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### **Geometric properties of soliton surfaces of the toda lattice equation**

The geometric properties of surfaces have found a number of applications in mechanics, engineering, and optics. Special classes of surfaces are studied by the methods of the soliton theory. Of great interest is the study of the relationship between integrable systems with differential geometry. One of the important surface characteristics is the first and second quadratic (fundamental) forms, Gaussian curvature, etc [1,2]. The first and second quadratic forms allow us to describe the internal and external surfaces of the investigated object. In the geometric interpretation of surfaces, the Sym-Tafel formula plays a significant role. Using these characteristics, it is possible to study soliton surfaces using the example of the Toda lattice equation. The Toda lattice is a non-linear evolution equation describing an infinite system of masses on a line that interacts through an exponential force. The Toda lattice is considered as a simple model of the nonlinear one-dimensional crystal in solid state physics. It is defined by a lattice of particles with the interaction of the nearest neighbor, described by the equations of motion [3,4].

The main transformation for the Toda lattice [5]

$$p_{nt} = v_n - v_{n+1}, v_{nt} = v_n(p_{n-1} - p_n) \quad (1)$$

which is derived from the compatibility conditions, i.e. the discrete zero-curvature equation

$$U_{nt} - V_{n+1}^1 + U_n V_n^1 = 0, \quad (2)$$

of the eigenvalue problem

$$EY_n(\lambda) = U_n(u, \lambda)Y_n(\lambda), U_n(u, \lambda) = \begin{pmatrix} 0 & 1 \\ -v_n & \lambda - p_n \end{pmatrix}, \tag{3}$$

and the auxiliary problem

$$\frac{d}{dt}Y_n(\lambda) = V_n^1(u, \lambda)Y_n(\lambda), V_n^1(u, \lambda) = \begin{pmatrix} \frac{1}{2}\lambda - p_{n-1} & -1 \\ -v_n & \frac{1}{2}\lambda \end{pmatrix}, \tag{4}$$

where  $u = (p_n, q_n)^T$  is a 2-component potential function vector and  $Y_n(\lambda) = (Y_n^1(\lambda), Y_n^2(\lambda))^T$ . Here, the shift operator  $E$  and the inverse of  $E$  are defined by

$$Ef(n) = f(n + 1) = f_{n+1}, E^{-1}f_n = f_{n+1}, n \in Z. \tag{5}$$

(3) and (4) equations are the Lax pairs of the Toda lattice (1).

We using (3) and (4) equations in quadratic forms. First quadraticform

$$I = \vec{r}_x^2 dx^2 + 2\vec{r}_x \vec{r}_y dx dy + \vec{r}_y^2 dy^2, \tag{6}$$

Second quadratic form

$$II = -d\vec{n} \cdot d\vec{r} = r_x^2, \tag{7}$$

where

$$r_x^\pm = r_{1x} \pm ir_{2x}, (r_1, r_2, r_3) = \vec{r},$$

$$\text{tr}(r_x^2) = 2(r_{3x}^2 + r_x^+ r_x^-) = 2(r_{1x}^2 + r_{2x}^2 + r_{3x}^2) = 2\vec{r}_x^2.$$

Often geometric problems associated with soliton equations use the Sym-Tafel formula, which is as follows

$$r = \Phi^{-1}\Phi_\lambda, \tag{8}$$

where,  $\Phi_x = U\Phi$  and  $\Phi_t = V\Phi$  and derivative of  $r$  by  $x$  and  $y$  gives

$$r_x = \Phi^{-1}U_\lambda\Phi, \tag{9}$$

$$r_y = \Phi^{-1}V_\lambda\Phi, \tag{10}$$

$$\vec{n} = \frac{\vec{r}_x \wedge \vec{r}_t}{|\vec{r}_x \wedge \vec{r}_t|} = \frac{\vec{r}_x \wedge \vec{r}_t}{\sqrt{g}}. \tag{11}$$

Build a soliton surface through  $r_x, r_y, \vec{n}$ . We need to find next  $\text{tr}$  for the Toda lattice:

$$\text{tr}(r_x^2) = 1, \text{tr}(r_t^2) = \frac{1}{2}, \text{tr}(r_x r_t) = \frac{1}{2}. \tag{12}$$

The first quadraticform of a soliton surface for the Toda lattice

$$I = dx^2 + dxdt + \frac{1}{2}dt^2. \quad (13)$$

Now you need to find the normal for the second quadratic form

$$n = \frac{\Phi^{-1}[U_\lambda, V_\lambda]\Phi}{\sqrt{-\frac{1}{2}\text{tr}([U_\lambda, V_\lambda]^2)}}, \quad (14)$$

for Toda lattice

$$[U_\lambda, V_\lambda] = U_\lambda V_\lambda - V_\lambda U_\lambda = 0, \quad (15)$$

so  $n = 0$  and the second fundamental form of a soliton surface for the Toda lattice equals zero:

$$II = 0. \quad (16)$$

Using the data obtained, you can get the surface area

$$S = \iint \sqrt{g} dxdt = \iint |\vec{r}_x \wedge \vec{r}_t| dxdt, \quad (17)$$

$$g = \det(g_{ij}) = \det \begin{pmatrix} \vec{r}_x^2 & \vec{r}_x \cdot \vec{r}_t \\ \vec{r}_x \cdot \vec{r}_t & \vec{r}_t^2 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 \\ 0 & \vec{r}_{xx}^2 \end{pmatrix} = \vec{r}_{xx}^2 = g. \quad (18)$$

Area of surfaces for Toda lattice:

$$S = \iint \sqrt{\frac{1}{2}\text{tr} \begin{pmatrix} v_n & 0 \\ 0 & v_n \end{pmatrix}} = \iint \sqrt{v_n} dxdt. \quad (19)$$

As a result of the study of soliton surfaces using the example of the classical Toda lattice, the first and second quadratic forms and surface area are obtained. An interesting result was obtained, where the second quadratic form vanished. Which does not allow to study the Gaussian curvature and requires the study of other variants of the Toda lattice equation. These results allow a better understanding of the nature and properties of surfaces in mathematical physics.

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