

Theorem 1. Let  $[\Delta]$  be complete for existential sentences (consists of  $\exists$ -complete theories),  $X$  be a countable  $cl$ -atomic Jonsson set. Then (1)  $\Rightarrow$  (2) and (2)  $\Leftrightarrow$  (3):

- (1)  $X$  is  $(\Sigma, \Sigma)$ -atomic set;
- (2)  $X$  is  $\Sigma^*$ -nice-algebraically prime set;
- (3)  $X$  is  $\Sigma$ -nice-algebraically prime set.

Theorem 2. Let  $[\Delta] \in JSp(K)$ , a class  $[\Delta]$  be universal and  $\exists$ -complete,  $X \subseteq C_{[\Delta]}$  be a countable algebraically prime  $cl$ -atomic Jonsson set. Then there exists a Jonsson set  $Y \subseteq C_{Fr(X)}$  such that  $cl(Y) = M$ ,  $M \in K$  and  $M$  is  $(\Sigma, \nabla)$ -atomic model for  $Fr(X)$ .

All necessary definitions can be found from (1; 3; 5)

**Acknowledgment:** This research has is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP23489523).

## References

- [1] Yeshkeyev A.R., Theories and their classes, in two volumes, Volume 1, KarU, 2024, 282.
- [2] Yeshkeyev A.R., Ulbrikht O.I., “JSp-cosemanticness and JSB-property of abelian groups”, Siberian Electronic Mathematical Reports, 13 (2016), 861–874.
- [3] Yeshkeyev A.R., Theories and their classes, in two volumes, Volume 2, KarU, 2024, 297.
- [4] Baldwin J.T., Kueker D.W., “Algebraically Prime Models,” *Annals of Mathematical Logic*, 20 (1981), 289–330.
- [5] Yeshkeyev A.R., Ulbrikht O.I., Issayeva A.K., “Algebraically prime and atomic sets”, Turkic World Mathematical Society Journal of Pure and Applied of Mathematics, 14:2 (2023), 232–245.

## ALGEBRAIC STRUCTURE OF THE JONSSON SPECTRUM

Yeshkeyev Aibat Rafhatovich<sup>1</sup>, Tungushbayeva Indira Orazbekovna<sup>2</sup>

<sup>1,2</sup>Karaganda Buketov University, Karaganda, Kazakhstan

<sup>1</sup>E-mail: yeshkeyev\_a@buketov.edu.kz

<sup>2</sup>E-mail: tungushbayeva\_i\_1@buketov.edu.kz

We consider Jonsson theories of a countable first-order language  $L$  of the signature  $\sigma$ . By theory, we mean a consistent set of  $L$ -sentences. We write  $E_\sigma$  for the class of  $L$ -structures that are existentially closed in the class of all  $L$ -structures.

Let  $T_1$  and  $T_2$  be Jonsson  $L$ -theories, and let  $M \in E_\sigma$  be a model of  $T_1 \cup T_2$ . Then  $T_1 \cup T_2$  is a Jonsson  $L$ -theory.

Let  $K$  be a class of  $L$ -structures. A Jonsson spectrum (1) of  $K$  is the following set of theories:

$$JSp(K) = \{T \mid T \text{ is a Jonsson theory and } K \subseteq Mod(T)\}.$$

Let  $K_\sigma$  be a class of  $L$ -structures containing a structure  $M \in E_\sigma$ . Then  $(JSp(K_\sigma), \cup)$  is a commutative monoid.

Two Jonsson theories are called cosemantic if their semantic models (2) coincide. Cosemanticness is an equivalence relation. Let  $K$  be any class of  $L$ -structures, and let  $JSp(K)_{/\sim}$  be a factor-set of

the Jonsson spectrum of  $K$  with respect to cosemanticness. Then  $[T]$  denotes the cosemanticness class of a theory  $T \in JSp(K)$ .

Let us introduce the operations "  $\wedge$  " and "  $\vee$  " for arbitrary  $L$ -theories  $T$  and  $T'$  as follows (3). Let

$$T \wedge T' = \{\varphi \wedge \varphi' \mid \varphi \in T, \varphi' \in T'\},$$

if this theory is consistent. Similarly, let

$$T \vee T' = \{\varphi \vee \varphi' \mid \varphi \in T, \varphi' \in T'\}.$$

Each cosemanticness class  $[T] \in JSp(K)_{/\infty}$  is a distributive lattice with respect to operations "  $\vee$  " and "  $\wedge$  ".

**Acknowledgements:** This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP23489523)

## References

- [1] Yeshkeyev A.R., Ulbrikht O.I., "JSp-cosemanticness and JSB-property of abelian groups", Siberian Electronic Mathematical Reports, 13 (2016), 861-874.
- [2] Ешкеев А.Р., Теории и их модели: монография в 2-х томах. Том 1, Издательство КарУ им. академика Е.А. Букетова, Караганда, 2024, 282 с.
- [3] Mustafin Ye., "Some properties of Jonsson theories", Journal of Symbolic Logic, 67:2 (2002), 528-536.

## WEIGHTED ESTIMATES FOR SOME CLASS OF QUASILINEAR OPERATORS

Zhangabergenova Nazerke Salmenkyzy<sup>1</sup>, Manarbek Makpal<sup>2</sup>

<sup>1</sup>L.N. Gumilyov Eurasian National University, Institute of mathematics and mathematical modeling, Astana, Kazakhstan

<sup>1</sup>E-mail: zhanabergenova.ns@gmail.com

<sup>2</sup>Institute of mathematics and mathematical modeling, Almaty, Kazakhstan

<sup>2</sup>E-mail: makpal9136@mail.ru

Let  $0 < q, p, r < \infty$  and  $\frac{1}{p} + \frac{1}{p'} = 1$ . Let  $u = \{u_i\}_{i=1}^{\infty}$  and  $v = \{v_i\}_{i=1}^{\infty}$  be weight sequences, i.e., positive sequences of real numbers. We denote by  $l_{p,v}$  the space of sequences  $f = \{f_i\}_{i=1}^{\infty}$  of non-negative real numbers such that

$$\|vf\|_p = \left( \sum_{i=1}^{\infty} (v_i f_i)^p \right)^{\frac{1}{p}} < \infty.$$

For any non-negative  $f \in l_{p,v}$  we consider the following iterated discrete Hardy-type inequality with three weights

$$\left( \sum_{n=1}^{\infty} u_n^q (K^{\pm} f)_n^q \right)^{\frac{1}{q}} \leq C \left( \sum_{i=1}^{\infty} (v_i f_i)^p \right)^{\frac{1}{p}}, \tag{1}$$