UDK 510.6

J.A.Tussupov[1], K.J.Jetpisov[2], A.J.Satekbaeva[1], L.M.Jalmagambetova[2]

[1]*Eurasian National University named after L.N.Gumilyov, Astana;*
[2]*Karaganda State University named after E.A.Buketov (E-mail: lauritta_88_@mail.ru)*

Definability a family of sets in the graph

We constructed a method of definability a family of sets to the oriented graph. Using this method we proved next results: Let *A* structure of the signature σ of the 2 nilpotent group. For each computable ordinal α there is a computable structure *A* of the signature σ that is Δ^0_α categorical but not relatively Δ^0_α (and without formally $\sum^0_\alpha$ Scott family). For each computable ordinal a there is a computable structure *A* of the signature σ that is R with a relation that is intrinsically $\sum^0_\alpha$ but not relatively intrinsically $\sum^0_\alpha$ on *A*.

Key words: the oriented graph, structure of the signature, computable function, isomorphic structures, a completely univalent functor.

A necessity background may be found in Ash, Knight [1]. In the [2] was proved next theorem:

Theorem 1. for $a = \omega$. *Let* $f \in \omega^\omega$ *be a strictly increasing computable function with values* ≥ 1. *There is a family* $S \subseteq \omega^\omega$ *such that:*

1. *S is discrete but not ω- discrete,*
2. *S has a unique anti-Friedberg* f *—* ω*-enumeration, up to strong* Δ^0_α *— equivalence.*

For the proof the definability we construct completely univalent functor from category of one structure to the category of other structure.

Definition 1. Let σ arbitrary signature, a category Mod^σ is a pair $\langle Ob^\sigma, Mor^\sigma\rangle$ where Ob^σ is the set of all structures of given signature and Mor^σ is a set of all morphisms between any pair structures from Ob^σ.

2. A *functor F* from category the $\Re$ to the category $\Im$ is the pair (F_0, F_M) maps F_0, F_M such that

1) $F_0 : Ob(\Re) \to Ob(\Im)$,

2) $F_M : Mor(\Re) \to Mor(\Im)$ with next properties:

a) $\forall\varphi[\varphi \in Mor(A,B) \Rightarrow F_M(\varphi) \in Mor(F_0(A), F_0(B))]$;

b) $\forall A \in Ob(\Re)[F_M(1_A) = 1_{F_0(A)}]$;

c) $\forall\varphi\psi[\varphi\psi \in Mor(\Re) \wedge rang(\varphi) = dom(\psi) \Rightarrow F_M(\psi\cdot\varphi) = F_M(\psi)\cdot F_M(\varphi)]$.

3. A functor *F* is *completely univalent* if the maps F_0 and F_M are bijections.

Let $S = \left\{\varphi_i^{\Delta^0_\alpha} : i \in \omega\right\}$ is a family of partial computable functions with Δ^0_α— oracle. We will construct a method of definability this family in the oriented graph,

The first author gratefully acknowledge support of the grant «Methods of the formal specifications and their applications for the design the the programs» and second and third authors gratefully acknowledge support of the grant «Models and Computability» Ministry of Education and Science in particularly in the bipartite graph.

Proposition 1. *Let S is a family of partial computable functions with* Δ0- *oracle. Then there is a bipartite structure* S_0 *of the signature* $< J^2(x, y) >$ *such that there is algorithm of the construction computable numeration* μ_v *of the structure* S_0 *by the computable numeration v of the S, and the following conditions hold*

1) numeration $v_{\mu o}$ *is not d-autoequivalent to* v_μ, *iff numeration* μ_0 *and* μ_1 *is not d-autoequivalent;*

2) for any numeration μ of the structure S_0 *there is the numeration v of the structure*
S such that μ and μ_v *are d-autoequivalent.*

