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FRACTAL MEASURES IN NANOELECTRONICS AND NEURODYNAMICS

The equation describing spatio-temporal evolution of additive physical quantities (measures) which depends on difference between fractal and topological dimensions of an object has been suggested. This equation is written in the form of two-dimensional map that takes into account a sign of derivative of measure. The notion for non-linear fractal measures with scale of measurement depending on the measure and external parameters has been proposed. Energy of excitonic formations and action potential of neurons have been considered as fractal measures. Results of the theory have been compared with experimental results.

*Keywords: non-linear fractal measures, topological dimensions, two-dimensional map, excitonic formation, action potential of neurons, n*anostructured semiconductor, *nanoelectronics, neurodynamics*

Introduction

Nanostructured semiconductors are perspective material for creation of quick-operating computer techniques, optoelectronic and photonic devices. Modern methods of microscopy demonstrate the nanocluster structure of semiconductor films. Such structures are irregular, self-similar and self-affine. Because of this, we describe such structures as fractal and multi-fractal objects.

For the description effects in optoelectronics and photonics we must take into account that wavelength of light is comparable to characteristic size of nanostructures, so the usual mean field approximation is supplemented with our corrections for the description of optical processes in nanostructured semiconductors. Nanostructures are fractal objects, so equations for the description of evolution of physical values contain fractional dimensions. Excitation energy of medium (i.e. sum of energy gap width and exciton binding energy) is considered to be a fractal measure depending on photon energy.

Light reflection, absorption and transmission factors can be defined via quantum form of fluctuation-dissipative relation and equation for fractal evolution of charge carriers. Correlations of charge carriers correspond to fluctuations of physical values. Dissipation can be defined via relation for equilibrium photon distribution [1-3].

Nowadays problems of nanoelectronics directly connected with physics of excitonic formations. Excitons and biexcitons can be described as two quantum bits which can interact each other. So, excitons and biexcitons can be used as units for quantum gates. Nano-sized semiconductors have irregular, chaotic structure. So, properties of excitons cannot be described universally via smooth regularities following, for example, from differential equations. Therefore, a common analogy between exciton and hydrogen-like atom is inapplicable for a full description of specifics of excitonic spectra in noncrystalline semiconductors with chaotic structure. Because of this a new nonlinear model for the description of fractal evolution of excitons depending on energy of exciting photons is necessary.

Systems of nanoelectronic elements characterized have properties similar to neural nets. So, such research areas as “neural networks”, “intelligence systems” are developing at the present time. The simple description of the similarity based on the fact that in nanoelectronics the usual spatial space division of conductivity, capacity, inductance and so on is losing its meaning. It is the result of functional behavior of nonlinear medium.

According to the statements of physics of open systems nonlinear dynamics of neural net characterized by scale-invariance, fractality, chaos, certain phase relations and so on. So, it is necessary to create a physical model for universal description of indicated properties. Different models used for the description of some properties of neurons are developed. Aim of the present work is construction of the simplest nonlinear model describing main regularities both of excitonic formations and neural dynamics (asymmetrical alternation, chaos, hierarchy and self-similarity, depolarization, phase reinstallation).

1. Universal map of alternation

As indicated below, from the criterion of fractal measure it is possible to obtain the universal map describing its alternated evolution as "accumulation-bursting". As distinct from all well-known differential and discrete models of dynamical systems the map describe chaotic oscillations with characteristics corresponding to criteria of self-organization.

Let us to consider evolution on time of $x(t)$ which is the module of a function related with fractal measure (i.e. additive value characterized by a measurable set) as

$$\frac{dx(t)}{dt} = sign(\frac{dx(t)}{dt})\frac{|\Delta x|}{|\Delta t|^{1-\gamma_0}}, \qquad (1.1)$$

where γ_0 is statistical characteristic of the set of t. γ_0 is used for supporting the Lipshitz–Hölder condition due to limitation of the derivative $\frac{dx}{dt}$. Module of increment of function $|\Delta x|$ can by replaced according to the condition of the fractal measure $x(t)$ as

$$x = x_0(|\Delta x|)^{-(D-d)}, \quad |\Delta x| = \left(\left|\frac{x}{x_0}\right|\right)^{-\frac{1}{\gamma}}, \quad \gamma = D - d, \qquad (1.2)$$

where x_0 is non-fractal regular measure, D is fractal dimension of the set of $x(t)$, d is topological dimension of measure carrier. By the substitution of (1.2) to (1.1) we obtain the equation with finite differences. Let us designate the discrete form of the sign function as μ_i. $\Delta t > 0$ always, so, $sign\left(\frac{dx(t)}{dt}\right)$ depends on Δx_i only. Its dependence on variable i can be described as

$$\mu_{i+1} = \frac{\Delta x_{i+1}}{\Delta x_i}\Big|_{x_i = x_{i+1}} = \frac{dx_{i+1}}{dx_i}. \qquad (1.3)$$

The equation (1.3) is written for a fixed point. Usually the values $\mu_{i+1} = \pm 1$ are used for linear description of evolution of perturbations. Let us define μ_{i+1} via μ_i and don't use any limitations for module of this value.

