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(E-mail: modth1705@mail.ru)**The similarity of forcing companions of positive Jonsson theories**

The main purpose of this work is to continue the study of model-theoretic properties of the basic concepts and results in connection with the notion of finite forcing Robinson for positive Jonsson theories, as well as of the language to get the criteria of syntactic and semantic similarity of the theories. All the basic information about positive Jonsson theories can be found in reference.

Key words: forcing companion, generic set, Δ -positively mustafinien, Jonsson stability, syntactic similarity, semantic similarity, the enriching of signatures, kosemanticness.

A distinctive feature of this article is the fact that, first, we are working with positive Jonsson theories that are not necessarily Jonsson, and secondly, in a special enrichment signatures, which allows arbitrary type definability in consideration of new kinds of stability theories [1–3]. Everything related to final determinations α -forcing and the corresponding forcing — a companion in the framework of this theory should be positive Jonsson found in [4]. In connection with the results of [5], we can use the above concepts positive morlization reduced to $B^+(At)$. It is essential that all definitions relating to both models and formulas, we shall deal only with the morphisms dives, in this case we instead Δ -PT theories we consider the theory of Δ - Γ ($\Gamma = \{J, R, M\}$). In [6] introduced a class of theories, which is the intersection with the class of theories Jonsson, summarizes it, and also contains generalized Jonsson theory, introduced in [7]. It is interesting to consider the relationship of forcing for such classes of theories, when we consider only the dive.

Recall the definition of this class.

Definition 1. The theory T is Δ — positively mustafinien (Δ -PM) — theory if:

- 1) The theory T has infinite models;
- 2) The theory T is Π_{n+2}^+ — axiomatizable;
- 3) The theory T admits Δ -JEP;
- 4) The theory T admits Δ -AP.

We call the theory Δ -mustafinien (Δ -M) — theory, if in the Definition 1 are consider only the definition of a dive.

Let L — first-order language. At — is the set of atomic formulas of the language. $B^+(At)$ — a relatively closed set of positive Boolean combinations (conjunction and disjunction) of all atomic formulas and their subformulae change of variables. $Q(B^+(At))$ — is the set of formulas in prenex normal form obtained through the use of quantifiers (\forall and \exists) to $B^+(At)$. We mean a formula positive if it belongs to $Q(B^+(At)) = L^+$. A theory is called positive axiomatizable if its axioms are positive. $B(L^+)$ — this is an arbitrary Boolean combination of formulas of L^+ .

Let $0 \leq n \leq \omega$. Let Π_n^+ — the set of all formulas of the language L^+ form $\forall \exists \dots \varphi$ (i.e. formulas of L^+ n variables quantifiers beginning with \forall). Let $\Delta \subseteq \Sigma_{n+1}^+ \subseteq L^+$. Suppose, further, C denotes the fixed countable infinite set of new constant symbols, T — arbitrary Δ -M theory of the countable language L^+ . The main logical connectives take $\exists, \wedge, \vee, \neg$. Through $AL^+(C)$ denote the set of all atomic sentences of the language $L^+(C)$, $\neg AL^+(C) = \{\neg \varphi : \varphi \in AL^+(C)\}$. Recall that we have the following result from [4].

Theorem 1. Let T — arbitrary Δ -M theory of the countable language L^+ . Then each B_α — generic model is $\Sigma_{\alpha+1}$ — closed model of T (positive analog existential isolation).

Standard defined forcing companion Δ -M theory.

T^f — is forcing companion of the theory T . $T^f = \{\varphi : T \Vdash \neg \neg \varphi\}$.

We recall some definitions of the various types of stability and the corresponding results for the above-mentioned positive Jonsson theories.

Definition 2. We say that the Jonsson theory T $J-\lambda$ — stable if for any T — existentially closed model A , for any subset X of A , $|X| \leq \lambda \Rightarrow |S^J(X)| \leq \lambda$.

The next result shows that the stability in the sense indicated above well with the classical notion of stability.

Theorem 2. Let T — complete for the existential proposals perfect Jonsson theory, $\lambda \geq \omega$. Then the following conditions are equivalent:

- 1) $T - J - \lambda$ — stable;
- 2) $T^* - \lambda$ — stable, where T^* — center of T .

