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ON NON-ESSENTIALITY OF AN O-STABLE EXPANSION OF $(\mathbb{Z}, <, +)$

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Model theory of ordered groups is a sufficiently important application of mathematical logic to algebra. Here we consider such a quite classical object as the ordered group of integers and its possible expansions.O. Belegradek, Y. Peterzil and F. Wagner proved in [1] that there is no quasi-o-minimal expansion of $(\mathbb{Z}, <, +)$. Later in this direction ordered groups were investigated in [2–4]. Quite recently E. Walsberg proved some results inthis context [5], in particular showing that a dp-minimal expansion $G$of a discretely ordered group is interdefinable with a model of the theory of$(\mathbb{Z}, <, +)$ if and only if G does not admit a nontrivial definable convex subgroup. Here we consider o-stable expansions of $(\mathbb{Z}, <, +)$.

Recall, that notion of o-stability was introduced in [6], where it was proved that quasi-o-minimal theories are o-superstable.

Definition.An ordered structure$M$ is o-λ-stable if for every $A \subseteq M$ of size at most λ and every cut (C, D) in M, at most λcomplete 1-types over A are consistent with (C, D). A theory T is o-stable if there is someinfinite λ such that every model of T is o-λ-stable, and it is o-superstable if there is some μsuch that for every $\lambda \geq \mu$, T is o-λ-stable.

Model theory of o-stable ordered groups has been developed in [7–9], where in particular it was proved that an o-stable ordered group is abelian and an o-stable ordered field without definable non-trivial convex subgroups of the additive group is weakly o-minimal and by Marker–Macpherson–Steinhorn theorem is real closed.More general theory of relative stability has been investigated in [10–11]. In [12] V. Verbovskiy proved that a dp-minimal theory with a definable linear order is o-stable, so the class of o-stable theories includes each of the following classes: o-minimal, weakly o-minimal, quasi-o-minimal [6], dp-minimal with a definable linear order.

Theorem. Any o-stable expansion of $(\mathbb{Z}, <, +)$is not essential, that is, if G is an o-stable expansion of $(\mathbb{Z}, <, +)$then any definable with parameters subset of G is also definable with parameters in $(\mathbb{Z}, <, +)$.

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