We can rewrite the equation (1.1) at $x_0 = 1$ with regard to (1.2) and (1.3) as

$$\frac{x_{i+1}}{\Delta t} = \frac{x_i}{\Delta t} + \mu_i |x_i|^{-\frac{1}{\gamma}} \Delta t^{\gamma_0 - 1} = \frac{x_i}{\Delta t} + \mu_i |x_i|^{-\frac{1}{\gamma}} \frac{\Delta t^{\gamma_0}}{\Delta t} = (x_i \Delta t^{-\gamma_0} + \mu_i |x_i|^{-\frac{1}{\gamma}})\frac{\Delta t^{\gamma_0}}{\Delta t}. \qquad (1.4)$$

Let us to eliminate the value of γ_0 from the equation (1.4) due to choosing of identical moments of time. We shall choose $\Delta t = 1$ according to discrete algorithm, but before this action we shall model the relation $x_i \Delta t^{-\gamma_0}$ via γ_0: just this relation, not $x_i \Delta t^{\gamma_0}$ is correspond to transition to chaos in values x_i.

Let us take into account that meaning of using of γ_0 is correspond to realization of the condition

$$\frac{\Delta x_i}{x_i}(\frac{\Delta t}{\tau})^{\gamma_0} = const \equiv C, \tag{1.5}$$

where τ is typical time of a process.

At $\gamma_0 = 0$ we calculate the Riemannian measure if $\Delta t = 1$. At $\gamma_0 \neq 0$ we have a possibility to find the Lebesgue measure with regard to the dependence of Δt^{γ_0} on increment of function $\Delta x_i(\gamma, x_i)$ as

$$(\frac{\Delta t}{\tau})^{-\gamma_0} = \frac{1}{C}\frac{\Delta x_i(\gamma, x_i)}{x_i} = \frac{(|x_i|)^{-1/\gamma}}{Cx_i} \tag{1.6}$$

where C is a constant value. Meaning of C can be explained as analogue of base (complexity) of a signal used for characterization of spectra:

$$B = \tau_k \Delta\omega, \tag{1.7}$$

where τ_k is typical correlation time and $\Delta\omega$ is frequency band width. According to the definition, value of C characterized complexity of choosing accuracy for the description of chaotic signals. Measure of a fractal object depends on accuracy of observation, therefore the constant C is included to theoretical results. Value of C corresponds to choosing of degree of accuracy of observation of a process and processes the values $C = 10^{-2}, 10^{-3}$ and so on. If sign of the derivative in (1.1) defined by external conditions (noise-type excitations), so we must use absolute values of Δx, Δt^{γ_0} in (1.5). Taking into account that $\Delta t = \tau$ we can finally rewrite the equation (1.4) as

$$x_{i+1} = (\frac{1}{C} + \mu_i)|x_i|^{-\frac{1}{\gamma}}. \tag{1.8}$$

If we differentiate (1.8) we have

$$\mu_{i+1} = (\frac{\Delta x_{i+1}}{\Delta x_i})_{x_i = x_{i+1}} = -\frac{1}{\gamma}(\frac{1}{C} + \mu_i)|x_i|^{-\frac{1}{\gamma}-1}. \tag{1.9}$$

Formulas (1.8) and (1.9) are required map of alternation, fractal evolution of measure. Parameter γ is fractional part of fractal dimension of a set of considered physical value. On the base of universal map of alternation (1.8) and (1.9) describing evolution of a system according to condition of fractality of measure we can model morphology of quantum dots ($\gamma > 2$), quantum wires ($\gamma > 1$) and quantum wells ($\gamma > 0$) located on a surface ($d = 2$).

Fractal dimensions are chosen for steady self-similar and self-affine sets via numbers I_1, I_2 [1, 2]. Using the present theory let us to describe results of experiments made by use of modern methods of microscopy.

The obtained map describes alternated, chaotic evolutionary processes. In contrast to well-known models the present map realizes asymmetrical alternation with strong bursts, i.e. signals belong to the type “accumulation - bursting” [4].

It is important that such signals satisfier to criteria of self-organization. Previously we have investigated such signals theoretically by circuit simulation and physical experiments on study of signals from radiotechnical generator with phase control.

(a) (b)

