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Т.Бәкібаев, Қ.Әбешев

Ашық ГИС карталарының қозғалыс сызықтарының графына бейімделуі

Мақалада «Open street maps» сайтынан алынған көше графтарының қозғалыс сызықтарының графтарына автоматты бейімделуінің алгоритмі ұсынылған. Жұмыс барысында бірнеше қиындықтар кездесті және олардың бірқатары шешімсіз қалды. Мақаланың негізгі мақсаты көше графтары қабырғаларының «оң жағынан» жаңа қабырғалар құру және оларды біріктіру болып табылады. Барлық шешім жолдары мәтінде сипатталған, сондай-ақ бағдарлама кодтары берілген.

Т.Бакибаев, К.Абешев

Адаптация открытых ГИС карт к графу полос движения

В статье описан алгоритм автоматической адаптации графа улиц, взятого с «Open street maps», к графу полос движения. Во время работы мы столкнулись с несколькими проблемами, некоторые из которых остались открытыми. Основная идея заключается в создании новых рёбер «справа» от рёбер графа улиц и соединении их. Все решения описаны текстом, затем следует программный код.

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B.Poizat

Claude Bernard Lion1 University, Camille Jordan Institute, France (E-mail: poizat@math.univ-lyon1.fr)

Fundaments of Positive Logic

The main purpose of this paper is to give the basic concepts of positive logic. The content of this work is the essence of the course of lectures given by me in the Academician Ye.A.Buketov Karaganda State University in the months of April and May 2013.

Key words: Homomorphisms, positive formulae, inductive limits, inductive classes, universal sentences, compactness, companion theories, model-complete theories.

Outlet

Chapter 0a.

Algebraically Closed Fields

K a field; a system of two polynomial equations, one variable, coefficients in K,

$$P(x) = a_m \cdot x^m + ... + a_1 \cdot x + a_0 = 0;$$

$$Q(x) = b_n \cdot x^n + ... + b_1 \cdot x + b_0 = 0.$$

If $m \geq n$, can be replaced by:

$$b_n \cdot P(x) - a_m \cdot x^{m-n} \cdot Q(x) = c_{m-1} \cdot x^{m-1} + ... = 0.$$

$$Q(x) = 0$$

Provided that $b_n \neq 0$!

We repeat and finally replace the two equations by one, gcd (P (x), Q (x)) = 0. The gcd has the expected form provided that some quantities polynomially expressible in function of the a_i and the b_j be $\neq 0$.

Distinguish cases!

For this reason, we shall speak of *conditionned* gcd.

To replace two inequations by one is trivial: $P(x) \neq 0 \wedge Q(x) \neq 0 \Leftrightarrow P(x) \cdot Q(x) \neq 0$.

Similarly, one equation and one inequation $P(x) = 0 \wedge Q(x) \neq 0$ can be replaced by one equation: divide P by gcd(P,Q) and repeat, till we obtain P_1 prime to Q. The system is equivalent to $P_1(x) = 0$.

Here again we distinguish cases.

Finite Boolean combination of polynomial equations, say with integer coefficients, in several variable $x_0, x_1, ..., x_n$.

Use the rules $\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$, $\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$, $\neg(\neg A) \Leftrightarrow A$, $A \wedge (B \vee C) \Leftrightarrow$ $\Leftrightarrow$ $(A \wedge B) \vee (A \wedge C)$, $A \vee (B \vee C) \Leftrightarrow A \vee (B \vee C) \Leftrightarrow A \vee B \vee C$ to express the Boolean combination as a disjunction (union) of systems (conjunction, intersection) of equations and in equations.

Single out the variable x_0; by the process of elimination described above, we can assume that each system in the disjunction contains at most one equation or one inequation involving x_0.

Repeating, we can even assume that each system has a triangular form.

Projections, or existential quantifications.

Assume now that K is algebraically closed, satisfying for each $d > 0$ the axiom:

$$(\forall c_{d-1}, ... c_0)(\exists x)\ x^d + c_{d-1} \cdot x^{d-1} + ... + c_0 = 0.$$

In K, $(\exists x)\ a_m \cdot x^m + ... + a_1 \cdot x + a_0 = 0$ is equivalent to $a_m \neq 0 \vee ... \vee a_1 \neq 0 \vee a_0 = 0$ and $(\exists x)\ b_n \cdot x^n + ... + b_1 \cdot x + b_0 \neq 0$ is equivalent to $b_n \neq 0 \vee ... \vee b_1 \neq 0 \vee b_0 \neq 0$.