Consider some enrichments of the signature. Enrichment will be called admissible if in this case we will ensure the stability of the considered types of definability. In the future, all considered acceptable enrichment. Let T — Jonsson arbitrary theory of the signature σ , C — its semantic model, A — subset of the model C , P new single predicate symbol. Consider the signature $\sigma_p(A) = \sigma_A \cup \{P\}$ following (generally incomplete) theory $T_p^J(A) = Th_{\exists}(C, a)_{a \in A} \cup \{P(C_a) | a \in A\} \cup \{P \subseteq\}$, where $\{P \subseteq\}$ — infinite number of sentences expressing the fact that the interpretation of the symbol P — is existentially closed submodel in the signature σ . The requirement of existential isolation from submodel significantly in the sense that it should not be the ultimate. Through S_p^J denote the set of all \exists — completions theory $T_p^J(A)$. Let λ — arbitrary cardinal.

Consider the concept of stability in enrichment.

Definition 3. Jonsson theory T is Jonsson $P-\lambda$ — stable (hereinafter, $J-P-\lambda$ — stable) if $|S^J(X)| \leq \lambda$ for any set A of cardinality $\leq \lambda$.

Definition 4. Jonsson theory T is $J-P$ — stable if T is $J-P-\lambda$ — stable for some λ .

The following concepts are related to a generalization of the concept of elementary Jonsson pair.

Let A, B — existentially closed model of T Jonsson and the inclusion $A \subseteq B$. Let $\sigma_p = \sigma \cup \{P\}$. And the interpretation of a single predicate symbol P in B there A .

Definition 5. The model (A, B) is called Jonsson elementary pair theory T .

Jonsson theory called elementary theory of the class K pairs T_p^J , where K — the set of all pairs Jonsson elementary theory T .

Lemma 1. If T — perfect Jonsson theory, then $T_p^J(A)$ — perfect Jonsson theory.

Theorem 3. Let T — perfect Jonsson \exists — a complete theory. Then the following conditions are equivalent:

- 1) The center of the theory T $P-\lambda$ — is stable (in the sense of [8]);
- 2) The theory T $J-P-\lambda$ — is stable.

Corollary 1. Let T — uncountable \exists -Jonsson — categorical complete theory. Then the following conditions are equivalent:

- 1) The theory of elementary par Jonsson T_p^J is \exists — complete of theories,
- 2) The theory of elementary par center complete theory T (in the sense of [9]).

Recall the definition of stability in the framework of the theory $\Delta-PM$. Let T an $\Delta-PM$ arbitrary theory in the language of the signature σ . Let C semantic model of the theory T . $A \subseteq C$. Let $\sigma_r(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$, where $\Gamma = \{P\} \cup \{c\}$. Consider the following theory $T_r^{PM}(A) = Th_{\Pi_{\alpha+2}^+}(C, a)_{a \in A} \cup \{P(c)\} \cup \{P \subseteq\}$, where $\{P \subseteq\}$ there is infinite number of sentences, which says that the interpretation of characters P have positively existentially closed submodel in the signature σ . This theory not necessarily complete. Through S_r^{PM} are denoted the set of all $\Sigma_{\alpha+1}^+$ — completions theory $T - P - \lambda$ - stable if $|S_r^{PM}| \leq \lambda$ for any A . Such that $|A| \leq \lambda$.

Consider all replenishment of the center T^* theory T in the new signature σ_Γ where $\Gamma = \{c\}$. By Δ -PM — completely theory T^* , there is its center, and we denote it as T^c . When restrictions T^c to the signature σ , the theory T^c becomes a complete type. This type we call the central type theory T .

Easy to see that the above definition can be considered in terms of the theory Δ - M . We've got the next result.

The main result of [4] is as next theorem.

Theorem 4. Let λ be an arbitrary infinite cardinal, T perfect theory Δ - M , complete of positive existential sentences. Then the following conditions are equivalent:

- 1) $T^* - P - \lambda$ -stable;
- 2) $(T^*)^F - \lambda$ — is stable in the classical sense, where $(T^*)^F$ — forcing companion of the theory T^* in the enriched signature T^* ;
- 3) $T^c - \lambda$ — is stable in the classical sense.