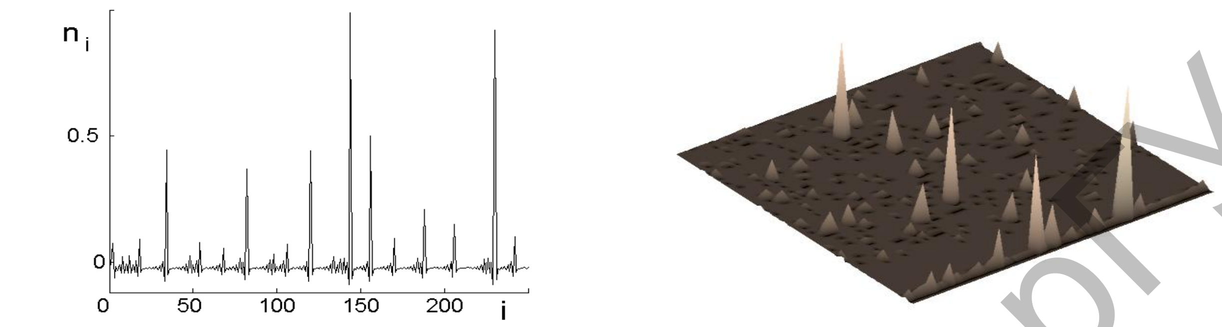

Figure 1. Realization of the map (a) and morphology (b) of quantum dots obtained at zero boundary conditions. $C_n = C_p = C_a = 0.9$; $\gamma_n = \gamma_p = \gamma_a = 3.8$; $X_{n,0} = X_{p,0} = 1/4$, $X_{a,0} = 1$; $\mu_0 = -1$.

(a) (b)

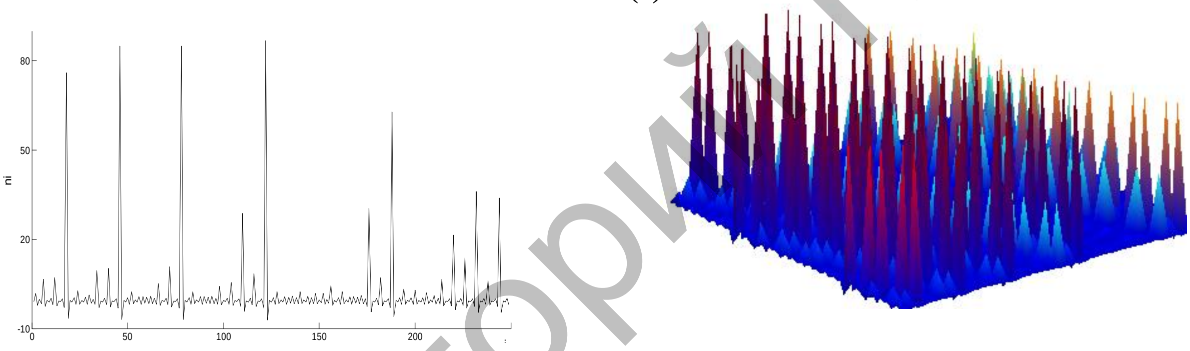

Figure 2. Realization of the map (a) and morphology (b) of quantum wires obtained at zero boundary conditions. $C_n = C_p = C_a = 0.9$; $\gamma_n = \gamma_p = \gamma_a = 2.8$; $X_{n,0} = X_{p,0} = 1$, $X_{a,0} = 1$; $\mu_0 = -1$.

(a) (b)

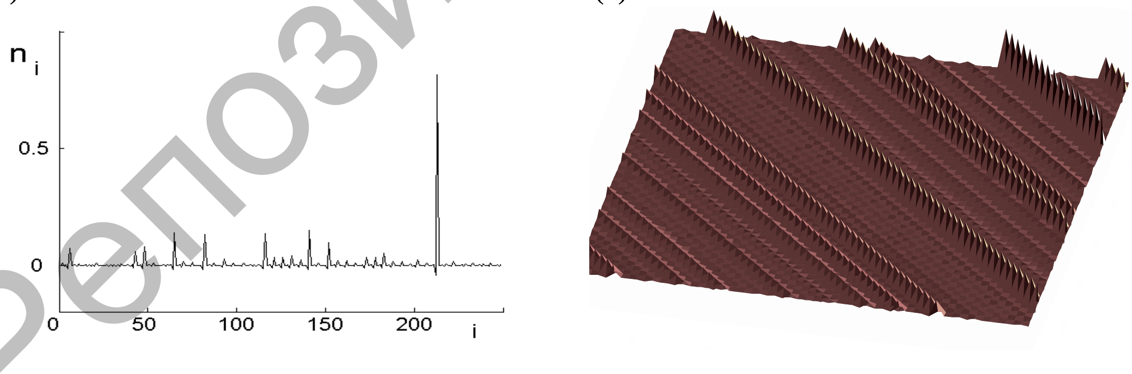

Figure 3. Realization of the map (a) and morphology (b) of linear structures (superlattices) obtained at zero boundary conditions. $\gamma_n = 1.618$, $\gamma_p = 1.194$, $\gamma_a = 1.806$; $C_n = C_p = C_a = 0.9$; $X_{n,0} = X_{p,0} = 1/4$, $X_{a,0} = 1$; $\mu_0 = -1$.

We explain the similarity of realizations in terms of physics. Fractality of processes used at development of the map is the main property of self-organized systems.

In the system of equations for the description of the generator of dynamical chaos we use nonlinear dependence of eigenfrequency of selective circuit on phase of feedback. This factor is also one of fundamental conditions of self-organization.