Using our elimination process and the rule $(\exists x)\ \varphi(x) \vee \psi(x) \Leftrightarrow (\exists x)\ \varphi(x) \vee (\exists x) \psi(x)$ we obtain.

To every Boolean combination $B(y, x)$ of polynomial equations with integer coeff. is associated another one $B^*(x)$ such that, in any ac field, $(\exists y)\ \ B(y, x) \Leftrightarrow B^*(x)$.

And even: To every (finite) formula $\varphi(\underline{x})$ constructed from pol. eq. with integer coeff. with the help of $\neg, \vee, \wedge$ and $\exists$ is associated a boolean comb. $B(x)$ of such equations such that, in any ac field, $\varphi(\underline{x}) \Leftrightarrow$ $B(x)$.

Note that, from a complexity point of view, our process of elimination of quantifiers is rather ineffective; but no truly efficient process of elimination is known [1–4].

A formula σ with no free variables is called a sentence, which is true or false in a given field. For ac fields, it is equivalent to a (finite) boolean combination of equations of the form $n = 0$, where n is an integer.

Since a field satisfies $n = 0$ iff n is divisible by its characteristic, we obtain the following three corollaries:

Corollary 1. Two ac fields of the same characteristic satisfy the same sentences.

Corollary 2. If a sentence is true in some ac field of char. p *for infinitely many primes* p, *then it is true in every ac field of char.* 0.

Corollary. 3 If a sentence is true in some ac field of char. 0, *then it is true in every ac field of char.* p *for all* p *except finitely many.*

Example: the Jacobian Conjecture

$F(x, y) = (P(x, y), Q(x, y))$ a polynomial map from $C \times C$ to $C \times C$, with complex coeff. We assume that its jacobian never vanishes, that is, is constant: $\frac{\partial P}{\partial x} \cdot \frac{\partial Q}{\partial y} - \frac{\partial P}{\partial y} \cdot \frac{\partial Q}{\partial x} = 1$.

Open question: is F injective?

Theorem 1. If *F* is injective, it is surjective.

Proof. Assume a counter-example of degree *d*; the following sentence, where $A \Rightarrow B$ is for $\neg A \vee B$, and $\forall$ for $\neg\exists\neg$, is true for some p in the algebraic closure of $F_p = Z/pZ$, which is locally finite:

$(\exists$ coefficients$)$ $\left[(\forall x, y, u, v)\ P(x, y) = P(u, v) \wedge Q(x, y) = Q(u, v) \Rightarrow x = u \wedge y = v\right] \wedge$ $\left[(\exists x, y)(\forall u, v)\ P(x, y) \neq P(u, v) \vee Q(x, y) \neq Q(u, v)\right]$.

Exercices

K is an ac field. Definable means a subset of a cartesian power of *K* which is defined by a formula $\varphi(\underline{x}, \underline{a})$ with parameters in *K*.

Exercices 1. (Ax's Thm.) Let *A* a definable set and f an injective definable map from *A* into *A*; show that it is surjective. (Hint: if *k* is a finite field containing *a*, then *k* is closed under *f*).

Exercices 2. For which characteristics there is a definable map from *K* to *K* whose each fiber has exactly two elements.

Exercices 3. Let *G* be a definable group, *q* a prime number, and *a* and *b* two elements of *G* of order *q*; show that there *c* in *G* of order *q* which comutes with *a* and a conjugate of *b*. (Hint: every sentence true in *G* is true in a locally finite group).

Exercices 4. (Jordan's decomposition) Show that, in a definable subgroup of GLn(*K*), every element can be written in a unique way as the product of a semi-simple element and a unipotent element which comute.

Note: It can be proven that a definable (the geometers say «constructible») group is definably isomorphic to an algebraic group.

Chapter 0b.

Existentially closed groups

Group equation: $W(x_1, \ldots x_n; a_1, \ldots a_m) = 1$ where W is a word in $x_1, \ldots x_n, a_1, \ldots a_m$ and their inverses; $x_1, \ldots x_n$ are unknowns and $a_1, \ldots a_m$ are parameters in *G*.

