THE COEFFICIENT CONDITION FOR BELONGING A FUNCTION IN THE LORENTZ SPACE

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Let $\{\varphi_n(x)\}_{n=0}^{+\infty}$, $x\in[0,1]$ the Price system [1] and $L_{p\theta}[0,1]$, $1<p<+\infty$, $1<\theta<+\infty$ Lorentz space [2]. Series $\sum\limits_{\nu=0}^{+\infty} a_\nu \varphi_\nu(x)$ is called the Fourier-Price series of the function $f\in L[0,1]$, where $a_\nu=\int\limits_0^1 f(x)\varphi_\nu(x)dx$ - Fourier-Price coefficients of the function $f(x)$ according to the multiplicative Price system.

Denote by M the class of nonnegative number sequences that decrease monotonically to zero. By QM, denote the class of quasimonotone number sequences, i.e. [3],

$$QM=\left\{a_n\in R:\quad \lim_{n\to+\infty} a_n=0 \ \ and \ \ \exists\tau\ge 0: \frac{a_n}{n^\tau}\downarrow 0\right\}.$$

Let $\{a_\nu\}_{\nu=0}^{+\infty}$ be sequence of positive numbers. Denote $\Delta_0 a_\nu=a_\nu$ and $\Delta_1 a_\nu=a_\nu-a_{\nu+1}$ -first order difference, $\Delta_k a_\nu=\Delta_1(\Delta_{k-1}a_\nu)$ - k-th order difference. Applying Pascal's triangle method $\Delta_k a_\nu$ can be written in the form $\Delta_k a_\nu=\sum\limits_{j=0}^{k}(-1)^j C_k^j a_{\nu+j}$, where $C_k^j=\dfrac{k!}{j!(k-j)!}$, $0!=1$ is called the binomial coefficient.

Let $D_n(x)=\sum\limits_{k=0}^{n-1}\varphi_k(x)$ be the Dirichlet kernel. Let's introduce the notation

$$D_n^{(0)}(x)=\varphi_n(x),\ D_n^{(1)}(x)=D_n(x)=\sum_{\nu=0}^{n-1}\varphi_\nu(x),\ D_n^{(k)}(x)=\sum_{\nu=0}^{n-1}D_\nu^{(k-1)}(x),\ k\ge 1,\ n\in N.$$

The Fourier-Price series $\sum\limits_{\nu=0}^{+\infty} a_\nu \varphi_\nu(x)$ of the function f∈ L[0, 1] can be represented as [4]

$$f=\sum_{\nu=0}^{+\infty}\Delta_{k-1}a_\nu D_{\nu+1}^{(k-1)}(x),\ \forall\, k\in N.$$

Let $\{p_i\}_{i=1}^{+\infty}$ be an arbitrary given sequence of natural numbers such that $p_i\le c$, $p_i\ge 2$ and let's assume that $m_0=1$, $m_\mu=\prod\limits_{k=1}^{\mu}p_k$, $\forall n\in N: n=\sum\limits_{k=0}^{l}n_k m_k$, where $n_k\in Z^+$, $0\le n_k\le p_{k+1}-1$, $k=0,1,2,\dots$.

Основным результатом данной работы является следующая теорема.

Theorem.Let $1 < p < +\infty$, $1 < \theta < +\infty$, $a = \{a_\nu\}_{\nu=0}^{+\infty} \in QM$ and $\Delta_k a_\nu \geq 0, \forall k \in N$, $\forall \nu \in Z^+$ k-th difference of Fourier-Price coefficients. Thegenerating sequence of the Price system $\{p_i\}_{i=1}^{+\infty}$ is limited, that is $\exists c \in N$: $p_i \leq c$, $p_i \geq 2$, $i \geq 1$. The numbers a_ν, $\forall \nu \in Z^+$ are the Fourier–Price coefficients of some function $f \in L_{p\theta}[0,1]$, $1 < p < +\infty$, $1 < \theta < +\infty$ if and only if the series

$$\sum_{\nu=0}^{+\infty} m_{\nu+1}^{k\theta - \frac{\theta}{p}} \left(\Delta_{k-1} a_{m_\nu}\right)^\theta,\ k \in N,\ \Delta_0 a_{m_\nu} = a_{m_\nu}$$

converges. In this case, the following inequality holds:

$$c'_{p\theta k} \left\{ a_0^\theta + \sum_{\nu=0}^{+\infty} m_{\nu+1}^{k\theta - \frac{\theta}{p}} \left(\Delta_{k-1} a_{m_\nu}\right)^\theta \right\} \leq \|f\|_{p\theta} \leq c_{p\theta k} \left\{ a_0^\theta + \sum_{\nu=0}^{+\infty} m_{\nu+1}^{k\theta - \frac{\theta}{p}} \left(\Delta_{k-1} a_{m_\nu}\right)^\theta \right\},$$

where the constants $c'_{p\theta k} > 0, c_{p\theta k} > 0$ depends only on the specified parameters.

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BOUNDARY CONJUGATION PROBLEM FOR PIECEWISE ANALYTIC FUNCTIONS IN BESOV SPACES

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The main aim of this work is to show solvability of the continuous boundary conjugation problem for analytic functions in the Besov space, which is embedded into the class of continuous functions.

As far as we know, (continuous) boundary value problems of conjugation of analytic functions are considered in Besov spaces for the first time in this work.

Until now, such and similar boundary value problems have been studied in spaces of functions continuous in the sense of H¨older [2, 3]. Our work proposes a method for solving the indicated boundary value problems in the class of continuous functions (H¨older property is not required) in terms of Besov spaces, which emphasises the novelty of the work.

References

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