

# BOUNDEDNESS OF THE HIGHER DIMENSIONAL HILBERT OPERATOR IN REARRANGEMENT INVARIANT QAUSI-BANACH SPACES

**Tulenov K. S.**

*Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*

E-mail: [tulenov@math.kz](mailto:tulenov@math.kz)

The talk is based on a recently published joint work [2] with F. Sukochev and D. Zanin where we obtained a weak (1, 1) type estimate for a higher dimensional Hilbert operator answering an open question by A. Osekowski [1]. This result together with results in [3] allow us to investigate boundedness of the Hilbert operator in rearrangement invariant quasi-Banach spaces.

## References

1. Osekowski A., Inequalities for Hilbert operator and its extensions: the probabilistic approach. *Ann. Probab.*, Vol. 45 No.1 (2017), 535–563.
2. Sukochev F., Tulenov K., and Zanin D., A weak type (1, 1) estimate for the Hilbert operator in higher-dimensional setting. *Studia Math.*, Vol. 265 No.3 (2022), 241–256.
3. Sukochev F., Tulenov K., and Zanin D., The optimal range of the Calderon operator and its applications. *J. Funct. Anal.*, Vol. 277 No.10 (2019), 3513–3559.

## ON JOINTLY CONCAVITY OF SOME TRACE FUNCTIONS

**Turdebek N. Bekjan**

*Department of Computational Mathematics and Database Analysis, Astana IT University, Nur-Sultan, Kazakhstan*

E-mail: [t.nurlybekuly@astanait.edu.kz](mailto:t.nurlybekuly@astanait.edu.kz)

Let  $\mathbb{M}_n$  be the von Neumann algebra of  $n \times n$  complex matrices, and let  $\mathbb{M}_n^+ = \{A \in \mathcal{M}: A \geq 0\}$  and  $\mathbb{M}_n^{++} = \{A \in \mathcal{M}: A > 0\}$ . In [3], Carlen and Lieb proved the following trace function on  $\mathbb{M}_n^+$ :

$$F_p(A_1, A_2, \dots, A_m) = [Tr((\sum_{k=1}^m A_k^p)^{\frac{q}{p}})]^{\frac{1}{q}} = \|(\sum_{k=1}^m A_k^p)^{\frac{1}{p}}\|_q \quad (1)$$

is jointly concave in  $(A_1, A_2, \dots, A_m) \in \mathbb{M}_n^+ \times \mathbb{M}_n^+ \cdots \times \mathbb{M}_n^+$ , for  $0 < p \leq q \leq 1$ . Bekjan [1] obtained that for  $0 < p \leq 1$ ,

$$F_p(A_1, A_2, \dots, A_m) = Tr((\sum_{k=1}^m A_k^{-p})^{-\frac{1}{p}}) \quad (2)$$

is jointly concave in  $(A_1, A_2, \dots, A_m) \in \mathbb{M}_n^{++} \times \mathbb{M}_n^{++} \cdots \times \mathbb{M}_n^{++}$ . In [4], Hiai extended these results as following: if  $\Phi_k: \mathbb{M}_{n_k}^+ \rightarrow \mathbb{M}_n^+$  is a strictly positive linear map ( $k = 1, 2, \dots, m$ ), either  $0 < p \leq 1$  and  $0 < s \leq \frac{1}{p}$ , or  $-1 \leq p < 0$  and  $\frac{1}{p} \leq s < 0$ , then

$$F_p(A_1, A_2, \dots, A_m) = \|(\sum_{k=1}^m \Phi_k(A_k^p))^s\|_1$$

is jointly concave in  $(A_1, A_2, \dots, A_m) \in \mathbb{M}_{n_1}^+ \times \mathbb{M}_{n_2}^+ \cdots \times \mathbb{M}_{n_m}^+$ , where  $\|\cdot\|_1$  is a symmetric anti-norm on  $\mathbb{M}_n^+$ .

Let  $\mathcal{M}$  be a finite von Neumann algebra with a normal faithful finite trace  $\tau$  and  $L_0(\mathcal{M})$  be the set of all measurable operators with respect to  $(\mathcal{M}, \tau)$ . The topology of  $L_0(\mathcal{M})$  is determined by the convergence in measure. Set  $L_0(\mathcal{M})^+ = \{x: x \in L_0(\mathcal{M}), x \geq 0\}$ ,  $\mathcal{M}^+ = \{x: x \in \mathcal{M}, x \geq 0\}$  and  $\mathcal{M}^{++} = \{x: x \in \mathcal{M}, x > 0\} = \{x: x \in \mathcal{M}, x \geq 0 \text{ and } x \text{ is invertible}\}$ . The aim of this talk is to prove that [(i)]

1. if  $f: [0, \infty) \rightarrow [0, \infty)$  is an operator concave function,  $0 < p \leq 1$ ,  $0 < s \leq \frac{1}{p}$  and  $\Phi_j$  is a continuous positive linear map from  $L_0(\mathcal{M}_j)$  to  $L_0(\mathcal{M})$  with  $\Phi_j(\mathcal{M}_j) \subset \mathcal{M}$ , where  $\mathcal{M}_j$  is finite von Neumann algebra,  $j = 1, 2, \dots, n$ , then for  $0 \leq t < \tau(1)$

$$\int_t^{\tau(1)} \mu_v((\sum_{j=1}^n \Phi_j(f(x_j^p)))^s) dv \quad \text{and} \quad \int_t^{\tau(1)} \mu_v((\sum_{j=1}^n \Phi_j(f(x_j)^p))^s) dv$$

are jointly concave in  $(x_1, x_2, \dots, x_n) \in L_0(\mathcal{M}_1)^+ \times L_0(\mathcal{M}_2)^+ \times \cdots \times L_0(\mathcal{M}_n)^+$ ,