G is existentially closed: every finite system of equations and inequations(several variables, coeff. in *G*) which has a solution in an overgroup of *G* has already a solution in *G*.

HNN-theorem. If *f* is an isomorphism between two subgroups *A* and *B* of *G*, then there is an overgroup of *G* with an element c such that, for every a in *A*, *c* - 1·a·*c* = *f*(*a*).

In an ec group, $a_1, \ldots a_n$ and $b_1, \ldots b_n$ are conjugated iff they generate isomorphic subgps iff they satisfy the same formulae. But the formula $(\exists u)\ u \cdot x = y \cdot u$ is equivalent to the conjunction of infinitely many quantifier-free formulae $x^n = 1 \Leftrightarrow y^n = 1$!

Exercices 5. A finitely generated group is isolated if its isomorphism type is determined by a finite number of equations and inequations.

(i) Show that the notion does not depends of the chosen finite set of generators.

(ii) Show that finite groups are isolated.

(iii) *G* is isolated iff it is finitely presented and there is a finite number of conjugacy classes $\neq 1$ such that every $a \neq 1$ has a product of conjugates in one of them.

Note: It is not known if there are isolated infinite groups.

Chapter 1.

Homomorphisms and positive formulae

Structure: a non-void set M, with some distinguished elements called *constants,* equipped with some *functions* (from M^n to M) and *relations* (an n-ary relation is a subset of M^n). The list of the constants, the functions and the relations, with their arities, is called the *language L* of the structure; it implicitely contains a binary relation symbol = denoting equality between element of M.

Exemples

Language of groups: one constant 1, one unary function x - 1, one binary function $x.y$ (and the binary relation $x = y$)

Language of rings: 1, 0, $-x$, $x + y$, $x.y$, =

Language of loose partial orders: ≤ ,=

Language of strict partial orders: < ,=

Homomorphism between L-structures: a map h from M to N s.t.:

- $h(c_M) = c_N$;
- $h(f_M(a_1,...a_n)) = f_N(h(a_1),...h(a_n))$;
- if $M \models r_M(a_1,...,a_n)$ then $N \models r_N(h(a_1),...,h(a_n))$.

Monomorphism (or embedding): h is injective and $M \models r_M(a_1,...,a_n)$ if and only if $N \models r_N(h(a_1),...,h(a_n))$.

Isomorphism: bijective monomorphism.

Exemples: homomorphisms of groups, of rings, of loose and strict partial orders.

Question: What is preserved by homomorphisms?

Answer: Positive formulae

Terms: variables, constants, and what is obtained from them by composing the functions of the language.

In groups, term = word; in rings, term = polynomial with integer coeff.; in orders, loose or strict, term = variable.

Atomic formulae: $t = t'$, $r(t_1,...t_n)$, where the t_i are terms.

In groups and rings, equation; in loose orders, $x \leq y$, $x = y$; in strict orders $x < y$, $x = y$.

Positive formulae: obtained from atomic formulae by the use of $\vee$, $\wedge$ and $\exists$.

$\varphi \wedge \psi$ true $\equiv$ φ true and ψ true.

$\varphi \vee \psi$ true $\equiv$ φ true or (non-exclusive) ψ true.

$(\exists x)\ \varphi(x)$ true $\equiv$ $\varphi(a)$ true for some a in M.

Using the rules $(\exists x)\ \varphi(x) \vee (\exists x)\ \psi(x) \Leftrightarrow (\exists x)\ \varphi(x) \vee \psi(x)$ and $(\exists x)\ \varphi(x) \wedge (\exists x)\ \psi(x) \Leftrightarrow$ $\Leftrightarrow$ $(\exists x, y)\ \varphi(x) \wedge \psi(y)$, a positive formula can be written $(\exists \underline{x})\ \varphi(\underline{x})$ where φ is quantifier-free.

Theorem 2. If h is a homomorphism from M to N and a is a tuple of elements of M, then every positive formula satisfied by a in M is satisfied by $h(a)$ in N.

If every a in M satisfies the same positive formulae than $h(a)$, we say that h is pure, or is a retromorphism (or an immersion).

We say that M is *positively existentially closed (pec)* in the class C of structure if every homomorphisme from M to any N in C is pure.

