2) \quad (15)$$

Now, we involve Laplace Transform ***(LT).*** *LT* reformulate differential equations into more resolvable forms, and overcoming complexity of especially with obtaining initial conditions. By transferring derivatives to a frequency domain, solutions could be demonstrated by standard algebraic treatment which are generally straightforward and easier to solve. Once the algebraic equation is solved, the inverse *LT* is used to recover the original time-dependent solution.

Solve the prior differential equations (1,10,11) using the *LT* to equations from [12-15]. Solve equation 12 using *LT*.

$$\frac{\partial^2 x_1}{\partial t^2} + C\, x_1 = 0 \quad (16)$$

$$x_0(t) = A\cos\left(t\sqrt{C}\right) + B\sin\left(t\sqrt{C}\right) \quad (17)$$

$$but\ x_0(0) = 1\,, A = 1$$
$$and\ x_0(0)' = 0\,, \qquad B = 0$$

$$x_0 = \cos(t\sqrt{C}) \quad (18)$$

Solve equation 13 using *LT*.

$$\frac{\partial^2 x_1}{\partial t^2} + C\, x_1 = -D\, x_0^3 = -D\, \cos^3\left(t\sqrt{Z}\right) \quad (19)$$

$$x_1(t) = D\mathcal{L}^{-1}\left[\frac{1}{s^2 + C}\mathcal{L}\left[\cos^3\left(t\sqrt{Z}\right)\right]\right] \quad (20)$$

$$x_1 = \frac{D}{32C}(\mathrm{Cos}(\sqrt{C}t) + \mathrm{Cos}(3\sqrt{C}t) - 12\sqrt{C}t\mathrm{Sin}(5\sqrt{C}t)) \quad (21)$$

Solve equation 14 using *LT*.

$$\frac{\partial^2 x_2}{\partial t^2} + C\, x_2 = 3D\, x_0^2\, x_1 \quad (22)$$

$$x_2(t) = -\frac{3D^2}{32\,C}\mathcal{L}^{-1}\left[\frac{1}{s^2 + C}\mathcal{L}\left[\ \mathrm{Cos}\left(\sqrt{C}t\right)^3 + cos\left(t\sqrt{C}\right)^2\mathrm{Cos}(3\sqrt{C}t) - 12\sqrt{C}tcos\left(t\sqrt{C}\right)^2\mathrm{Sin}(3\sqrt{C}t)\right]\right] \quad (23)$$

$$x_2(t) = -\frac{3D^2}{32\,C}\mathcal{L}^{-1}\left[\frac{1}{s^2 + C}\left(\left(24\,C\,s\frac{(21C^2 + 6s^2 + s^4)}{(C + s^2)^2(9C + s^2)^2}\right) + \frac{s(7C + s^2)}{(C + s^2)(9C + s^2)} + \frac{1}{4}s\left(\frac{2}{9C + s^2} + \frac{1}{25C + s^2} + \frac{1}{C + s^2}\right)\right)\right] \quad (24)$$

$$x_2 = \frac{D^2}{1024\,C^2}\left[-72\,t^2\,C\cos\left(t\sqrt{C}\right) + 96\,t\,\sqrt{C}\sin\left(t\sqrt{C}\right) - 36\,t\,\sqrt{C}\sin\left(3t\sqrt{C}\right) + 23\cos\left(t\sqrt{C}\right) - 24\cos\left(3t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right] \quad (25)$$

Solve equation 15 using *LT*.

$$\frac{\partial^2 x_3}{\partial t^2} + Cx_3 = -3D(\, x_0^2\, x_2 + \, x_0\, x_1^2) \tag{26}$$

