BOUNDEDNESS OF THE HIGHER DIMENSIONAL HILBERT OPERATOR IN REARRANGEMENT INVARIANT QAUSI-BANACH SPACES

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The talk is based on a recently published joint work [2] with F. Sukochev and D. Zanin where weobtained a weak (1, 1) type estimate for a higher dimensional Hilbert operator answering an openquestion by A. Osekowski [1]. This result together results in [3] allow us to investigate boundednessof the Hilbert operator in rearrangement invariant quasi-Banach spaces.

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ON JOINTLY CONCAVITY OF SOME TRACE FUNCTIONS

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Let $\mathbb{M}_n$ be the von Neumann algebra of $n \times n$ complex matrices, and let $\mathbb{M}_n^+ = \{A \in \mathcal{M}\colon A \geq 0\}$ and $\mathbb{M}_n^{++} = \{A \in \mathcal{M}\colon A > 0\}$. In [3], Carlen and Lieb proved the following trace function on $\mathbb{M}_n^+$:

$$F_p(A_1, A_2, \ldots, A_m) = [Tr((\sum_{k=1}^m A_k^p)^{\frac{q}{p}})]^{\frac{1}{q}} = \| (\sum_{k=1}^m A_k^p)^{\frac{1}{p}} \|_q \qquad (1)$$

is jointly concave in $(A_1, A_2, \cdots, A_m) \in \mathbb{M}_n^+ \times \mathbb{M}_n^+ \cdots \times \mathbb{M}_n^+$, for $0 < p \leq q \leq 1$. Bekjan [1] obtained that for $0 < p \leq 1$,

$$F_p(A_1, A_2, \ldots, A_m) = Tr((\sum_{k=1}^m A_k^{-p})^{-\frac{1}{p}}) \qquad (2)$$

is jointly concave in $(A_1, A_2, \cdots, A_m) \in \mathbb{M}_n^{++} \times \mathbb{M}_n^{++} \cdots \times \mathbb{M}_n^{++}$. In [4], Hiai extended these results as following: if $\Phi_k\colon \mathbb{M}_{n_k}^+ \to \mathbb{M}_n^+$ is a strictly positive linear map ($k = 1,2,\cdots,m$), either $0 < p \leq 1$ and $0 < s \leq \frac{1}{p}$, or $-1 \leq p < 0$ and $\frac{1}{p} \leq s < 0$, then

$$F_p(A_1, A_2, \ldots, A_m) = ||(\sum_{k=1}^m \Phi_k(A_k^p))^s||_!$$

is jointly concave in $(A_1, A_2, \cdots, A_m) \in \mathbb{M}_{n_1}^+ \times \mathbb{M}_{n_2}^+ \cdots \times \mathbb{M}_{n_m}^+$, where $||.||_!$ is a symmetric anti-norm on $\mathbb{M}_n^+$.

Let $\mathcal{M}$ be a finite von Neumann algebra with a normal faithful finite trace τ and $L_0(\mathcal{M})$ be the set of all measurable operators with respect to $(\mathcal{M}, \tau)$. The topology of $L_0(\mathcal{M})$ is determined by the convergence in measure. Set $L_0(\mathcal{M})^+ = \{x\colon x \in L_0(\mathcal{M}),\ x \geq 0\}$, $\mathcal{M}^+ = \{x\colon x \in \mathcal{M},\ x \geq 0\}$ and $\mathcal{M}^{++} = \{x\colon x \in \mathcal{M},\ x > 0\} = \{x\colon x \in \mathcal{M},\ x \geq 0\ \ and\ \ x\ \ is\ invertible\}$. The aim of this talk is to prove that [(i)]

1. if $f\colon [0,) \to [0,)$ is an operator concave function, $0 < p \leq 1$, $0 < s \leq \frac{1}{p}$ and Φ_j is a continuous positive linear map from $L_0(\mathcal{M}_j)$ to $L_0(\mathcal{M})$ with $\Phi_j(\mathcal{M}_j) \subset \mathcal{M}$, where $\mathcal{M}_j$ is finite von Neumann algebra, $j = 1,2,\cdots,n$, then for $0 \leq t < \tau(1)$

$$\int_t^{\tau(1)} \mu_v((\sum_{j=1}^n \Phi_j(f(x_j^p)))^s)dv \quad and \quad \int_t^{\tau(1)} \mu_v((\sum_{j=1}^n \Phi_j(f(x_j)^p))^s)dv$$

are jointly concave in $(x_1, x_2, \cdots, x_n) \in L_0(\mathcal{M}_1)^+ \times L_0(\mathcal{M}_2)^+ \times \cdots \times L_0(\mathcal{M}_n)^+$,