Proof: Let *S* is a family of partial computable functions with Δ0 — oracle. We construct the structure S_0 of the signature $\sigma_0 =< J^2(x, y) >$. We eliminate sets *L*1, L_2 for the according indeces of other sets. The construction: $S_0 = L_0 \cup L_3 \cup L_4 \cup M_1 \cup M_2 \cup M_3 \cup M_4 \cup Q$ where

$$L_i^j = \left\{l_i^j : i \in \omega\right\}, \text{for } j = 0, 3, 4 \text{ and } M_i^j = \left\{m_i^j : i \in \omega\right\}, \text{ for } j = 1,2,3,4.;\ Q = \left\{q_{(i,j)} : i, j \in \omega\right\},$$

where (i, j) is the Cantor's function for the numeration the pair of natural number. We will construct next connections between sets:

$$L_0 \to M_1 \leftarrow Q;$$
$$L_0 \to M_2 \leftarrow Q;$$
$$L_3 \to M_3 \leftarrow L_4 \to M_4 \leftarrow Q.$$

We define the predicate *J (x, y)* on the set *S₀* in the following way: For any *i, j Eω* we have the next conditions:

1. $L_0 \to M_1$;

a) $J(l_i^0, m_i^1)$, b) $J(l_{i+1}^0, m_i^1)$, c) $J(l_1^0, m_i^1)$, d) $J(l_3^0, m_i^1)$.

2. $L_0 \to M_2$;

a) $J(l_i^0, m_i^2)$, b) $J(l_0^0, m_i^2)$, c) $J(l_2^0, m_i^2)$.

3. $Q \to M_2$; $J(q_{(i,j),} m_i^2)$.

4. $L_3 \to M_3$;

a) $J(l_i^3, m_i^3)$,

b) $J(l_i^3, m_{i+1}^3)$.

5. $L_4 \to M_3$; $J(l_i^4, m_i^3)$.

6. $L_4 \to M_4$;

a) $J(l_i^4, m_i^4)$,

b) $J(l_{i+1}^4, m_i^4)$,

c) $J(l_0^4, m_1^4)$.

7. $Q \to M_4$; $J(q_{(i,j),} m_j^4)$.

8. $Q \to M_1$; If $\varphi_i(j){\downarrow} \downarrow$ then $S_0 \mathrm{I} = J(q_{(i,j)}, m_i^1)$.

Construction is complete.

Lemma 1. The sets *L₀, L₃, L₄, M₁, M₂, M₃, M₄, Q* are definable by the 3-formulas.

Proof. We define the next auxiliary formulas which describe some cycles:

$$Z_4(x,y,z,w) \leftarrow J(x,y) \wedge J(z,w) \wedge J(x,w) \wedge J(z,y) \text{ and it doesn't have proper cycles;}$$

$$Z_4(x) = \exists y \exists z \exists \omega (Z_4(x,y,z,\omega) \vee Z_4(y,x,z,\omega));$$

$$Z_6(x,y,z,w,r,s \leftarrow J(x,y) \wedge J(z,y) \wedge J(x,w) \wedge J(z,s) \wedge J(r,s) \text{ and it doesn't have proper cycles;}$$

$$Z_6(x) = \exists y_0 \exists z_0 \exists z_1 \exists \omega_0 \exists \omega_1 (Z_6(x, y_0, z_0, \omega_0, z_1, \omega_1) \vee Z_6(y_0, x, z_0, \omega_0, z_1, \omega_1));$$

We define some elements and sets:

$$\Phi_0(x) = \exists y \exists \upsilon_0 \exists \upsilon_1 \exists \upsilon_2 \exists \upsilon_3 (\underset{i \neq j}{\wedge} (\upsilon_i \neq \upsilon_j) \wedge \underset{i \neq j}{\wedge} Z_4(x, \upsilon_i, y, \upsilon_j));$$

$$\Phi_0(S_0) = \left\{ l_0^0, l_1^0, l_2^0, l_3^0 \right\};$$

$$\Phi_1(x) = \exists x_0 \exists \upsilon_0 \exists \upsilon_1 \exists \upsilon_2 (x_0 \neq x \wedge \underset{i \neq j}{\wedge} (\upsilon_i \neq \upsilon_j) \wedge \underset{\leq i \leq 2}{\wedge} \Phi_0(\upsilon_i) \wedge \underset{\leq i \leq 2}{\wedge} (J(x_0, \upsilon_i) \wedge J(x, \upsilon_i)));$$