2. Nonlinear fractal measures

Fractals have different properties on different scales of measurement. Fractal is self-similar object if characterized by only one value of similarity factor on variables, and self-affine if several similarity factors need for its description. Semiconductor films have self-affine structure because of anisotropy, i.e. films characterized by a set of fractal dimensions. Fractal objects can be formed of dot, linear, spatial and volume structures with topological dimensions $d = 0, 1, 2, 3$. Fractal dimension of *n*-dimensional object is

$$D = d + \gamma,\ 0 \le d \le n,\ 0 < \gamma < 1. \tag{2.1}$$

We can describe all types of fractal structures by use of different combinations of d and D. Values of γ can be defined by the following considerations.

Let us use the well-known relation between fractal dimension D of intersecting surface of a chaotic object and information entropy S developed by B. Mandelbrot:

$$D = \tilde{d} - S, \tag{2.2}$$

where $\tilde{d}$ is topological dimension of space with embedded object. So, we have

$$\gamma = \tilde{d} - d - S,\ 1 - S \le \gamma \le \tilde{d} - S,\ S = (I_1, I_2),\ \tilde{d} = 2, 3, \tag{2.3}$$

where $I_1 = 0.567$, $I_2 = 0.806$ are fixed points of normalized information and entropy. Theory of these numbers is described in our previous works.

Main properties of fractals are self-similarity and dependence of measure on scale of measurement. We mean that measure is a physical value which can be characterized by additive measurable set. For example, measures of a geometrical fractal are its length, square and volume. Nanoobjects have surprising variety of physical properties because their measures depend on their values according to nonlinear laws. This fact brings out clearly to necessity of fractal analysis in nanoscience.

By use of well-known theories of fractals we choose minimal scale of measurement (size of cells covering an object) independently on value of defining measure. For the description of evolution of measure in dependence upon given parameter of order which is determining variable of a physical process, we choose the scale of measurement via this parameter and desired measure. Hence, fractal measure is a nonlinear function depending on the process.

The traditional definition of fractal measure M can be written as

$$M = M_0 \left(|\Delta M| / M_* \right)^{-\gamma},\ \gamma = D - d,\ \gamma > 0, \tag{2.4}$$

where M_0 is a regular (non-fractal) measure, ΔM is a scale of measurements, M_* is norm of M, D is fractal dimension of the set of values of M, d is topological dimension of norm carrier. ΔM is independent on M, therefore, measure defining by (2.4) can be tentatively called the linear value.

Dependence ΔM on M indicates to the existence of a external excitation, in common case it means the existence of order parameter. Let us consider λ as parameter of order. So, we can choose ΔM as

$$\Delta M_M = \frac{|M - \lambda|}{M} = \left| 1 - \frac{\lambda}{M} \right|,\ \Delta M_\lambda = \frac{|M - \lambda|}{\lambda} = \left| 1 - \frac{M}{\lambda} \right|, \tag{2.5}$$

where indexes M and λ correspond to the norms ΔM. According to (2.5) we can rewrite (2.4) as

$$M_M = M_0\left(\left|1-\frac{\lambda}{M}\right|\right)^{-\gamma},\ M_\lambda = M_0\left(\left|1-\frac{M}{\lambda}\right|\right)^{-\gamma}. \qquad (2.6)$$

At $\gamma \to 0$ we have $M_M = M_\lambda = M_0$, it corresponds to the meaning of M_0. At $\lambda = 0$ we have $M_M = M_0$, $M_\lambda = 0$. It means that the fractal measure defined by its own norm exists in a case when external influence characterized by parameter λ is absent.

3. Nonlinear map for modeling of morphology of nanostructures semiconductor films

From the system of nonlinear differential equations describing distribution of electrons, holes and impurities in nanostructured semiconductors we have the following nonlinear map [2]:

$$Y_{i+1} = \left(Y_i + sign\left(\psi_Y\right)\right)\left|\psi_Y\left(n_i, a_i, p_i, i\right)\right|^2 \left|\frac{Y_i}{Y_0}\right|^{-\frac{1}{\gamma_Y}}, \qquad Y = \left(n, a, p\right). \qquad (3.1)$$

Here n, p, a are concentrations of quasi-particles which are electrons, holes and clusters (defects of different types) in semiconductor, γ_Y are fractal dimensions of sets with self-similar and self-affine properties, Y_0 are equilibrium concentrations of electrons, holes and impurities. Generally, we can define wave functions from Shrödinger equation or by use of wave functions of corresponding regular ($\gamma = 0$) objects. In the case of closely coupled interface of electrons to clusters it is possible to work with wave functions centered on the centers of clusters $\vec{R}$, i.e. to use the analogues of Wannier functions $\varphi\left(\vec{r}-\vec{R}\right)$:

$$\psi_{\vec{k}}\left(\vec{r}\right) = \sum_{\vec{R}} \varphi\left(\vec{r}-\vec{R}\right)\exp\left(i\vec{k}\vec{R}\right). \qquad (3.2)$$

At the case of loosely coupled interface of electron to cluster we can use the plane waves as

$$\psi_n\left(x\right) = \psi_p\left(x\right) = \psi_a\left(x\right) = \cos\left(k_n x\right)\cos\left(k_p x\right)\cos\left(k_a x\right). \qquad (3.3)$$