We proceed to the main part of this work. Our purpose in the above definitions of positivity and forcing companion of this theory is to determine the syntactic and semantic similarity of these theories.

Issues related to the similarity of these theories are discussed in the following article [10].

And in conclusion, we note that the above link stability perfect central type Δ - M theory in the study of this theory forcing companion naturally generates a series of questions regarding the above results of the first author (Theorem 2, Theorem 3, and Corollary 1).

We assume that the necessary facts about the different types of similarity are not mentioned here can be found either in [11] or [10].

In [12] T.G. Mustafin determined the exact syntactic notion [12, Def.1] and semantic similarity [12, Def.4] complete theories. Moreover, the language of these definitions and related concepts (for example, the shell theory [12, Def.12], semantic property (theories, models, element) [12, Def.8]), he proved that for an arbitrary complete theory there is syntactically similar to it a theory of polygons [12, Th.4, Th.5]. It was also proved that the semantic similarity retains a number of useful properties [12]. In the class of the Jonsson theories this approach to the classification of the corresponding objects is acceptable, but required some changes in the relevant definitions in the sense T.G. Mustafin. This is due, firstly, to the fact that, generally speaking, the Jonsson theory is not complete, and, secondly, that in the class of the models Jonsson theory homogeneous — universal model, generally speaking, is not saturated.

Recall of determination of syntactic and semantic similarity [12]. Let T is the complete theory, then $F(T) = \bigcup_{n < \omega} F_n(T)$, where $F_n(T)$ — is a Boolean algebra formulae with n free variable.

Definition 6. [12, Def.1]. Let T_1 and T_2 are complete theory.

We will say that T_1 and T_2 are syntactically similar if there is a bijection $f : F(T_1) \rightarrow F(T_2)$ such that

- 1) restriction f to $F_n(T_1)$ is isomorphism of the Boolean algebra $F_n(T_1)$ and $F_n(T_2)$, $n < \omega$;
- 2) $f(\exists v_{n+1} \varphi) = \exists \varphi_{n+1} f(\varphi)$, $\varphi \in F_{n+1}(T)$, $n < \omega$;
- 3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 7. [12, Def.2].

1) The clean triple are called $\langle A, \Gamma, M \rangle$ where A not empty Γ — is group of the permutation A , and M — a family of subsets A , such that $M \in M \Rightarrow g(M) \in M$ for each $g \in \Gamma$;

2) If $\langle A_1, \Gamma_1, M_1 \rangle$ and $\langle A_2, \Gamma_2, M_2 \rangle$ are clean triple, and $\psi : A_1 \rightarrow A_2$ — bijection, then ψ is isomorphism, if

- (i) $\Gamma_2 = \{\psi g \psi^{-1} : g \in \Gamma_1\}$;
- (ii) $M_2 = \{\psi(E) : E \in M_1\}$.

Definition 8. [12, Def.3]. The clean triple $\langle |C|, G, N \rangle$ is called semantic theory the complete triple T , where $|C|$ — monst.pov carrier model C of the theory T , $G = \text{Aut}(C)$, N — class all subsets $|C|$, each corresponding to a carrier which elementary submodels C .

Definition 9. [12, Def.4]. Complete of Theory T_1 and T_2 called semantically similar if their semantic triple isomorphous with each other.

[12] the following result is known.

Suggestion 1 [12, Prop 3.1] If the theory T_1 and T_2 syntactically similar, and then T_1 and T_2 semantically similar, converse is not true.

Analogues give the above definitions for Jonsson theories.

The following definitions from [11] are generalizations previous definitions.

Let T is the arbitrary Jonsson theory, then $(T) = \bigcup_{n < \omega} E_n(T)$, where $E_n(T)$ — is a lattice \exists — formulas with n free variables T^* -Jonsson center theory T , i.e. $T^* = Th(C)$, there C — Jonsson semantic model theory T .

Definition 10. Let T_1 and T_2 -Jonsson theories.