$$\Phi_1(S_0) = \left\{ m_1^1, m_2^1 \right\};$$

$$\Phi_2(\mathrm{x}) \exists \mathrm{y}_0 \exists y_1 \exists y_2 \exists \upsilon_0 \exists \upsilon_1 (\underset{i \neq j}{\wedge} (y_i \neq y_j) \wedge \upsilon_0 \neq \upsilon_1 \wedge \underset{0 \leq i \leq 1}{\wedge} \Phi_1(\upsilon_\mathrm{i}) \wedge \underset{0 \leq \mathrm{i} \leq 2}{\wedge} \Phi_0(\mathrm{y}_\mathrm{i}) \wedge$$
$$\wedge Z_4(y_0, x, y_1, \upsilon_0) \wedge Z_4(y_0, x, y_1, \upsilon_1) \wedge \underset{0 \leq i \leq 2}{\wedge} J(y_i, x));$$

$$\Phi_2(\mathrm{S}_0) = \left\{ m_1^2 \right\};$$

$$\Phi_3(\mathrm{x,y}) = \exists y_0 \exists y_1 \exists y_2 \exists x_0 \exists x_1 (\underset{i \neq j}{\wedge} (y_i \neq y_j) \wedge x_0 \neq x_1 \wedge \underset{0 \leq i \leq 1}{\wedge} ((x \neq x_i) \wedge \Phi_0(x_i)) \wedge$$
$$\wedge \Phi_0(\mathrm{x}) \wedge \Phi_2(y_0) \wedge \underset{1 \leq i \leq 2}{\wedge} \Phi_1(\mathrm{y}_\mathrm{i}) \wedge \mathrm{Z}_4(x_0, y_0, x, y) \wedge Z_4(x_0, y_0, x_1, y_1) \wedge Z_4(x_0, y_0, x_1, y_2));$$

$$\Phi_3(S_0)=\left\{(l_0^0,m_1^0\right\};\ \Phi_4=\exists y\Phi_3(x,y);$$

$$\Phi_4(x,y)=\exists y\Phi_3(x,y);$$

$$\Phi_5(x)=\exists y\Phi_3(y,x);$$

$$\Phi_5(x)=\exists y\Phi_3(x,y);$$

We define sets by the definable elements:

$$M_1(x)=J(l_1^0,x)\wedge J(l_3^0,x);$$

$$M_2(x)=J(l_0^0,x)\wedge J(l_2^0,x);$$

$$L_0(x)=\Phi_0(\mathrm{x})\vee\exists y\exists z(M_2(y)\wedge Z_6(l_1^0,m_1^2,l_2^0,y,x,z));$$

Now, we consider definable elements from the set L_3 U M_3 U L_4 U M_4.

$$\Phi_{\mathrm{l}_0^3}(x)=\exists x_0\exists x_1\exists y_0\exists y_1\exists z_0\exists z_1\underset{0\le i\le 1}{\wedge}Z_6(x,x_0,y_0,z_i,y_1,x_1);$$

$$\varphi_{m_1^3,l_1^4}(x)=\exists x_0\exists x_1\exists y_0\exists y_1\exists z_0\exists z_1\exists w_0\exists w_1(x_0\ne x_1\wedge y_0\ne y_1\wedge z_0\ne z_1\wedge$$

$$\wedge w_0\ne w_1\wedge\Phi_{\mathrm{l}_0^3}(w_0)\wedge Z_6(w_0,x_0,y_0,z_0,y,x)\wedge Z_6(w_1,x,y,z_1,y_1,x_1));$$

$$\Phi_{\mathrm{m}_1^3}(x)=\exists y\Phi_{\mathrm{m}_1^3,l_1^4}(x,y);$$

$$L_4(x)=\exists x_0\exists x_1\exists x_2\exists z_0\exists z_1\exists w(\underset{i\ne j}{\wedge}(x_i\ne x_j)\wedge Z_6(w,x_0,x,x_1,z_0,z_1)\wedge\underset{0\le i\le 2}{\wedge}J(x_i,x));$$

$$M_4(x)=\exists x_0\exists x_1\exists x_2\exists x_3\exists y_0\exists y_1\exists w(\underset{i\ne j}{\wedge}(x_i\ne x_j)\wedge Z_6(w,y_0,x_0,x,x_1,y_1)\wedge\underset{0\le i\le 3}{\wedge}J(x,x_i));$$