Models of surfaces of semiconductor films shown on Figures 1-3 were obtained by use of formulas (3.1-3.3). Morphology of a film surface depends on fractal dimension γ_Y. By use of different values of γ_Y, Y_0 we cal plot images similar to photos of surfaces of semiconductor thin films obtained by use of modern methods of microscopy. Scanning tunneling microscopy image of 5 monolayers of *Ag* deposited on a *Si* substrate is shown in Figure 4(a) [5]. Model of the surface plotting according to our approach is shown in Figure 4(b). In Figure 5(a) we have shown a photo of $Si_{1-x}Ge_x$ ($x \approx 0.25$) on a silicon substrate obtained by use of atomic-force microscopy [6], model of the surface is shown in Figure 5(b). Authors of the works have noted that we have self-organized formations of nanostructures on surfaces of samples. Photos of surface of *GaN* with a layer of *InGaN* obtained by scanning tunneling microscopy are shown on Figure 6(a, c) [7]. The corresponding models of surfaces of the films are shown in Figures 6(b, d).

It is remarkable that the map (3.1) is a version of the universal map (1.8). We use the sign function of wave function ψ_Y with probability of realization equal to $(\psi_Y)^2$ instead of sign function μ_i. At the development of (1.8) we have noted that if sign of derivative defines by external conditions (existence of de Broglie waves for microparticles), so accuracy of observation C defines via absolute values of measurable values. Therefore, in (3.1) we use Y_i instead of $1/C$.

As follows from the figures, by use of system (3.1) we can describe different types of structures of semiconductor surfaces. Chaotic and fractal models describe in detail different types of microscopy imaged of surfaces of thin films. By use of our theory we take into account fractals dimensions of clusters, sets of electrons and holes, their non-equilibrium distribution, relative concentration of equilibrium components, type of wave functions of electrons. Using of our approach let us to plot models with dot, linear, spatial and volume structures at concrete parameters. Such types of structures exist on semiconductor surfaces.

(a) (b)

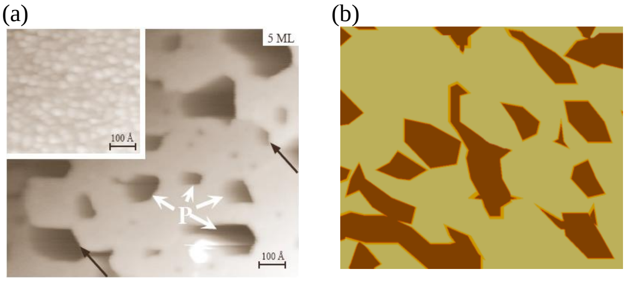

Figure. 4. Image for the Ag adsorption on a Si surface. (a) experimental data [5]; (b) theoretical results: $\gamma_n = \gamma_p = \gamma_a = I_2$, $n_0 = p_0 = a_0 = 5$, $n_1 = p_1 = 1$, $a_1 = 100$

(a) (b)

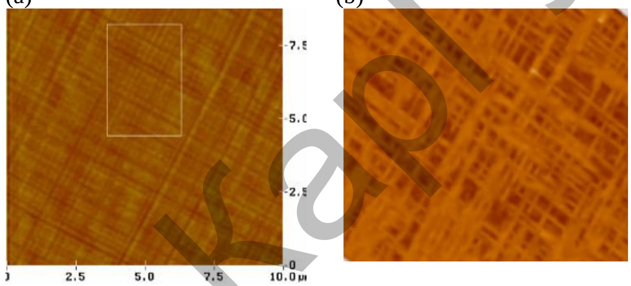

Figure 5. Morphology of a surface of $Si_{1-x}Ge_x$ on silicon substrate.(a) experimental data [6]; (b) theoretical results: $n_1 = p_1 = a_1 = 1$, $\gamma_n = \gamma_p = \gamma_a = 3 + I_2$, $n_0 = p_0 = a_0 = 1$

(a) (b) (c) (d)

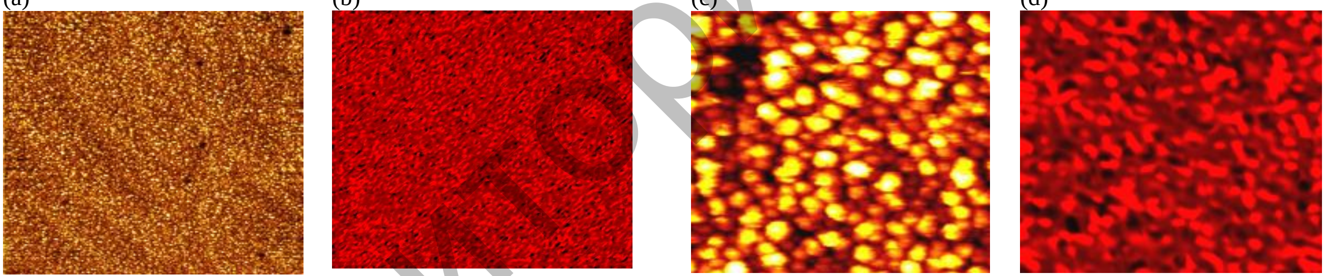

Figure 6. Image of a surface of $InGaN$ on GaN substrate.(a, c) experimental data [7]; (b, d) theoretical results: $n_1 = p_1 = 1$, $a_1 = 5$, $\gamma_n = \gamma_p = \gamma_a = I_2$, $n_0 = p_0 = a_0 = 5$.