We say that T_1 and T_2 J — syntactically similar if there is a bijection $f : E(T_1) \rightarrow E(T_2)$ such that

- 1) there is a limit f to $E_n(T_1)$ the lattice isomorphism $E_n(T_1)$ and $E_n(T_2)$, $n < \omega$;
- 2) $f(\exists v_{n+1} \varphi) = \exists \varphi_{n+1} f(\varphi)$, $\varphi \in F_{n+1}(T)$, $n < \omega$;
- 3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 11. The clean triple $\langle C, AutC, SubC \rangle$ is called J semantic triple where C is semantic model T , $AutC$ — is group of the avtomopfizms C , $SubC$ is class of all subsets carrier eccentricity C are carriers appropriate submodels.

Definition 12. Two the Jonsson theories T_1 and T_2 J — are called semantically similar if their J semantic triple isomorphous is as clean triple.

The correctness of this definition follows from the fact that the theory of perfect Jonsson semantic model is unique up to isomorphism. Otherwise, all semantic models only elementary are equivalent together.

Recall that [13] two Jonsson theories are kosemantichny between themselves if they have the same semantic model.

The following definitions introduce us to the theory of the positive Jonsson theories.

Let L be a first-order language. At $B^+(At)$ — is the set of atomic formulas of the language. $B^+(At)$ — is close set of positive Boolean combinations (conjunction and disjunction) of all atomic formulas and their subformulae change of variables. $Q(B^+(At))$ — is a set of formulas in normal form prenex obtained by applying quantifiers (\forall и \exists) to $B^+(At)$. We mean a formula positive if it belongs to $Q(B^+(At)) = L^+$. Axiomatizable theory is called positive if its axioms are positive. $B(L^+)$ — it is an arbitrary Boolean combination of formulas L^+ .

Following [14, 15] we define Δ - morphisms between structures.

Let M and N of language structure $\Delta \subseteq B(L^+)$. Mapping $h : M \rightarrow N$ is called Δ -homomorphism (symbolically $h : M \xrightarrow{\Delta} N$), if for any $\varphi(\bar{x}) \in \Delta$, $\forall \bar{a} \in M$ of that $M \models \varphi(\bar{a})$, it follows that $N \models \varphi(h(\bar{a}))$. Following [14, 15], the model M is called the beginning in N and we say that M continues in N , with N , $h(M)$ is called an extension of M . If this is true, and the opposite, i.e. for all $\varphi(\bar{x}) \in \Delta$, $\forall \bar{a} \in M$ $M \models \varphi(\bar{a}) \Leftrightarrow N \models \varphi(h(\bar{a}))$, then we say that h immerses M in N (symbolically $h : M \xleftrightarrow{\Delta} N$). In the future, we will use the term Δ - and continued Δ -dive. Under this definition (Δ -homomorphism), we easily see that isomorphic embedding and embedding are elementary — dives, and when, respectively.

Definition 13. Model M — theory T is Δ -positively existentially closed if for every Δ -homomorphism $h : M \xrightarrow{\Delta} N$, $N \in ModT$ and $\forall \bar{a} \in M$ and $\varphi(\bar{x}, y) \in \Delta$: $N \models \exists y \varphi(f(\bar{a}), y) \Rightarrow M \models \exists y \varphi(\bar{a}, y)$.

The class of all Δ — positively existentially closed models of T is denoted by E_T^+ ; under E_T we understand class of existentially closed models of the theory T .

Definition 14. We say that a theory T admits Δ -JEP, if for any two $A, B \in ModT$ exists $C \in ModT$ and Δ -homomorphisms $h_1 : A \rightarrow_{\Delta} C, h_2 : B \rightarrow_{\Delta} C$.

Definition 15. We say that the theory T admits Δ -AP, if for any $A, B, C \in ModT$, what $h_1 : A \rightarrow_{\Delta} C, g_1 : A \rightarrow_{\Delta} B$, where h_1, g_1 — Δ -homomorphisms exists $D \in ModT$ and $h_2 : C \rightarrow_{\Delta} D, g_2 : B \rightarrow_{\Delta} D$ where h_2, g_2 — Δ -homomorphisms such that $h_2 \circ h_1 = g_2 \circ g_1$.