$$M_3(x)=\exists x_0\exists x_1\exists w\exists y\exists z\exists w(L_4(x_0)\wedge L_4(x_1)\wedge M_4(w)\wedge Z_6(z,x,x_0,w,x_1,y));$$

$$L_3(x)=\exists y_0\exists y_1\exists z_0\exists z_1\exists w(M_3(y_0)\wedge M_3(y_1)\wedge L_4(z_0)\wedge L_4(z_1)\wedge M_4(w)\wedge Z_6(x,y_0,z_0,w,z_1,y_1));$$

$$Q(x)=\exists y\exists z(M_2(y)\wedge M_4(z)\wedge J(x,y)\wedge J(x,z)).$$

We define some formulas for the definition a function-successor $Succ(x)$ by induction:

Let $\psi_0(x,y,z)=\Phi_{\mathrm{l}_0^3}(x)\wedge\Phi_{\mathrm{m}_0^3}(y)\wedge\Phi_{\mathrm{m}_1^3}(z);$

$$\psi_{k+1}(x,y,z)=\exists x_0\exists y_0\exists z_0\exists z_1\exists z_2\exists w_0\exists w_1(\underset{i\ne j}{\wedge}z_i\ne z_j\wedge\underset{0\le i\le 3}{\wedge}L_4(z_i)\wedge Z_6(x_0,y_0,z_0,w_0,z_1,y)\wedge$$

$$Z_6(x,y,z_1,w_1,z_2,z)\wedge\psi_k(x_0,y_0,y));$$

Then we define a function-successor $Succ(x)$ such that $k=0\Leftrightarrow x=l_0^3;$

$$Succ(k)=k+1\Leftrightarrow S_0\perp=\exists y\exists z\psi_{k+1}(x,y,z).$$

So, we coded structure $N=\langle N,0,Succ(x)\rangle$ in the substructure $S_0^*=\langle S_0^*,J(x,y)\rangle$ of the structure S_0 where $S_0^*=(L_3\cup M_3\cup L_4\cup M_4)$.

Lemma 2. If S_0 is a computable structure then $L_0,L_3,L_4,M_1,M_2,M_3,M_4,Q$ are computable sets.

Proof. It follow from the partition the computable set $|S_0|$ by the sets $L_0,L_3,L_4,M_1,M_2,M_3,Q$ which $\exists$— definable.

Lemma 3. The map from $I_z(S,R)$ to $I_z(S_0,R_0)$ is a bijection.

Proof. Let structure $S=\left\{\varphi_i^{\Delta_\alpha^0}:i\in\omega\right\}$. When we constructed structure S_0 by given structure S by the next manner $i\to l_i^0$ for every index $i\in\omega$ and Let s^0 is sign of the element from $|S_0|$ then

$$s_i^0=\begin{cases} l_k^0, & if \quad i=8*k; \\ m_k^1, & if \quad i=8*k+1; \\ l_k^3, & if \quad i=8*k+2; \\ m_k^2, & if \quad i=8*k+3; \\ l_k^4, & if \quad i=8*k+4; \\ m_k^3, & if \quad i=8*k+5; \\ m_k^4, & if \quad i=8*k+6; \\ q_k, & if \quad i=8*k+7. \end{cases}$$

Let S and R are isomorphic structures and φ — isomorphism. We define isomorphism φ^* from S_0 to R_0, which induced by φ by the next manner:

$$\varphi^*(l_i^k) = l_{\varphi(i)}^k, \quad k = 0, \quad 3, \quad 4; \quad \varphi^*(m_i^j) = m_{\varphi(i)}^j; j = 1, ...4 \quad \varphi^*(q_{(i,j)} = q_{(\varphi(i),j)}).$$

Let S_0 and R_0 are isomorphic structures. Let φ^* a map from S_0 to R_0 which a isomorphism of the numerated structures Let $f_0 : \omega \to L_0^{S_0}$ and $f_1 : \omega \to L_0^{R_0}$ are numeration sets $L_0^{S_0}$ and $L_0^{R_0}$ respectively and $f_1(i) = \varphi^*(f_0(i))$. Then the map $\varphi = f_1^{-1} \cdot \varphi^* \cdot f_0$ is the isomorphism from S to R.