This fact confirmed by photos of semiconductor thin films obtained by modern methods of microscopy. Thus, we can model topology of nanostructured surfaces via (1.8), (1.9) and (3.1). We expect that the first method can be used for the description of chaos appeared due to dynamics of nanostructures, but the second method can be used for the description of chaos appeared due to wave dynamics.

4.Exciton absorption

Excitons and exciton formations such as biexcitons and trions can be used for distinguishing quick-changing information signals. So, excitons are the subject of many studies on nanoelectronics. Exciton energy can be defined as nonlinear fractal measure. For this aim we choose different scales of measurement which are required energy and energy of exiting photon. So, energy of a single exciton E_1 can be described via equation (2.6) as

$$E_1 = E_0 \left(\left| 1 - \frac{\hbar w - E_g}{E_1} \right| \right)^{-\gamma} \equiv f\left(E_0, E_1\right), \quad E_{1,w} = E_{0,w} \left(\left| 1 - \frac{E_{1,w}}{\hbar w - E_g} \right| \right)^{-\gamma} \equiv f\left(E_{0,w}, E_{1,w}\right), \qquad (4.1)$$

where E_0 is exciton energy on the excitation threshold at $\hbar w = E_g$. In this case $E_{0,w} = 0$.

We can describe biexcitons, trions and other clusters as hierarchical structures:

$$E_n = f\left(\dots f\left(\frac{E_0}{n}, E_n\right)\dots\right), \tag{4.2a}$$

$$E_{n,F} = f\left(\dots f\left(\frac{E_{0,w}}{n}, E_{n,w}\right)\dots\right), n = 1,2,\dots, \tag{4.2b}$$

where number of brackets is equal to n. Equations (4.2*a*) correspond to the choice of scale of measurements of fractal measure relative to the measure and describe evolution of excitons, biexcitons and other structures existing in a ground state, i.e. without of external radiation. Equations (4.2*b*) correspond to the choice of scale of measurements of fractal measure relative to photon energy and describe excited states only. By use of equations (4.1), (4.2*a*), (4.2*b*) we can define energy of a system consisting of excitonic formations as

$$E = \sum_{i=1}^{n} E_i, \quad n = 1, 2, \dots \tag{4.3}$$

Here $n=1$ describes an exciton, $n=2$ describes biexciton, $n=3$ describes trion and so on.

In the simplest case, photoluminescence intensity and absorption coefficient $\alpha(w)$ are defined via density of number of states at the absence of nonradiative transitions in a semiconductor with intrinsic conductivity. So,

$$\alpha(w) = \alpha\left(E_g/\hbar\right)E(w)^{\frac{1+\gamma}{2}}. \tag{4.4}$$

Formula (4.4) includes γ due to taking into account fractality of impulse space for which we determine density of states. In this way, we can take into account influence of external field P on relative photoluminescence intensity of biexciton $\alpha_2(P)$ and exciton $\alpha_1(P)$ as

$$\alpha_2(x,w) = \alpha_2(w)\left(\frac{\left\langle \alpha_1(w)^2\right\rangle}{\left\langle \alpha_2(w)^2\right\rangle}\right)^{1/2} th(x)^{\frac{1+\gamma}{2}}, \quad x = \frac{P}{P_0}, \tag{4.5}$$

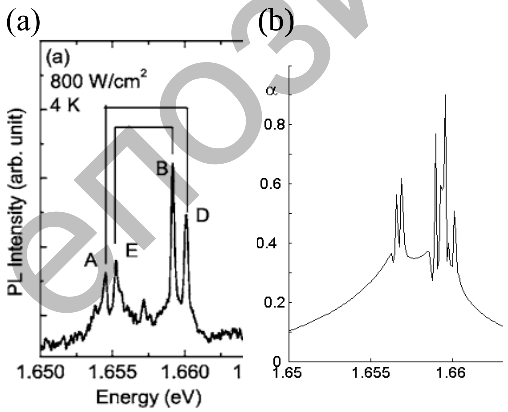

Figure 7. Photoluminescence spectrum from bilayer *InGaAs* quantum dots.
(a) experimental data [10], (b) theoretical results:
$E_g = 1.65\ eV, \gamma = 1 - I_2,\ E_0 = 4\cdot 10^{-3}\ eV$.