Exercices.6 (i) Who are the pec groups? (For the usual notion of ec groups, we must use — positively — the language 1, x^{-1}, $x.y$, =, ≠)

(ii) Who are the pec loose partial orders?

Who are the pec strict partial orders?

Who are the pec rings?

Chapter 2.

Inductive limits and inductive classes

Consider an ascending sequence of groups

$$G_0 \subseteq G_1 \subseteq ... \subseteq G_n \subseteq G_{n+1} \subseteq ...$$

G_n being a subgroup of G_{n+1}; on the union $G = \cup G_n$ we obtain a group structure, called the *inductive limit* of the sequence.

When we have a sequence of embeddings

$$G_0 \overset{h_0}{\to} G_1 \overset{h_1}{\to} \dots \overset{h_{n-1}}{\to} G_n \overset{h_n}{\to} G_{n+1} \overset{h_{n+1}}{\to} \dots$$

we can assimilate $h_n(G_n)$ to a subgroup of G_{n+1} and do the same, but the construction works even when the h_n are non-injective homomorphisms: take the disjoint union of the G_n and identify a in G_n with $h_n(a)$.

It works also for rings, and in fact for every sequence of homo. between *L*-structures:

$$M_0 \overset{h_0}{\to} M_1 \overset{h_1}{\to} \dots \overset{h_{n-1}}{\to} M_n \overset{h_n}{\to} M_{n+1} \overset{h_{n+1}}{\to} \dots$$

atomic formulae (in particular equalities) being true in the inductive limit *M* if they are true for large enough *n*.

Limits along the integers are not enough: one has to consider M_i indexed by a totally ordered set *I*, with h_{ji} from M_i into M_j if $i < j$, s.t. h_{kj} o $h_{ji} = h_{ki}$ si $i < j < k$.

An *inductive* class of *L*-structures is a class closed under inductive limits of homomorphisms (Ex.: groups, rings, orders).

Axiom of Choice

In an inductive class, every point can be continued into a pec element.

Proff Setting $M = M_0$, construct a sequence M_n such that every quant. free pos. formula $\varphi(\underline{a}, \underline{x})$, $\underline{a} \in M_n$, with a solution in a continuation of M_{n+1} has a solution in M_{n+1}; then take the limit.

Exercices 7. Let *E* be an equivalence relation between elements of a set $A \neq \varnothing$. The language *L* consists in a unary rel. *X*(*x*) and a constant symbol for each element of *A*. The class *C* of *L*-structures is as follows: all the symbols in *A* are interpreted by distinct elements, and *X* does not contains two points in *A* which are congruent modulo *E*. Show that *C* is inductive and non-void. Who are the pec elements of *C*?

Chapter 3.

Inductive and universal sentences

Terminus: one-point structure satisfying all the relations in the language.

To avoid ending in Terminus, and produce more interesting structures, one has to respect a minimum of negative conditions.

Question The negative thing in groups with ≠ and in strict orders is clear, but where it is in rings?

An *h*-universal sentence declares that a certain positively defined set is void; it has the form: $\neg(\exists \underline{x})\, \varphi(\underline{x})$, equivalent to $(\forall \underline{x})\, \neg\varphi(\underline{x})$, where φ is positive qtf.-free. It is the negation of a positive sentence!

If there is an homomorphism from *M* to *N* and it is true in *N*, then it is true in *M*.

Exercices 8. Show that all the groups (without ≠) satisfy the same *h*-universal sentences.

A basic *h*-inductive sentence says that a positively defined set is included into another. It has the form $(\forall \underline{x})[(\exists \underline{y})\, \varphi(\underline{x}, \underline{y}) \Rightarrow (\exists \underline{z})\, \psi(\underline{x}, \underline{z})]$, or $(\forall \underline{x}, \underline{y})(\exists \underline{z})\, \neg\, \varphi(\underline{x}, \underline{y}) \vee \psi(\underline{x}, \underline{z})$, or $(\forall \underline{u})(\exists \underline{v})\ \neg\varphi'(\underline{u}) \vee \psi(\underline{u}, \underline{v})]$.

General *h*-inductive sentences are conjunction of basics. They go through inductive limits: if the *h*-inductive sentence is true in each of the *Mj*, it is true in the limit *M* !