Proposition 1 is proved.

Let A is a countable oriented graph with predicate $J^2(x,y)$. We will construct a method of definability this oriented graph to symmetric irreflexive graph of signature $F^2(x,y)$ with next espesial properties:

(i) for any non equal n nodes there is no more then one common node;

(ii) for any *x, y* such that $F(x,y)$ there is no node z that $F(x,z)$ and $F(y,z)$;

(iii) for any x there are y,z such that $y \neq z$ and $F(x,y), F(x,z)$.

Proposition 2. *Let A is a oriented graph. Then there is a espesial graph A_0 of the signature $\langle F^2(x,y)\rangle$ such that there is algorithm of the construction computable numeration μ_v of the structure A_0 by the computable numeration v of the A, and the following conditions hold:*

1) numeration v_{μ_0} is not d-autoequivalent to $v_{\mu 1}$, iff numeration μ_0 and μ_1 is not d-autoequivalent;

2) for any numeration μ of the structure A_0 there is a numeration ν of the structure A such that μ and μ_v are d-autoequivalent.

Proof: Let A is a structure of the signature $\sigma = \langle J^2(x,y)\rangle$ and $A = \{i : i \in w\}$. We construct the structure A_0 of the signature $\sigma_0 = \langle F^2(x,y)\rangle$ with next espesial properties by the following way:

$A_0 = M_0 \cup M_1 \cup M_2 \cup M_3 \cup M_4$ where $M_i^j = \{M_i^j : i \in \omega\}$.

We define the predicate $F(x,y)$ on the set A_0 in the following way: For any $i, j \in \omega$ we have the next conditions:

1. $F(m_i^j, m_i^{j+1})$;
2. $F(m_i^0, m_{i+1}^1)$;
3. $F(m_0^0, m_i^1)$;
4. $F(m_1^0, m_0^1)$;
5. $F(m_i^4, m_{i+1}^3)$;
6. If $J(i,j)$ then $F(m_i^1, m_j^3)$;
7. If $\neg J(i,j)$ then $F(m_i^1, m_j^4)$.

Lemma 1. The sets M_0, M_1, M_2, M_3, M_4 are definable by the $\exists$ — formulas.

Proof. We use some cycles of length $4, 5$ and 6 under the condition that they don't have proper cycles. We define some elements and sets:

$$\Phi_{\mathrm{m}_0^0}(x) = \exists x_0 \exists x_1 \exists y_0 \exists y_1 \exists y_2 \exists y_3\ (x_0 \neq x_1 \wedge \underset{i \neq j}{\wedge}(y_i \neq y_j) \wedge Z_4(x, y_1, x_0, y_0) \wedge Z_4(x, y_1, x_0, y_2) \wedge$$

$$\wedge Z_4(x, y_2, x_1, y_3)).$$

$$\Phi_{\mathrm{m}_1^1} x) = \exists x_0 \exists x_1 \exists y (x_0 \neq x_1 \wedge Z_4(m_0^0, x, y, x_0) \wedge Z_4(m_0^0, x, y, x_1);$$

$$\Phi_{\mathrm{m}_1^0}(x) = \exists y \exists y_0 \exists y_1 (x_0 \neq x_1 \wedge Z_4(m_0^0, y, x, y_0) \wedge Z_4(m_0^0, y, x, y_1);$$

$$\Phi_{\mathrm{m}_0^1}(x) = \exists y_0 \exists y_1 \exists z_0 \exists z_1 \exists w_0 \exists w_1 (y_0 \neq y_1 \wedge z_0 \neq z_1 \wedge w_0 \neq w_1 \wedge ((Z_5(m_0^0, x, y_0, z_0, m_1^1) \wedge$$

$$\wedge Z_5(m_0^0, x, z_1, y_1, m_1^1)) \vee (Z_6(m_0^0, x, y_0, z_0, w_0, m_1^1) \wedge Z_6(m_0^0, x, w_1, z_1, y_1, m_1^1)));$$