Definition 16. Theory T is called Δ -positive Jonsson (Δ -PJ) theory if it satisfies the following conditions:

- 1) T has an infinite model;
- 2) T is positively $\forall\exists$ -axiomatizable;
- 3) T admits Δ -JEP;
- 4) T admits Δ -AP.

From the definition of the theory Δ -PJ- T should be noted that $ModT$ closed under homomorphisms, but the theory T is not always Jonsson. If it is Jonsson, the continuation Δ - in determining the theory T is Δ -immersions.

Let T be an arbitrary Δ -PJ-theory, then $E^+(T) = \bigcup_{n < \omega} E_n^+(T)$, where $E_n^+(T)$ — is a lattice of positive existential formulas with n free variables.

Definition 17. Let T_1 and T_2 are Δ -PJ-theory.

We say that T_1 and T_2 are Δ -PJ-syntactically similar if there is a bijection $f : E^+(T_1) \rightarrow E^+(T_2)$ such that:

- 1) limit f to $E_n^+(T_1)$ have a lattice isomorphism $E_n^+(T_1)$ and $E_n^+(T_2)$, $n < \omega$;
- 2) $f(\exists v_{n+1} \varphi) = \exists v_{n+1} f(\varphi), \varphi \in E_n^+(T), n < \omega$;
- 3) $f(v_1 = v_2) = (v_1 = v_2)$.

In [11] under these definitions the following result, which connects the above definition of syntactic similarity and J -syntactic similarity.

Theorem 5. Let T_1 and T_2 — \exists -complete committed Jonsson theory.

Then the following conditions are equivalent:

- 1) T_1 and T_2 are J -syntactically similar;
- 2) T_1^* and T_2^* are syntax similar (in the sense of [12]), and where T_1^* and T_2^* are the centers T_1 and T_2 respectively.

In [16] in Δ -PJ under these definitions the following result, which connects the above definition of syntactic similarity and PJ-syntactic similarity.

Theorem 6. Let T_1 and T_2 are Δ -PJ-theory, Jonsson, committed, complete of positive existential sentences.

Then the following conditions are equivalent:

- 1) T_1 and T_2 are PJ-syntactically similar;
- 2) T_1^* and T_2^* are syntactically similar (in the sense of [12]).

Definition 18. Let T_1 and T_2 are Δ -PJ-theory.

We say that T_1 and T_2 are PJ-syntactically similar if there is a bijection $f : E^+(T_1) \rightarrow E^+(T_2)$ such that:

- 1) limit f to $E_n^+(T_1)$ have a lattice isomorphism $E_n^+(T_1)$ and $E_n^+(T_2)$, $n < \omega$;
- 2) $f(\exists v_{n+1} \varphi) = \exists v_{n+1} f(\varphi), \varphi \in E_n^+(T), n < \omega$;
- 3) $f(v_1 = v_2) = (v_1 = v_2)$.

In [13], [17] it is considered property of kosemantichnost models respectively for the Jonsson theories and Δ -PJ — theories.

Definition 19. [13] [T.G. Mustafin]. We say that the Jonsson theories T_1 and T_2 are kosemantichny ($T_1 \triangleright \triangleleft T_2$), if they have a common semantic model, i.e. $C_{T_1} = C_{T_2}$.

Definition 20. [13] We say that the model A and B are kosemantichny ($A \triangleright \triangleleft B$) if for any Jonsson theory T such that $A \models T$, there is Jonsson theory T' kosemantichny with T such that $A \models T'$. And conversely.

Note 1. For all two models, A and B runs

$$A \equiv B \Rightarrow A \equiv_J B \Rightarrow A \triangleright \triangleleft B.$$

Definition 21. [17] Two Δ -PJ-theories T_1 and T_2 are called Δ -PJ — kosemantichny $T_1 \triangleright \triangleleft_{PJ}^\Delta T_2$ if they have the common semantic model when T_1 and T_2 Jonsson theories and have the common and universal field in the case where they are not Jonsson.

Definition 22. [18] Models A and B are called Δ -PJ-equivalent if for any Δ -PJ-theory T . $A \models T \Leftrightarrow B \models T$ and are denoted by $A \equiv_{PJ}^\Delta B$.