When we add to the language a special 0-ary symbol $\perp$ to define positively the empty set, it is apparent that *h*-universal sentences are a special case of *h*-inductive sentences, of the form: $(\forall \underline{x})\left[(\exists \underline{y})\varphi(\underline{x}, y) \Rightarrow \perp\right]$.

Exercices 9. Check that the notions of group, ring, strict and loose order, and the class in Ex. 7 are axiomatized by *h*-ind. sentences.

Other special cases:

– positive $\forall\exists$-sentences, $(\forall \underline{x})\ \ (\exists \underline{y})\varphi(\underline{x}, y)$, of the form $(\forall \underline{x})\left[(\exists u)u = u \Rightarrow (\exists \underline{y})\varphi(\underline{x}, y)\right]$, in particular $(\forall \underline{x})\varphi(\underline{x})$, which is *not* *h*-universal, and $(\exists \underline{y})\varphi(\underline{y})$;

– «negation»: $\neg(\exists \underline{x})\varphi(\underline{x}) \wedge \psi(\underline{x})$ together with $(\forall \underline{x})\varphi(\underline{x}) \vee \psi(\underline{x})$;

– «graph»: $(\forall \underline{x})\ \ (\exists y)\varphi(\underline{x}, y)$ together with $(\forall \underline{x}, y, z)\varphi(\underline{x}, y) \wedge \varphi(\underline{x}, z) \Rightarrow x = y$.

A *theory T* is a set of sentences, in a given language *L*; the *models* of *T* are the *L*-structure satisfying all the sentences in *T*.

T is consistent, or non-contradictory, if it has a model. A sentence is a consequence of *T* if it is true in every model of *T*.

Conclusion The models of an *h*-inductive theory form an inductive class; if Ti is consistent, it has pec models.

Exercices 10. A structure *A* is said *h*-maximal inside a class *C* if every homomorphism from *A* into any element of *C* is an embedding. What are the *h*-maximal rings, the *h*-maximal irreflexive graph, the *h*-maximal cycle-free graphs?

Exercices 11. Show that the pec and the *h*-maximal elements of an inductive class form inductive classes.

Chapter 4.

Compacity of First Order Logic

Theorem 3. A theory of First Order Logic (will full use of negation) is consistent provided each of its finite subets is so.

First step: replace functions by graphs

Second step: interpret Negative Logic inside Positive Logic, so that it will be enough to prove the theorem for h-inductive theories in a language with only relations and constants

For that, given a negative theory *T* in a language *L*, we consider the theory *T** in the language *L** obtained by adding for each formula (*p*(*x*) a new relation cp*(*x*):

– for φ atomic, $(\forall \underline{x})\varphi(\underline{x}) \Leftrightarrow \varphi^*(\underline{x})$ is in *T**

– if $\chi = \varphi \vee \psi$, $(\forall \underline{x})\varphi^*(\underline{x}) \vee \psi^*(\underline{x}) \Leftrightarrow \chi^*(\underline{x})$

– if $\chi = \varphi \wedge \psi$, $(\forall \underline{x})\varphi^*(\underline{x}) \wedge \psi^*(\underline{x}) \Leftrightarrow \chi^*(\underline{x})$

– if $\psi = (\exists \underline{y})\varphi(y,\underline{x}),(\forall \underline{x})\left[\psi^*(\underline{x}) \Leftrightarrow \varphi^*(y,\underline{x})\right]$

– if $\psi \in \varphi$, we put in *T** $\psi^* \Leftrightarrow \neg\varphi^*$

– last, we put in *T** all the σ^* for $\sigma^* \in T$

*T** is h-inductive; *T** has basically the same models as *T* (but the homomorphisms are not the same !); *T** is consistent, or finitely consistent, if and only if *T* is so.

Remains to prove two easy lemmata:

1. Compacity for set of sentences which are either *h*-universal or atomic.

2. If *Ti* is *h*-inductive, and *Tu* is the set of *h*-universal sentences which are consequences of a finite fragment of *Ti*, then any pec model of *Tu* is a model of *Ti*.

Lemma 1. Un ensemble d'enonces *h*-universels ou atomiques *Tu* $\cup$ *Ta* est consistant pourvu que toutes ses parties finies le soient.