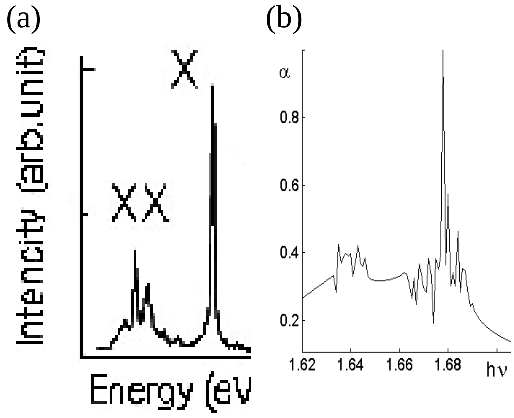

Figure 8. Exciton-biexciton photoluminescence spectrum from an isolated InGaAs quantum dot.
(a) experimental data [11], (b) theoretical results:
$E_g = 1.60\ eV, \gamma = 0.244,\ E_0 = 5\cdot 10^{-3}\ eV$.

Here P_0 is relative minimal intensity for saturation of $\alpha_2(x,w)$. Function $th(x)$ is used for accounting of well-known expression for variation of number of coherent oscillators, angle brackets mean frequency averaging.

Figure 7(a) shows experimental exciton photoluminescence spectrum from *InGaAs* quantum dots fabricated by metalorganic vapor epitaxy [10]. The sample has two quantum dot layers. Separation between each layer is about 5 nm. Quantum dots were formed by self-organization. Each peak in the spectrum corresponds to excited state of an exciton or biexciton. Figure 7(b) shows result of simulation by equations (4.2*a*), (4.3) and (4.4).

Figure 8(a) shows exciton-biexciton spectrum of an isolated quantum dot of *InGaAs* [11] recorded at 4 K . Figure 8(b) illustrates the exciton-biexciton spectrum, which was calculated by equations (4.2a), (4.3) and (4.4). Figures 7(b) and 8(b) are plotted in $\Delta\hbar w = 10^{-4}$ eV steps.

Comparison of our theoretical results t corresponding experimental data have shown that our theory can be used for the description basic spectral regularities.

5. Neural networks

Main problem in neural dynamics is simulation of self-coordinated operations in an ensemble of interacting elements with different coupling under external excitation. The modeling neural network must be characterized by properties of associativity, resistance to noise, distributed character of information storage, adaptability to forming of couplings between elements, i.e. by properties of function of brain. In respect to laws of physics of open systems, nonlinear dynamics of neural network must be characterized by scale invariance, fractality, presence of chaos, defined phase relations and so on. So, it is necessary to develop a universal physical model for the description of the indicated properties. One of the possible version in this direction is study of nonlinear fractal models of excitons and theirs formations (biexcitons, trions and so on). We have shown the possibility of the description of modern experiments by use of this method [15]. It is possible to use the base equation for the description of dynamics of neurons and nerve cells.

Experiments on measurements of time dependence of neurons potential show the presence of the following laws. Under the influence of stimulus (external potential), neurons move from the ground state to an excited state. The transition is observed in the form of spike potential and their clusters. This phenomenon is called the "accumulation - bursting" phenomenon. Neurons can generate not only burst fluctuations but also a variety of periodic, chaotic and noise-like oscillations. Therefore, time realizations of the potential of a neuron, in general, can have asymmetrical intermittent (alternating with the relative order of the explosions) character.

Different models for the description of some properties of neurons have been developed [13]. We shall consider a possibility of creation of simplest nonlinear model for the description of main regularities of neural dynamics (asymmetrical alternation, presence of chaos, hierarchy and self-similarity, depolarization, phase reinstallation).
Neurons have inherent properties of quasi-particles, they can’t exist without movement and outside of medium, in which they are fluctuations. Neurons communicate by sending action potentials. Therefore it is natural to assume that the action potentials has modulation - periodic nature, and we can take the external field in the form

$$F(t) = A(1 + B\sin(\Omega t)), \qquad (5.1)$$

considering only the low-frequency variation of the modulated oscillations amplitude. For a system of *N* neurons in order to simplify the numerical analysis of equation, equations (4) can be written in iterative form

$$V_{i+1}^{(k)} = V_0^{(k)}\left(\left|1 - F^{(k)}(t)\Big/\sum_{k=1}^{N} V_i^{(k)}\right|\right)^{-\gamma_{\mathbf{k}}}, \qquad (5.2a)$$

$$V_{i+1,F}^{(k)} = V_{0,F}^{(k)}\left(\left|1-\sum_{k=1}^{N} V_{i,F}^{(k)} \Big/ F^{(k)}(t)\right|\right)^{-\gamma_k}. \qquad (5.2b)$$

Here k – number of a neuron. By use of equation (5.2*a*) we can take into account the possibility of realization of intrinsic subthreshold oscillations of neurons at $F(t)=0$ By use of (5.2*b*) we can take into account presence of the stimulus $F(t)\neq 0$ only.