Definition 23. Model A and B are the models of the signature σ are called Δ -PJ-kosemantichny $A \triangleright \triangleleft_{PJ}^\Delta B$ if any Δ -PJ-theory T_1 such that $A \models T_1$ there exists Δ -PJ-theory T_2 , Δ -PJ-kosemantichny with T_1 such that $B \models T_2$. And conversely.

In this article we will discuss similar issues for the central type of positive generalizations Jonsson theories. In case Δ -PJ — we have the following theory, in connection with enrichment of the signature.

Let T there be any Δ -PJ-theory in the language of first-order of the signature σ . Let C is a semantic model theory T . $A \subseteq C$. Let $\sigma_r(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$ where $\Gamma = \{P\} \cup \{c\}$. Let $T_r^{PJ}(A) = Th_{\forall\exists^+}(C, a)_{a \in A} \cup \{P(c_a) \mid a \in A\} \cup \{P(c)\} \cup \{P \subseteq\}$, where $\{P \subseteq\}$ there are an infinite number of sentences expressing the fact that the interpretation of the symbol P is existentially closed submodel language of the signatures σ . We consider all replenishment of the theory T^* for theory T in the language of the signature σ_r , where $\Gamma = \{c\}$. As T^* is Δ -PJ-theory, it has its center and we denote it by T^c . When we restrict T^c to the signature σ of the theory, the theory T^c becomes a complete type. This type we call the theory of central type T . Note that all the semantic models are elementary equivalent between themselves.

In view of this theory and perfect definition of central type is correctly.

In [5] in connection with the above concepts in the rich signature it is obtained the following result.

Theorem 7. Let T_1 and T_2 are Δ -PJ-theories, the Jonsson theories, committed, are complete for positive existential sentences.

Then the following conditions are equivalent:

- 1) T_1^* and T_2^* are PJ-syntactically similar;
- 2) T_1^c and T_2^c are syntactically similar (in the sense of [12]).

Let $0 \leq n \leq \omega$. Let Π_n^+ is the set of all formulas L^+ of the form $\forall\exists\dots\phi$ (i.e. the formula of L^+ a change n of quantifiers beginning \forall). Let $\Delta \subseteq \Pi_n^+ \subseteq L^+$. In [19], there is a class of theories, which is a generalization of the theory Δ -PJ. In particular, if $n = 0$ we obtain a special case Δ -PJ-theories, if $n = \omega$ we are dealing with positive-axiomatizable complete theory.

Definition 24. [19] The theory T is called Δ positive Mustafinien — (Δ -PM) — theory, if

- 1) The theory T has infinite models;
- 2) The theory T is Π_{n+2}^+ axiomatizable;
- 3) The theory T admits Δ -JEP;
- 4) The theory T admits Δ -AP.

From the definition Δ -PM of theory T it should be noted that $ModT$ is the relatively closed Δ -homomorphisms, but the theory T is not always α -Jonsson. If it is α -Jonsson, the Δ -continuation in determining of the theory T is Δ -dives.

In this paper we present the results associated with the likes of Δ -PM-theories in the enrich signature.

Let T is arbitrary Δ -PM-theory in the language of the signature σ . Let C is semantic model theory T , $A \subseteq C$. Let $\sigma_r(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$ where $\Gamma = \{P\} \cup \{c\}$. We consider the following theory $T_r^{PM}(A) = Th_{\Pi_{\alpha+2}^+}(C, a)_{a \in A} \cup \{P(c)\} \cup \{P \subseteq\}$, where $\{P \subseteq\}$ is the infinite number of sentences, which says that the interpretation of characters P have positively existentially closed submodel in the signature σ .

Existential isolation requirement is essential in the sense that the interpretation of symbols P should be infinite. It is clear that many proposals considered a theory and this theory, generally speaking, is not complete. We consider all replenishment center T^* theory T in the new signature σ_r , where $\Gamma = \{c\}$. By Δ -PM completely theory T^* , there is its center, and we denote it as T^c . When T^c is restricted to the signature σ , the theory T^c becomes a complete type. This type, as in the Δ -PJ-case, we called the central type theory T . Let T -be any Δ -PM theory, then $E^+(T) = \bigcup_{n,m < \omega} E_{n,m}^+(T)$, where $E_{n,m}^+(T)$ is lattice of positive existential formulas with n free variables and m variables with quantifiers.