Demonstration. On peut supposer que l'ensemble *C* des individus du langage (sans fonctions) riest pas vide. On considere la structure de base *C*, qui ne satisfait que les enonces atomiques de Ta; si elle ne satisfait pas un enonce $\neg(\exists \underline{x})\varphi(\underline{x})$ de *Tu*, c'est qu'il y a un contre-exemple $\varphi(\underline{c})$ dans ce modele minimum de *Ta*, et que $\neg(\exists \underline{x})\varphi(\underline{x})$ est contradictoire avec un fragment fini de *Ta*. Fin

Lemma 2. Si *Ti* est *h*-inductive, et *Tu* est *Г* ensemble des enonces *h*-universels qui sont consequences d'un de ses fragments finis, tout modele existentiellement clos de *Tu* est modële de *Ti*.

Demonstration. Consicterons un modele *M* existentiellement clos de *Tu*, et un enonce σ dans *Ti* $(\forall \underline{x})\left[(\exists \underline{y}) f(\underline{x},\underline{y}) \Rightarrow (\exists \underline{z}) g(\underline{x},\underline{z})\right]$. Nous pouvons ecrire $g(\underline{x},\underline{z})$ sous la forme $g_1(\underline{x},\underline{z}) \vee \ldots \vee g_n(\underline{x},\underline{z})$, ou chaque $g_i(\underline{x},\underline{z})$ est une conjonction finie de formules atomiques.

Soit a un uple d'elements de *M* qui ne satisfait pas $(\exists \underline{z}) g(\underline{x},\underline{z})$; cela est aussi vrai pour l'image de a dans toute continuation de *M* qui soit modele de *Tu*, ce qui signifie que, pour chaque *i*, la theorie formee de *Tu*, de $g_i(\underline{a},\underline{z})$ et de l'ensemble *D*(*M*) des formules atomiques vraies dans *M* est inconsistante (le langage de cette th6orie est celui de *Ti*, augmente d'un symbole d'individu par element de *M*, et du uple $\underline{z}$ d'individus).

D'apres le Lemme 1, dans chaque cas un fragment fini de $D(M)$ suffit a cette inconsistance, et on en dëduit l'existence d'une formule libre positive $h(\underline{a},\underline{b})$ satisfaite dans M et incompatible avec $g(\underline{a},\underline{z})$ et un certain fragment fini $T'u$ de Tu.

En consequence, $T'u$ implique $\neg\left[\left(\exists \underline{x},\underline{z},\underline{u}\right) g(\underline{x},\underline{z}) \wedge h\left(\underline{x},\underline{u}\right)\right]$, si bien que cet enonce h-universel est consequence d'un fragment fini $T'i$ de Ti, dans lequel on peut inclure a.

Dans ces conditions, $T'i$ a pour cons6quence $\neg\left[\left(\exists \underline{x},\underline{z},\underline{u}\right) f(\underline{x},\underline{y}) \wedge h\left(\underline{x},\underline{u}\right)\right]$, et comme ce dernier enonce est h-universel, il fait partie de Tu, si bien que a ne peut satisfaire $\left(\exists \underline{y}\right) f(\underline{x},\underline{y})$; autrement dit, M ne contient pas de contre-exemple a σ. Fin

Chapter 5.

Companions Theories, Model Complete theories

Positive diagram Add to L names for the elements of M; Diag(M) is the set of atomic sentences true in M.

A model of Diag(M) is the same thing as a *continuation* of M, i.e. a structure containing an homomorphic image of M.

Companion theories Two h-inductive theories are *companions* if each model of one of them can be continued into a model of the other iff they have the same h-universal consequences iff they have the same pec models.

An h-inductive theory Ti has a minimal companion Tu, which is the set of its h-universal consequences, and a maximal companion Tk, which is the h-inductive theory of its pec models.

Caracterisation of the pec models In a pec model M of Ti, if a in M does not satisfy a positive formula $\left(\exists \underline{y}\right)\varphi\left(\underline{x},\underline{y}\right)$, it is because it satisfies another positive formula $\left(\exists \underline{z}\right)\psi\left(\underline{x},\underline{z}\right)$ which is contradictory to it: Ti, in fact Tu, implies that $\neg\left[\left(\exists \underline{x},\underline{z},\underline{u}\right)\varphi(\underline{x},\underline{y}) \wedge \psi\left(\underline{x},\underline{z}\right)\right]$.