2. if $f:(0,)\to(0,)$ is an operator concave function, Φ_j is a strictly positive linear map from finite von Neumann algebra $\mathcal{M}_j$ to $\mathcal{M}$, $j=1,2,\cdots,n$, $0<p\le 1$ and $0<s\le\frac{1}{p}$, then for $0\le t<\tau(1)$,

$$\int_t^{\tau(1)}\mu_v((\sum_{j=1}^n\Phi_j(f(x_j^{-p})))^{-s})dv\quad and\quad \int_t^{\tau(1)}\mu_v((\sum_{j=1}^n\Phi_j(f(x_j)^{-p}))^{-s})dv$$

are jointly concave in $(x_1,x_2,\cdots,x_n)\in\mathcal{M}_1^{++}\times\mathcal{M}_2^{++}\times\cdots\times\mathcal{M}_n^{++}$,
where $\mu_t(z)$ is the generalized singular number of $z\in L_0(\mathcal{M})$.

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ОЦЕНКИ НАИЛУЧШИХ *M*-ЧЛЕННЫХ ПРИБЛИЖЕНИЙ НА КЛАССАХ ФУНКЦИЙ С ОГРАНИЧЕННОЙ СМЕШАННОЙ ПРОИЗВОДНОЙ В ПРОСТРАНСТВЕ ЛОРЕНЦА

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Через $L_{p,\tau}$ обозначается пространство Лоренца всех вещественнозначных измеримых по Лебегу функций f, которые имеют 2π-период по каждой переменной и для которых

$$\|f\|_{p,\tau}=\left\{\frac{\tau}{p}\int_0^1\left(f\quad(t)\right)^{\tau}t^{\frac{\tau}{p}-1}dt\right\}^{\frac{1}{\tau}}<+\infty,1<p<\infty,1\le\tau<\infty,$$

$f\quad(t)$ – невозрастающая перестановка функции $|f(2\pi\overline{x})|$, $\overline{x}\in[0,1)^m$. Пусть $\overline{r}=(r_1,\dots,r_m)$, $r_j>0, j=1,\dots,m$ и $F_{\overline{r}}(\overline{x})$ –m-мерное ядро Бернулли (см. [2], [3]). Рассмотрим функциональный класс $W_{p,\tau}^{\overline{r}}=\{f:f=\varphi\star F_{\overline{r}},\|\varphi\|_{p,\tau}\le 1\}$, где $1<p<\infty,1\le\tau<\infty$,

$$(\varphi\star F_{\overline{r}})(\overline{x})=\frac{1}{(2\pi)^m}\int_{T^m}(\varphi(\overline{x}-\overline{u})F_{\overline{r}})(\overline{u})d\overline{u},\ T^m=[0,2\pi)^m.$$

В случае $\tau=p$ класс $W_{p,\tau}^{\overline{r}}$ рассмотрен в [1], [2]. $e_M(f)_{p,\tau}$ – наилучшее -членное тригонометрическое приближение функции $f\in L_{p,\tau}, M\in N$.

В докладе будут представлены точные по порядку оценки наилучших -членных приближений функций класса$W_{q,\tau_1}^{\overline{r}}$ в пространстве L_{p,τ_2} при различных соотношениях между параметрами p,q,τ_1,τ_2. В частности,

Теорема. Пусть $0<r_1=\cdots=r_\nu<r_{\nu+1}\le\cdots r_m$, $1<q\le 2<p<\infty,1<\tau_1<\tau_2<\infty$ и $b\in R$. Если $1<q<2$ и $r_1>\frac{1}{q}$, то

$$e_M\left(W_{q,\tau_1}^{\overline{r}}\right)_{p,\tau_2}\succ\prec M^{-\left(r_1+\frac{1}{2}-\frac{1}{q}\right)}(log_2M)^{(\upsilon-1)\left(r_1-\frac{1}{q}+\frac{1}{\tau_1'}\right)}.$$

Если $q=2$ и $r_1>\frac{1}{2}$, то

$$e_M\left(W_{2,\tau_1}^{\overline{r}}\right)_{p,\tau_2}\le CM^{-r_1}(log_2M)^{(\upsilon-1)\left(r_1-\frac{1}{2}+\frac{1}{\tau_1'}\right)+\frac{1}{2}-\frac{1}{\tau_1}}.$$

В случае $\tau_1=q$, $\tau_2=p$ из полученных результатов следуют теорема 3 в [1] и теоремы 2.1 и 2.2 в [2], а также теорема 1.2 в [3].