We will write the equation for the system of neurons, in which the action potential of neuron depends only on the neighboring neuron. The modulation-periodic external field will affect only the first neuron ($V^{(1)}=F, k\geq 2$). So, we have

$$V_{i+1}^{(k)} = V_{0}^{(k)}\left(\left|1-V_i^{(k-1)} \Big/ \sum_{k=1}^{N} V_{i}^{(k)}\right|\right)^{-\gamma_k}, \qquad (5.3a)$$

$$V_{i+1,F}^{(k)} = V_{0,F}^{(k)}\left(\left|1-\sum_{k=1}^{N} V_{i,F}^{(k)} \Big/ V_{i,F}^{(k-1)}\right|\right)^{-\gamma_k}. \qquad (5.3b)$$

More complex clusters are defined as ensembles of neurons which have a hierarchical structure of order n:

$$V_n = f\left(\dots f\left(\frac{V_0}{n}, V_n\right)\dots\right), \qquad (5.4a)$$

$$V_{n,F} = f\left(\dots f\left(\frac{V_{0,F}}{n}, V_{n,F}\right)\dots\right), n=1,2,\dots, \qquad (5.4b)$$

where the number of brackets is equal to n.

Experimental results of R.Baker, W.Precht и R.Llinas [14] describing normal and deserebellate animals were obtained from cats anesthetized with an initial dose of sodium pentobarbital. (Figure 9). A similar picture of the temporal evolution of the action potential of neurons can be modeled by (5.2*a*).

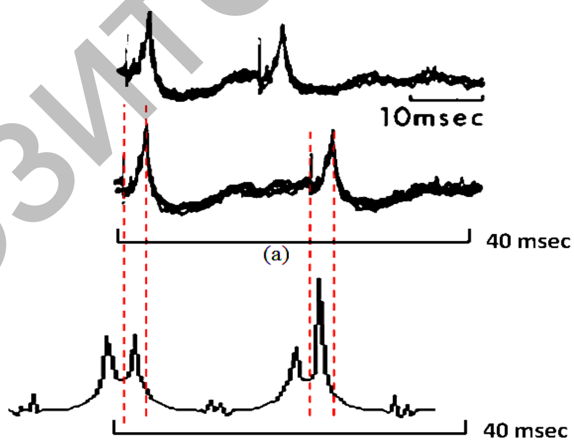

Figure 9. a) Interaction potential in normal and decerebellate animal [14]. b) realization of (5.2*a*), (5.4*a*) at n=2. $\gamma=\gamma_1=\gamma_2=\gamma_3=1-I_2, V_0^{(1)}=0.194, V_0^{(2)}=0.191, V_0^{(3)}=0.199, A^{(1)}=0.373, A^{(2)}=0.512, A^{(3)}=0.397, B^{(1,2,3)}=0.5129$.

Phase fluctuations is an important characteristic of neural systems. Figure 10*a* 6 presents six action potentials of neurons with different phases according to (5.2*a*). We notice that after depolarization of the action under stimulus phase oscillations are captured. This regime is closed to coherent regime (Figure 10*b*). The same effect is obtained by use of the formula (5.2*b*). This result observed in experiments is called the phase reinstallation.

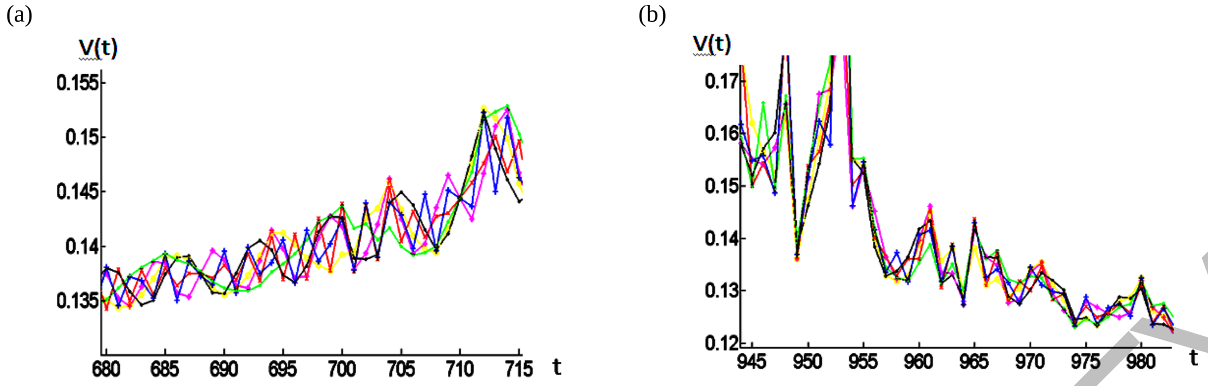

Figure 10. Realization of phase reinstallation of a neural network at $\gamma=\gamma_1=\gamma_2=\gamma_3=\gamma_4=\gamma_5=\gamma_6=1-I_2$, $V_0^{(k)}=0.1$, $1\le k\le 6$, $\Omega_1=1, \Omega_2=3, \Omega_3=5, \Omega_4=7, \Omega_5=10, \Omega_6=13$.

Conclusions

For the first time the map for the description of spatial-time evolution of fractal measure is suggested in the present paper. It is shown that the map can be applied for modeling of morphology of nanostructured surfaces, dynamics of excitonic formations, optical processes in nanostructures, basic regularities of neural networks. Results of the present work can be used as physical base for investigations of nanostructures, for development and perfection of optimal technology of nanoelectronics, optoelectronics and computer technologies.

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