$$M_1(x) = F(x, m_0^0);$$

$$M_0(x) = \exists y_0 \exists y_1 \exists z (M_1(y_0) \wedge M_1(y_1) \wedge Z_4(x, y_0, z, y_1));$$

$$M_2(x) = \exists y_0 \exists y_1 \exists z \exists w (M_1(y_0) \wedge M_1(y_1) \wedge (Z_5(m_0^0, y_0, x, z, y_1) \vee Z_6(m_0^0, y_0, x, z, w, y_1));$$
$$M_3(x) = \exists y_0 \exists y_1 \exists z \exists w (M_1(y_0) \wedge M_1(y_1) \wedge Z_5(m_0^0, y_0, x, z, y_1) \vee Z_6(m_0^0, y_0, x, z, w, y_1));$$
$$M_4(x) = \exists y_0 \exists y_1 (M_3(y_0) \wedge M_3(y_1) \wedge y_0 \neq y_1 \wedge F(x, y_0) \wedge F(x, y_1)).$$

So, we have sets by given property.

Lemma 2. If A_0 is a computable structure then M_0, M_1, M_2, M_3, M_4 are computable sets.

Proof. It follow from the partition the computable set |*A*0| by the sets M_0, M_1, M_2, M_3, M_4 which definable 3 formulas. So, each of these sets is computable.

Lemma 3. The map from $I_z(A,B)$ to $I_z(A_0,B_0)$ is a bijection.

Proof. Let structure $A = \{i : i \in \omega\}$. When we constructed structure A_0 by given

Structure A by the next manner $i \to m_i^0$ for every $i \in \omega$ and Let a^0 is sign of the element from $|A_0|$ then

$$a_i^0 = \begin{cases} m_k^0, & if \quad i = 8*k; \\ m_k^1, & if \quad i = 8*k+1; \\ m_k^2, & if \quad i = 8*k+2; \\ m_k^3, & if \quad i = 8*k+3; \\ m_k^4, & if \quad i = 8*k+4. \end{cases}$$

Let A and B are isomorphic structures and φ — isomorphism. We define isomorphism φ^* from A_0 to B_0, which induced by φ by the next manner: $\varphi^*(m_i^k) = m_{\varphi(i)}^k$, $k = 0,1,2,3,4$.

Let A_0 and B_0 are isomorphic structures. Let φ^* a map from A_0 to B_0 which a isomorphism of the numerated structures Let $f_0 : \omega \to M_0^{A_0}$ and $f_1 : \omega \to M_0^{B_0}$ are numeration sets $M_0^{A_0}$ and $M_0^{B_0}$ respectively and $f_1(i) = \varphi^*(f_0(i))$.. Then the map $\varphi = f_1^{-1} \cdot \varphi^* \cdot f_0$ is the isomorphism from ***A*** to ***B.***

Proposition 2 is proved.

In the [Tuss] was constructed a completely univalent functor from category of the class of computable copies of the espesial symmetric irreflexive graph to the category of the class of computable copies of the 2 nilpotent group. So, we have sequiences of functors:

F_0 from category the % to the category $<3_0$; *F*1 from category the $<3_0$ to the category $<3_1$; F_2 from category the $(S_1$ to the category $(3_2$; where # is a category of the class of computable numerations of the family *S*, $(S_0$ is a category of the class of computable copies of the oriented graph, 61 is a category of the class of computable copies of the espesial symmetric irreflexive graph, $<3_2$ is a category of the class of computable copies of the 2 nilpotent group. Then the functor *F* such that $F = F_2 \circ F_1 \circ F_0$ *is* the functor from category the # to the category $<3_2$. Tussupov used results from [3] for the proving results from [4] for the computable successor ordinals *α.*

Our method of coding allows prove next results. Let *A* structure of signature *σ* of the 2 nilpotent groups.

Theorem. *For each computable ordinal α there is a computable structure A of signature* σ_i *that is* Δ_α^0 *categorical but not relatively* Δ_α^0 *(and without formally* $\sum_\alpha^0$ *Scott family).*

Theorem 2. *For each computable ordinal α there is a computable structure A of signature* σ_1 *with additional relation R that is intrinsically* $\sum_\alpha^0$ *but not relatively intrinsically* $\sum_\alpha^0$ on *A.*

References

1 *Ash C.J., Knight J.F.* Computable Structures and the Hyperarithmetical Hierarchy. — Elsevier, 2000.