$$\begin{aligned}
x_3(t) = -\frac{D^3}{C^3}\Bigg[&\frac{69}{32768}\left(12t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(3t\sqrt{C}\right)\right) \\
&- \frac{3}{4096}\left(12\,t\sin\left(t\sqrt{C}\right) + 7\cos\left(t\sqrt{C}\right) - 6\cos\left(t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right) \\
&+ \frac{1}{65536}\left(11\cos\left(t\sqrt{C}\right) - 6\cos\left(5t\sqrt{C}\right) - 4\cos\left(5t\sqrt{C}\right) - \cos\left(7t\sqrt{C}\right)\right) \\
&+ \frac{9}{4096}\left(-8t^2C\cos\left(t\sqrt{C}\right) + 8t\sqrt{C}\sin\left(t\sqrt{C}\right) - 4t\sqrt{C}\sin\left(3t\sqrt{C}\right)\right. \\
&\left.+ 3\cos\left(t\sqrt{C}\right) - 3\cos\left(3t\sqrt{C}\right)\right) \\
&- \frac{27}{32768}\left(32\,t^3\,C^{\frac{3}{2}}\sin\left(t\sqrt{C}\right) + 48\,t^2\,C\cos\left(t\sqrt{C}\right) - 8t^2C\cos\left(3t\sqrt{C}\right)\right. \\
&- 48\,t\sqrt{C}\sin\left(t\sqrt{C}\right) + 12t\sqrt{C}\sin\left(3t\sqrt{C}\right) - 7\cos\left(t\sqrt{C}\right) \\
&\left.+ 7\cos\left(3t\sqrt{C}\right)\right) \\
&- \frac{3}{32768}\left(-72\,t^2C\cos\left(t\sqrt{C}\right) + 72t\sqrt{C}\sin\left(t\sqrt{C}\right) - 72t\sqrt{C}\sin\left(3t\sqrt{C}\right)\right. \\
&\left.- 12\,t\sqrt{C}\sin\left(5t\sqrt{C}\right) + 59\cos\left(t\sqrt{C}\right) - 54\cos\left(3t\sqrt{C}\right) - 5\cos\left(5t\sqrt{C}\right)\right) \\
&+ \frac{1}{16384}\left(-32\,t^3\,C^{\frac{3}{2}}\sin\left(t\sqrt{C}\right) - 48\,t^2C\cos\left(t\sqrt{C}\right)\right. \\
&- 24\,t^2C\cos\left(3t\sqrt{C}\right) \\
&+ 48t\sqrt{C}\sin\left(t\sqrt{C}\right) + 36t\sqrt{C}\sin\left(3t\sqrt{C}\right) - 21\cos\left(t\sqrt{C}\right) \\
&\left.+ 21\cos\left(3t\sqrt{C}\right)\right) \\
&- \frac{1}{16384}\left(72t^2\,C\cos\left(t\sqrt{C}\right) - 72t\sqrt{C}\sin\left(t\sqrt{C}\right) - 12t\sqrt{C}\sin\left(5t\sqrt{C}\right)\right. \\
&\left.+ 5\cos\left(t\sqrt{C}\right) - 5\cos\left(5t\sqrt{C}\right)\right) \\
&- \frac{1}{16384}\left(36\,t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right) \\
&+ \frac{9}{16384}\left((-8)t^2C\cos\left(t\sqrt{C}\right)\right. \\
&\left.+ 8t\sqrt{C}\sin\left(t\sqrt{C}\right) - 4t\sqrt{C}\sin\left(3t\sqrt{C}\right) + 3\cos\left(t\sqrt{C}\right) - 3\cos\left(3t\sqrt{C}\right)\right) \\
&+ \frac{3}{32768}\left(12t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(3t\sqrt{C}\right)\right) \\
&+ \frac{1}{65536}\left(48t\sqrt{C}\sin\left(t\sqrt{C}\right) + 3\cos\left(t\sqrt{C}\right) - 2\cos\left(5t\sqrt{C}\right)\right. \\
&\left.- \cos\left(7t\sqrt{C}\right)\right)\Bigg]
\end{aligned} \tag{27}$$

We solve the equation using Equation $\mathrm{x(t)} = \lim_{\mathrm{p}\to 1} \mathrm{x} = \mathrm{x}_0 + \mathrm{x}_1 + \cdots$

.

