It is clear that $JSpQV\ (K) \subseteq JSp(A)$ for any model $A \in K$, where $JSp(A)$ is the Jonsson spectrumof the model A .

We say that class $K_1$ $JSpQV$ -cosemantic to class $K_2(K_1 \overset{\bowtie}{_{JSpQV}} K_2)$ if $JSpQV\ (K_1) /_{\bowtie} = JSpQV\ (K_2) /_{\bowtie}$ .

In the framework of studying the properties of perfect Jonsson varieties of algebras, the following result is obtained.

Theorem. Let $K$ be a class of algebras of some signature $\sigma$, and let $T$be a complete inductivetheory,$E_T \neq \emptyset$. Then, the class $K$fromexistentially closed models of the theory $T$,$K \subseteq E_T$. Let $[T] \in PJSp(K)$. $E_{[T]}$ coincide with $Mod(Th_{\forall\exists}(E_{[T]}))$.

Also for all who have an interest in the particular case of the above-considered materials, one canfind out in the following resources [1-3].

This work was supported by the Science Committee of the Ministry of Education and Science of theRepublic of Kazakhstan (grant AP09260237).

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## THE PROPERTIES OF EXISTENTIALLY PRIME SUBCLASSES OF PERFECT JONSSON THEORIES

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Definition 1 [1, p. 80]. A theory $T$ is called a Jonsson theory if the following conditions hold:

1. $T$ has at least one infinite model,
2. $T$ is $\forall\exists$-axiomatizable,
3. $T$ possesses AP and JEP.

Definition 2 [2]. A theory $T$ is called existentially prime, if $AP_T \cap E_T \neq \emptyset$, where $AP_T$ is the class of algebraically prime models of $T$.

Let $T$ be a Jonsson theory. For any model $\mathfrak{M} \in E_T$ there are the inner world $IW(\mathfrak{M})$ and the outer world $OW(\mathfrak{M})$ defined in [3].

Definition 3 [3]. Let $T$ be a Jonsson theory and let $\mathfrak{M}$ be an existentially closed model of $T$. $IW_T(\mathfrak{M}) = \{\mathfrak{N} \in E_T \mid f: \mathfrak{N} \to \mathfrak{M}$ is said to be the inner world of $\mathfrak{M}$ for the theory $T$, where $f$ is an isomorphism.

Definition 4 [3]. Let $T$ be a Jonsson theory. $\left(OW_T(\mathfrak{M})\right) = \{\mathfrak{N}' \in E_T \middle|$ there exists$\mathfrak{N} \cong \mathfrak{M}, \mathfrak{M} \subseteq \mathfrak{N}\}$ is called the outer world of $A$ for the theory $T$.

The result of the paper is the following Theorem, where some properties of the inner world of a fixed theory $T$are formulated.

Theorem. Let $T$ be a perfect Jonsson existentially prime theory and let $\mathfrak{A}, \mathfrak{B} \in E_T$. Then

1. $OW(\mathfrak{A}) \subseteq OW(\mathfrak{B})$ whenever $\mathfrak{A} \subseteq \mathfrak{B}$;
2. $OW(\mathfrak{A}) = OW(\mathfrak{B})$ whenever $\mathfrak{A} \prec \mathfrak{B}$;
3. $OW(\mathfrak{A}) = \cap\, \{B \mid \mathfrak{B} \subseteq \mathfrak{A}$ and $\mathfrak{B} \models T$ whenever $\mathfrak{A}$ is a universal model of $T$.

This work was supported by the Science Committee of the Ministry of Education and Science of theRepublic of Kazakhstan (grant AP09260237).

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# THE CONNECTION OF AP AND JEP-THEORIES IN THE LANGUAGE OF ∃-FORMULAS

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Definition 1 [1, p. 80] A theory $T$ has the joint embedding property (JEP) if for any models $U$, $B$ of the theory $T$ there exists a model $M$ of the theory $T$ and isomorphic embeddings $f: U \to M$, $g: B \to M$.

Definition 2 [1, p. 68] A theory $T$ has the amalgam property (AP) if for any models $U$, $B_1$, $B_2$ of the theory $T$ and isomorphic embeddings $f_1: U \to B_1$, $f_2: U \to B_2$ there are $M \vDash T$ and isomorphic embeddings $g_1: B_1 \to M$, $g_2: B_2 \to M$ such that $g_1 \circ f_1 = g_2 \circ f_2$.

Initially, the AP and JEP are algebraic properties. In [1], one can find their definitions, commonly used in Model Theory in semantic key. In 1988, Mustafin T.G. proved the statements (Theorem 1 and Theorem 2), where he formulated amalgam and joint embedding properties in a syntactic way.

Let $L$ be a first-order language and $T$ be a theory of $L$. $(*)$ is a statement that states the following:

Let $\overline{x} \cap \overline{y}$ be empty and $p(\overline{x})$, $q(\overline{y})$ be such arbitrary sets of $\Sigma$-formulas that $T \cup p(\overline{x})$ and $q(\overline{x})$ are consistent, $\overline{x} = \langle x_1, x_2, \ldots x_n \rangle$ and $\overline{y} = \langle y_1, y_2, \ldots y_n \rangle$. Then $T \cup p(\overline{x}) \cup q(\overline{y})$ is also consistent.

Theorem 1 [3]. The following conditions are equivalent: $T$ has JEP if and only if $(*)$ is true.

Let $(**)$ be a statement that says the following:

Let $\overline{x} = \langle x_1, x_2, \ldots x_n \rangle$ and $\overline{y} = \langle y_1, y_2, \ldots y_n \rangle$. If $p(\overline{x})$ and $q(\overline{x})$ are the sets of $\Sigma$-formulas such that the sets $T \cup p(\overline{x})$, $T \cup q(\overline{x}), T \cup \{\neg\phi(\overline{x}) \in \Sigma,\ \phi(\overline{x}) \notin p(\overline{x}) \cap q(\overline{x})$ are consistent, then the set $T \cup p(\overline{x}) \cup q(\overline{x})$ is consistent.

Theorem 2 [3].$T$ has AP if and only if $(**)$ is true.

The study of AP, JEP and their connection is of great importance in the development of Model Theory and related fields. It is well known that amalgam property and joint embedding property are independent of each other. This fact can be supported by the theories of various classes of unars [4]. However, when a theory has both AP and JEP, one can reveal some theory's syntactic and semantic specificity that causes the possession of the properties on consideration.

The following definition was introduced by Yeshkeyev A.R. with regard to the connection between AP and JEP in some theories:

Definition 3 [2, p. 130]. Let $T$ be an inductive theory. A theory $T$ is called to be

1. an AP-theory if in the theory $T$ amalgam property implies joint embedding property;
2. a JEP-theory if in the theory $T$ joint embedding property implies amalgam property;
3. an AJ-theory if in the theory $T$ both properties are equivalent.
4. Otherwise, we say that for the theory $T$, the properties of $AP$ and $JEP$ are independent of each other.

The described classes of theories in Definition 3 form the corresponding subclasses within Jonsson theories. In [2], the authors considered some classical algebras as examples of AP-theories. In this manner, it was shown that the theory of differential fields of characteristic 0, and the theory