Definition 25. Suppose T_1 and T_2 are Δ -PM-theories. We shall say that T_1 and T_2 are Δ -PM syntactically similar if there is the bijection $f : E^+(T_1) \rightarrow E^+(T_2)$ such that:

- 1) There is a limit $f : E_{n,m}^+(T_1)$ the lattice isomorphism $E_{n,m}^+(T_1)$ and $E_{n,m}^+(T_2)$, $n, m < \omega$;
- 2) $f(\exists v_{n+1}\varphi) = \exists v_{n+1}f(\varphi)$, $\varphi \in E_n^+(T)$, $n < \omega$;
- 3) $f(v_1 = v_2) = (v_1 = v_2)$.

One of the results obtained under the above definitions is as follows:

Theorem 8. Let T_1 and T_2 are Σ_{m+1}^+ -complete, perfect, Jonsson Δ -PM theories. Then the following conditions are equivalent:

- 1) T_1^* and T_2^* are Δ -PM-syntactically similar;
- 2) T_1^c and T_2^c are syntactically similar (in the sense of [12]).

Proof. Note that because of some of the results of ([14, 15] the positive morlization), it suffices to considered case when n — is arbitrary and $m = 0$ in the definition 25, i.e. it is proved that everything can be reduced to the case when $\Delta = \Delta_0$. We also need the following facts

Fact 1 [19]. Jonsson for any theory T the following conditions are equivalent:

- 1) T is perfect;
- 2) T^* is model-complete .

Fact 2 [19]. For any complete proposals for the existential theory T . Jonsson following conditions are equivalent:

- 1) T^* is model-complete;
- 2) For each $n < \omega$ $E_n(T)$ is Boolean algebra, where $E_n(T)$ — is a lattice of existential formulas with n free variables.

Definition 26. Following [20], we say that A model of K is simple in the class K , if for any B of K such that there exists a homomorphism $h: A \rightarrow B$, it follows that h is an embedding.

Fact 3 [19, Chapter 3, Theorem 6.5]. Assume that the class K is closed under ultraproducts, and let each algebraic system of K can be embedded in the simple system of K . If A is algebraically closed in K and non-trivial (i.e., power 2, then A is existentially closed in K and A simple in K .

Fact 4 [21, Theorem 1.4]. Suppose Σ is the class of relational structures. Then there is a theory T such that $\Sigma = \Sigma_T \Leftrightarrow \Sigma$ satisfies the following conditions:

- 1) Σ is closed under isomorphisms;
- 2) Σ is closed under substructures;
- 3) Σ is closed under ultraproducts .

Note 2. Easy to see that if $\Delta = B^+(At)$ that $\Delta = \Delta_0$, that $A \in E_r^+$ if and only if A is algebraically closed. As well as $A \in E_r \Rightarrow A \in E_r^+$, in virtue of the fact 3 in the Theorem 8, we note that if the theory Δ -PM -Jonsson as in Theorem 8 that in the rich language of the theorem concerning the conditions of its center will be the same, i.e. Δ -PM -Theory, this is achieved as follows: constants will move into images constants, implementation of the predicate in the image of implementation. Necessary images are obtained by the corresponding maps that we provide Δ -JEP and Δ -AP out Δ -PM -Jonsson original theory. Therefore T_1^* and T_2^* are Δ -PM -Jonsson theory.

Show 1) \Rightarrow 2). We have that for each $n < \omega$ $E_n^+(T_1^*)$ is isomorphic to $E_n^+(T_2^*)$. Let this isomorphism is on f_{1n} . By hypothesis and facts 1 and 2 for each $n < \omega$ $E_n(T_1^*)$ and $E_n(T_2^*)$ are Boolean algebras. But by

