INTEGRATION OF THE KAUP-BOUSSINESQ TYPE SYSTEM VIA INVERSE SCATTERING METHOD.

Babajanov B.A., Azamatov A.Sh.
[1]*Urgench State University, Urgench, Uzbekistan.*
E-mail:a.murod@mail.ru, azizbek.shavkatovich@gmail.com

Nonlinear evolution equations are widely used as models to describe complex physical phenomena in various fields of sciences, especially in fluid mechanics, solid-state physics, plasma physics and biology. In [1], D.J. Kaup proved that the nonlinear system of equations

$$\begin{cases} \eta_\tau = \Phi_{xx} + \beta^2 \Phi_{xxx} - \varepsilon \cdot \left(\Phi_x \eta\right)_x \\ \eta = \Phi_\tau + \dfrac{1}{2}\varepsilon \cdot \Phi_x^2 \end{cases} \qquad (1)$$

is completely integrable.This system was first derived by Boussinesq in the theory of wave propagation in shallow water [2] and therefore it is called the Kaup-Boussinesq system. One of the basic physical problems for this model is to obtain their soliton solutions. In [3], multisoliton solutions were found, and the asymptotic behavior of these solutions was investigated. In papers [4, 5], real finite-zone regular solutions of the Kaup-Boussinesq system are studied. In [6], the Kaup system with self-consistent sources is studied by means of the inverse problem for the quadratic pencil of Sturm-Liouville equations.

In recent years, in connection with intensive research of problems optimalmanagement of the agroecosystem, for example, the problem of long-termforecasting and regulation of the level of groundwater and soil moisture, therehas been a significant increase in interest in loaded equations. Among the works devoted to loaded equations, one should especially note the works of A. Kneser [7], L. Lichtenstein [8], A. M. Nakhushev [9], and others. It is known that the loaded differential equations contain some of the traces of an unknown function.

In this work, we consider the following loaded Kaup-Boussinesq type system

$$\begin{cases} v_t - v_{xxx} - 6uu_{xxx} - 18u_x u_{xx} + 6vv_x + 24vuu_x + 6v_x u^2 = \mu(t)v(0,t)u(0,t)v_x, \\ u_t - u_{xxx} + 6vu_x + 6v_x u + 30u_x u^2 = \mu(t)v(0,t)u(0,t)u_x \end{cases} \qquad (2)$$

under initial condition

$$v(x,t)\big|_{t=0} = v_0(x),\ u(x,t)\big|_{t=0} = u_0(x),\ x \in \mathbb{R}\ . \qquad (3)$$

where $\mu(t)$ are given arbitrary continuous function and the functions $v_0(x)$, $u_0(x)$ satisfy the following conditions:

(i) $u_0(x)$ is absolutely continuous on each finite segment $[\alpha,\beta] \subset (-\infty,\infty)$ and the inequalities hold

$$\int_{-\infty}^{\infty} |u_0(x)|\,dx < \infty, \quad \int_{-\infty}^{\infty} (1+|x|)[|v_0(x)| + |u_0'(x)|]dx < \infty, \qquad (3)$$

(ii) the operator generated by the differential expression

$$T(0,k) := -\frac{d^2}{dx^2} + v_0(x) + 2ku_0(x) - k^2$$

has exactly $2N$ simple eigenvalues $k_1(0), k_2(0), ..., k_{2N}(0)$.

The main aim of this work is to derive representations for the solutions $v(x,t)$ and $u(x,t)$ of the Cauchy problem (2)-(3) within the inverse scattering method for the quadratic pencil of Sturm-Liouville operators:

$$T(t,k)y := -y'' + v(x,t)y + 2ku(x,t)y - k^2 y = 0,\ x \in \mathbb{R}$$

References

1. Kaup D. J., A Higher-Order Water-Wave Equation and the Method for Solving It// Progress of Theoretical Physics, 1975, vol. 54, issue 2, pp. 396-408.
2. Boussinesq J., Theorie de l'itumescence liquide appelee onde solitarie ou de translation, sepropageant dans un canal rectangulaire// Comptes Rendus Hebdomadaires des Seance de l'Academie des Sciences, 1871, 72, pp.755-759.
3. Matveev V.B., Yavor M. I., Solutions Presque Periodiques et a N-solitons de l'Equation Hydrodynamique Nonlineaire de Kaup// Ann.Inst. Henri Poincare, Sect., 1979, A. 31, no. 1, pp. 25-41.
4. Smirnov A.O., Real Finite-Gap Regular Solutions of the Kaup-Boussinesq Equation// Theor. Math. Phys. 1986, vol.66, no.1, pp.19-31.
5. Mitropolsky Yu., Bogolyubov N. Jr., Prykarpatsky A., Samoilenko V., Integrable dynamical system: spectral and differential-geometric aspects// Naukova Dunka, Kiev, 1987.
6. Jaulent M., Miodek I., Nonlinear Evolution Equation Associated with Energy-Dependent Schrodinger Potentials// Lett. Math.Phys., 1976, vol. 1, no. 3, pp. 243-250.
7. Kneser A. Rendicon ti del Circolo Matematico di Palermo, 1914, t. 37, p. 169–197.
8. Lichtenstein L., Vorlesungen uber einege Klassen nichtlinear Integralgleichungen und Integral differential gleihungen nebst, Anwendungen, Berlin: Springer, 1931.
9. Nakhushev A.M. Equations of Mathematical Biology, Vishaya shkola, Moscow, 1995, 302 p.

ON THE PERIODIC CAMASSA – HOLM EQUATION WITH A SOURCE

Babajanov B.A., Atajonov D.O.

Urgench State University, Urgench, Uzbekistan

E-mail: a.murod@mail.ru, diwa_4848@mail.ru

We consider the Camassa – Holm (CH) equation with a self-consistent source

$$u_t - u_{xxt} = uu_{xxx} + 2u_x u_{xx} - 3uu_x + \sum_{k=0}^{\infty} \alpha_k(t) s(\pi, \lambda_k, t) q_x(x,t) \psi^2(x, \lambda_k, t) + 2q(x,t)\left(\psi^2(x,\lambda_k,t)\right) \quad (1)$$

in the class of real-valued π- periodic on the spatial variable x function $u = u(x,t)$ which satisfy the regularity of assumption $u \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0)$ with the initial condition

$$u(x,0) = u_0(x), x \in R, \quad (2)$$

where $q(x,t) = u(x,t) - u_{xx}(x,t)$ and $u_0(x) \in C^3(R)$ is the given real-valued π- periodic function and $\psi(x, \lambda_k, t)$ are the Floquet solution (normalized by the condition $\psi(0, \lambda_k, t) = 1$ of the weighted Sturm-Liouville equation.

$$y'' = \frac{1}{4} y + \lambda q(x,t) y, \ \ x \in R. \quad (3)$$

Here λ_k is zeros of the function $\Delta^2(\lambda) - 4$, where $\Delta(\lambda) = c(\pi, \lambda, t) + s'(\pi, \lambda, t)$. We denote by $c(x, \lambda, t)$ and $s(x, \lambda, t)$ the solutions of equation (3) satisfying the initial conditions $c(0, \lambda, t) = 1, c'(0, \lambda, t) = 0$ and $s(0, \lambda, t) = 0$, $s'(0, \lambda, t) = 1$ respectively. In system (1), the functions $\alpha_k(t)$, $k \in Z$, can be chosen freely within the class of real-valued continuousfunctions having uniform asymptotic decay $\alpha_k = O\left(\frac{1}{k^2}\right), k \to +\infty$, thus providing uniform convergence of the series in equation (1).

The aim of this work is to provide a procedure for constructing the solution $u(x,t)$, $\psi(x, \lambda_k, t)$ of problem (1)-(3) using the inverse spectral theory for the weighted Sturm-Liouville equation (3).

For a discussion of integration of the CH equation we refer to works [1-4]. With regard to their applications we refer to works [5-6].

In [7], the CH equation with a self-consistent source was constructed and investigated using the Darboux transformation. In [8], the CH equation with a self-consistent source was integrated by the method of inverse scattering theory.