2 *Chisholm J., Fokina E.B. et al.* Intrinsic bounds on complexity and definability at limit levels // J. of Symbolic Logic. 2009. — Vol. 74. — № 3. — P. 1047–1060.

3 *Goncharov S.S.* Isomorphisms and definable relations on Computable Models, Proceeding of the Logic Colloquium. — 2005. — Athens. — P. 26–45.

4 *Tussupov J.A.* Isomorphisms, definable relation and Scott family of class 2 nilpo-tent groups. Algebra and Logic. — 2007. — Vol. 46 (4). — P. 281–286 (English translation). www.springer.com/math/algebra/journal/10469

Ж.А.Түсіпов , Қ.Ж.Жетпісов, А.Ж.Сәтекбаева, Л.М.Жалмағамбетова

Графтағы жиындар үйірінің анықталғандығы

Мақалада бағдарланған графтағы жиындар үйірінің анықталғандық әдісі ұсынылды. Бұл әдісті қолданып, келесі тұжырымдарды дәлелдеуге болады. Айталық, A 2-сатылы-нильпотентті группаның құрылымы және σ берілген группаның сигнатурасы болсын. Әрбір есептелетін α ординалы үшін σ A сигнатурасының есептелетін құрылымын табуға болады және ол Δ^0_α болады, бірақ Δ^0_α қатысты категориялы болмайды (яғни формалды емес $\sum^0_\alpha$ Скотт үйірісіз). Әрбір есептелетін α ординалы үшін σ A сигнатурасының есептелетін құрылымы табылады және оған қатынасын R қосу ішкі $\sum^0_\alpha$ болады, бірақ ол құрылымда $\sum^0_\alpha$ қатынасы A-да ішкі қатынас болып табылмайды.

Ж.А.Тусупов, К.Ж.Жетписов, А.Ж.Сатекбаева, Л.М.Жалмагамбетова

Определимость семейства множеств в графе

В статье предложен метод определимости семейства множеств в ориентированный граф. Используя этот метод, можно доказать следующие результаты. Пусть A — структура 2-ступенно-нильпотентных групп и σ — сигнатура данных групп. Для каждого вычислимого ординала α существует вычислимая структура A сигнатуры σ такая, что она Δ^0_α, но не относительно Δ^0_α категорична (т.е. без формального $\sum^0_\alpha$ семейства Скотта). Для каждого вычислимого ординала α существует вычислимая структура A сигнатуры σ такая, что добавленное отношение R является внутренне $\sum^0_\alpha$, но не относительно внутренне $\sum^0_\alpha$ на A.

УДК 517.95

Б.Х.Турметов, Н.З.Байметова

Международный казахско-турецкий университет им. Х.А.Ясави, Туркестан (E-mail: baymetovanigora@mail.ru)

О разрешимости некоторых краевых задач с оператором типа Адамара-Маршо

В настоящей работе в классе гармонических функций изучены свойства некоторых интегро-дифференциальных операторов, обобщающих операторы дробного дифференцирования Адамара и Адамара-Маршо. В качестве применения полученных свойств рассмотрены некоторые краевые задачи для уравнения Лапласа в круге.

Ключевые слова: дробное дифференцирование, краевые задачи, уравнение Лапласа, дробный интеграл Адамара-Маршо.

1. Введение

Пусть $\Omega=\left\{x\in R^2 : |x|<1\right\}$ — единичный круг, $u(r,\varphi)$ — гармоническая функция в Ω, $r=|x|, \varphi=arctg\frac{x_2}{x_1}$, $0<\alpha<1$, $0\le\mu$ — действительные числа.

Рассмотрим операторы

$$J^{\alpha}_{\mu}[u](r,\varphi)=\frac{1}{\Gamma(\alpha)}\int_0^1 |\ln s|^{\alpha-1}\, s^{\mu-1}u(sr,\varphi)ds, \tag{1}$$