virtue of perfectness T_1^* and $T_2^* \Rightarrow (T_1^*)^*$ and $(T_2^*)^*$ are model complete and in virtue of the fact 1, and so for each $n < \omega$, for any formula $\varphi(\bar{x})$ from $F_n(T_1^*)$ of the remark 2, there is a formula $\psi(\bar{x})$ from $E_n^+(T_1^*)^*$ in such a way that $(T_1^*)^* \models \varphi \leftrightarrow \psi$. Due to the fact that the theory T_1^* is complete for positive existential sentences and $E_n^+(T_1^*) \subseteq E_n^+(T_1^*)^*$ (since $T_1^* \subseteq (T_1^*)^*$), it follows that $E_n^+(T_1^*) = E_n^+(T_1^*)^*$. Due to the fact that the theory T_2^* is complete for positive existential sentences, and $E_n^+(T_2^*) \subseteq E_n^+(T_2^*)^*$ (because it $T_2^* \subseteq (T_2^*)^*$ implies that $E_n^+(T_2^*) = E_n^+(T_2^*)^*$). For each $n < \omega$, we define for each $\varphi_1(\bar{x})$ from $F_n(T_1^*)^*$ of the following mapping between $F_n(T_1^*)^*$ and $F_n(T_2^*)^*$: $f_{2n}(\varphi_1(\bar{x})) = f_{1n}(\psi_1(\bar{x}))$, where, $(T_1^*)^* \models \psi_1 \leftrightarrow \varphi_1$, $\psi_1 \in E_n^+(T_1^*)$. Easy to see that by properties f_{1n} and above f_{2n} it is a bijection, defining an isomorphism between $F_n(T_1^*)^*$ and $F_n(T_2^*)^*$. Consequently, $(T_1^*)^*$ and $(T_2^*)^*$ are syntactically similar (in the sense of [12]). But these $(T_1^*)^* = T_1^c$ and $(T_2^*)^* = T_2^c$, respectively, defining the core types of these theories.

Show 2) \Rightarrow 1). Trivial, since $F_n(T_1^*)^*$ is isomorphic to $F_n(T_2^*)^*$ for each $n < \omega$, and by virtue of the hypothesis and the facts 1, 2 and 3, this isomorphism extends to all subalgebras.

We are now ready to formulate the main result of this paper in the form of the following corollary of Theorem 8.

Corollary 2. Let T_1 and T_2 are Σ_{m+1}^+ — complete, perfect, Δ – M –theories. Then the following conditions are equivalent:

- 1) $(T_1^*)^F$ and $(T_2^*)^F$ are Δ – PM –syntactically similar;
- 2) T_1^c and T_2^c are syntactically similar (in the sense of [12]).

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Позитивті йонсондық теориялардың форсинг компаньондардың ұқсастығы

Мақаланың басты мақсаты — негізгі ұғымдардың моделді-теоретикалық қасиеттерін және позитивті йонсондық теориялар үшін Робинсонның шекті форсинг ұғымдарына қатысты нәтижелердің, сонымен қатар осы тілдегі қарастырылып отырған теориялардың синтаксистік және семантикалық критерийлерін тауып, зерттеу жұмысын жалғастыру. Барлық негізгі позитивті йонсондық теориялар туралы мәліметтер жан-жақты талданып, келтірілген.

А.Р.Ешкеев, И.А.Репкина

Подобие форсинг компаньонов позитивных йонсоновских теорий

Основная цель данной работы — продолжение изучения теоретико-модельных свойств основных понятий и результатов в связи с понятием конечного форсинга Робинсона для позитивных йонсоновских теорий, а также получение на этом языке критерий синтаксического и семантического подобия рассматриваемых теорий. Все основные сведения о позитивных йонсоновских теориях можно найти в рассмотренной литературе.

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Small models of positive convex Jonsson theories in the enriched signature

The main results of this paper are descriptions of small models of positive convex jonsson theories in the enriched signature. These descriptions are given in the language of the central types in enriching under automorphism by predicate and constant and also using simple algebraic extensions uncountable categoricity of the above theories was studied.

Key words: positive jonsson theory, enrichment of signature, automorphism, unary predicate, constant, simple algebraic extension, atomic model.

This paper presents the results of the authors on the subject of positive jonsson theory in the enriched signature. The results obtained in this paper differ from similar studies in this area from the previous results that now we are considering the $\Delta - M$ -theory and their centers in the enriched signature by automorphism, constant and predicate.

The paper studies the countable small models and the concept of existential closeness, generally speaking, of incomplete theories in the enriched signature.