$$
\begin{aligned}
x(t) = {} & \cos\left(t\sqrt{C}\right) - \frac{D}{32C}\left(12t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(3t\sqrt{C}\right)\right) \\
& + \frac{D^2}{1024\,C^2}\left[-72\,t^2\,C\cos\left(t\sqrt{C}\right) + 96\,t\,\sqrt{C}\sin\left(t\sqrt{C}\right)\right. \\
& \left. - 36\,t\,\sqrt{C}\sin\left(3t\sqrt{C}\right) + 23\cos\left(t\sqrt{C}\right) - 24\cos\left(3t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right] \\
& - \frac{D^3}{C^3}\left[\frac{69}{32768}\left(12t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(3t\sqrt{C}\right)\right)\right. \\
& - \frac{3}{4096}\left(12\,t\sin\left(t\sqrt{C}\right) + 7\cos\left(t\sqrt{C}\right) - 6\cos\left(t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right) \\
& + \frac{1}{65536}\left(11\cos\left(t\sqrt{C}\right) - 6\cos\left(5t\sqrt{C}\right) - 4\cos\left(5t\sqrt{C}\right) - \cos\left(7t\sqrt{C}\right)\right) \\
& + \frac{9}{4096}\left(-8t^2C\cos\left(t\sqrt{C}\right) + 8t\sqrt{C}\sin\left(t\sqrt{C}\right) - 4t\sqrt{C}\sin\left(3t\sqrt{C}\right)\right. \\
& \left. + 3\cos\left(t\sqrt{C}\right) - 3\cos\left(3t\sqrt{C}\right)\right) \\
& - \frac{27}{32768}\left(32\,t^3\,C^{\frac{3}{2}}\sin\left(t\sqrt{C}\right) + 48\,t^2\,C\cos\left(t\sqrt{C}\right) - 8t^2C\cos\left(3t\sqrt{C}\right)\right. \\
& - 48\,t\sqrt{C}\sin\left(t\sqrt{C}\right) + 12t\sqrt{C}\sin\left(3t\sqrt{C}\right) - 7\cos\left(t\sqrt{C}\right) \\
& \left. + 7\cos\left(3t\sqrt{C}\right)\right) \\
& - \frac{3}{32768}\left(-72\,t^2C\cos\left(t\sqrt{C}\right) + 72t\sqrt{C}\sin\left(t\sqrt{C}\right) - 72t\sqrt{C}\sin\left(3t\sqrt{C}\right)\right. \\
& \left. - 12\,t\sqrt{C}\sin\left(5t\sqrt{C}\right) + 59\cos\left(t\sqrt{C}\right) - 54\cos\left(3t\sqrt{C}\right) - 5\cos\left(5t\sqrt{C}\right)\right) \\
& + \frac{1}{16384}\left(-32\,t^3\,C^{\frac{3}{2}}\sin\left(t\sqrt{C}\right) - 48\,t^2C\cos\left(t\sqrt{C}\right)\right. \\
& - 24\,t^2C\cos\left(3t\sqrt{C}\right) \\
& + 48t\sqrt{C}\sin\left(t\sqrt{C}\right) + 36t\sqrt{C}\sin\left(3t\sqrt{C}\right) - 21\cos\left(t\sqrt{C}\right) \\
& \left. + 21\cos\left(3t\sqrt{C}\right)\right) \\
& - \frac{1}{16384}\left(72t^2\,C\cos\left(t\sqrt{C}\right) - 72t\sqrt{C}\sin\left(t\sqrt{C}\right) - 12t\sqrt{C}\sin\left(5t\sqrt{C}\right)\right. \\
& \left. + 5\cos\left(t\sqrt{C}\right) - 5\cos\left(5t\sqrt{C}\right)\right) \\
& - \frac{1}{16384}\left(36\,t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(5t\sqrt{C}\right)\right) \\
& + \frac{9}{16384}\left((-8)t^2C\cos\left(t\sqrt{C}\right)\right. \\
& \left. + 8t\sqrt{C}\sin\left(t\sqrt{C}\right) - 4t\sqrt{C}\sin\left(3t\sqrt{C}\right) + 3\cos\left(t\sqrt{C}\right) - 3\cos\left(3t\sqrt{C}\right)\right) \\
& + \frac{3}{32768}\left(12t\sqrt{C}\sin\left(t\sqrt{C}\right) + \cos\left(t\sqrt{C}\right) - \cos\left(3t\sqrt{C}\right)\right) \\
& + \frac{1}{65536}\left(48t\sqrt{C}\sin\left(t\sqrt{C}\right) + 3\cos\left(t\sqrt{C}\right) - 2\cos\left(5t\sqrt{C}\right)\right. \\
& \left.\left. - \cos\left(7t\sqrt{C}\right)\right)\right] .
\end{aligned}
\tag{28}
$$

Significantly, these partition solutions agree completely one hundred percent with those provides using LDM [11]. The four particular solutions are demonstrated in figure 1. The figure shows a convergence of four mathematical functions. Each curve corresponds to a distinct equation, and their interactions weave a intricate tapestry. By varying the "t" variable from 0 to 10, the graph unveils the dynamic behavior of these equations. The contours of the curves, their points of intersection, and the overarching patterns offer valuable insights into the functions' behaviors and interconnections. However, the first portion of solution x_0; shows a behavior of simple harmonic motion where the amplitude, wave length and periodic time all are constant. Otherwise, this state presents the stability of atom, where no nonlinear term (physically no additional external field) appears.