Caveat: Ti does not implies necessarily that the each of yhe two formulae is the negation of the other.

Model-complete theories A h-inductive Ti is *modele-complete* if every homomorphism between models of Ti is pure iff every model of Ti is pec iff to every positive formula $\varphi(\underline{x})$ is associated another one, $\varphi^*(\underline{x})$, such that Ti declares that $\varphi(\underline{x})$ and $\varphi^*(\underline{x})$ are negations of each other. The homomorphisms between models of a model-complete theory respect the satisfaction of the formulae of the full First Order Logic with negation: they are *elementary embeddings.*

Classical First Order Logic with negation corresponds exactly to the model-complete theories of Positive Logic.

Algebraically closed fields have a model-complete theory; ec groups have not.

Chapter 6.

Spaces of types

Given a h-inductive Ti and a tuple of variables $\underline{x}=\left(x_1,...,x_n\right)$, a *complete n-type* is a maximal set of positive formulae $\varphi(\underline{x})$ which is consistent with Ti.

Examples: prime ideals of $K\left[\underline{x}\right]$ in ac fields, isomorphism types in ec groups.

Every type can be realized in some pec model, every tuple in a pec model has a type.

We put a topology on the sets Sn(Ti) of types, by declaring that the type satisfying a given positive formula form a closed set. We obtain a compact set (french sense), not necessarily Hausdorff.

Examples: the prime spectrum of $K\left[\underline{x}\right]$ with the constructive topology; ec groups are Hausdorff.

All properties of the pec models are recoverable from the spaces of types. For instance, we can caracterize model-complete theories as follows: each Sn have a Hausdorff totally disconnected topology, generated by clopen sets; equality is clopen; the projection from Sn+1 onto Sn is open.

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Б.Пуаза

Позитивті логиканың негіздері

Мақаланың басты мақсаты — позитивті логиканың негізгі ұғымдарын беру. Бұл жұмыстың мазмұны академик Е.А.Бөкетов атындағы Қарағанды мемлекеттік университетінде 2013 жылдың сәуір–мамыр айларында мен оқыған дәріс курсының квинтэссенциясы болып табылады.

Б.Пуаза

Основы позитивной логики

Основная цель статьи — дать основные понятия позитивной логики. Содержание данной работы есть квинтэссенция курса лекций, прочитанных мною в Карагандинском государственном университете имени академика Е.А.Букетова в апреле–мае 2013 года.

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N.S.Imanbayev

Kh.A.Yasawi International Kazakh-Turkish University, Turkestan (E-mail: imanbaevnur@mail.ru)

On the stability and instability of the basis properties of root functions of the Schrödinger equation with nonlocal perturbations of boundary condition

In paper Samarsky-Ionkin spectral problem for the Schrodinger equation with an integral perturbation in the boundary conditions is considered. It is assumed that the unperturbed problem has a system of eigenfunctions forming Riesz basis in $L_2(0,1)$. It is shown that the basis property of the systems of root functions of a problem can be varied under any arbitrarily small variation of the kernel of the integral perturbation.

Key words: a singularly perturbed problem, integral-differential equation, regularization of the problem, iterative task, the asymptotic convergence.

We consider the operator L_1, defined by the differential expression

$$L_1 u = -u'' + q(\dot{x})u = \dot{\lambda}u, \dot{x} \in (0,1), q(x) \in C[0,1] \tag{1}$$

and regular but not strongly regular boundary conditions [1] with an integral perturbation

$$\begin{aligned} &U_1(u) \equiv a_{11}u'(0) + a_{12}u'(1) + a_{13}u(0) + a_{14}u(1) = 0; \\ &U_2(u) \equiv a_{23}u(0) + a_{24}u(1) = \int_0^1 \overline{P(x)}u(x)dx, P(x) \in L_2(0,1). \end{aligned} \tag{2}$$

Here $U_j(u)$ are the independent linear forms with complex constant coefficients satisfying the conditions of unboosted regularity

$$|a_{11}| + |a_{12}| \neq 0, a_{11}a_{24} + a_{12}a_{23} = \pm[a_{11}a_{23} + a_{12}a_{24}] \neq 0.$$