The second part of solution x1, shows growing in the wave amplitude as the time increase. Also, it shows a slight decrease in periodic time.

The third part of solution x_2, also shows growing in the wave amplitude and a recognizable decrease in periodic time as the time increase.

The effect of the fourth part of solution is very small and doesn't provide a recognizable contribution on the general solution.

Lastly, all nonlinear parts of solutions (x_1, x_2 and x_3) show differ from the linear x_0 solution with about (π/2) phase shift.

On the other hand, figure 2, demonstrates the total solution where the four partition solutions construct to investigate the charge oscillation as a function of time.

The chart illustrates a intricate physical system or phenomenon, showcasing prominent characteristics such as oscillations with fluctuations in both amplitude and frequency. These oscillations hint at the system's periodic behavior. The intricacy of the graph implies the possible involvement of multiple interacting components or forces within the system. The variations in amplitude and frequency of the oscillations may suggest a non-linear or chaotic nature of the system.

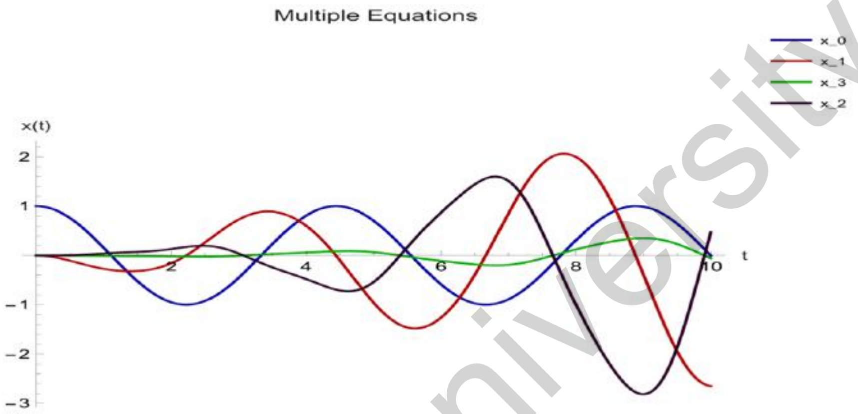

Fig. 1. The relation between the partition solutions as a function of time.

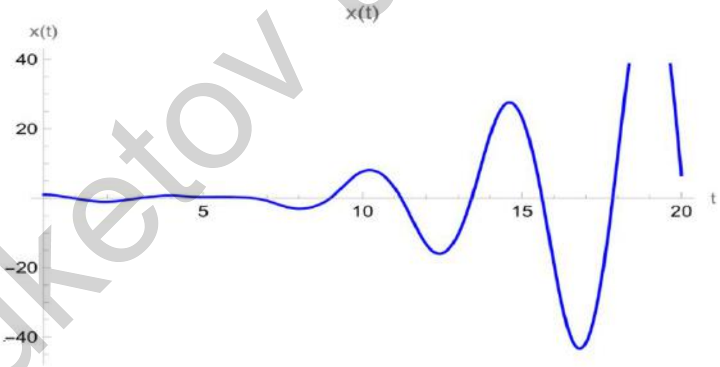

Fig.2. The total solution x(t) as a function of time.

4. Conclusion

We encountered intricacies and demonstrating explicitly the non-linear harmonic oscillation for charged particles affected by electrical field. The HP method is utilized to involve initially the initial four boundaries before progressing to ascertain the comprehensive solution. The HP method with a significant accuracy investigate physical properties of the wave function and demonstrate regular mathematical variables as control switch buttons to recognize and understand the change in physical properties of the wave motion such amplitude and frequency variance. This complex dynamic model which expresses a nonlinear oscillator where the restoring force is not purely linear but involving a cubic term, is demonstrating analytically and graphically. The presence of the linear term displays the predatory in the motion while the cubic term provides the system's stiffness changes with displacement. Otherwise, the result provides an identical partition solutions comparing to those provide by LDM. The outcomes proved to be noteworthy, as illustrated by the conclusive graphs and sufficient solution.

Conflict of interest statement
The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

CRediT author statement
Bashar and Emad: Conceptualization, Data Curation, Writing Original Draft; Bashar and Omar: Methodology, Investigation; Abdulrahman and Emad.: Writing Review & Editing, Supervision.
The final manuscript was read and approved by all authors.

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