UDC 517.9

A.Sh.Akysh

Institute of mathematics and mathematical modeling MES CS RK, Almaty
(E-mail: akysh41@mail.ru)

Simplified maximum principle of the Navier-Stokes equation

In the work the validity of principle of maximum for the Navier-Stokes equations (NSE) is shown. On what basis in the chosen space are proved uniqueness of weak generalized solutions and existence of strong solutions of a problem for NSE as a whole on time $t \in [0,T]$, $\forall T < \infty$.

Key words: nonlinear Navier-Stokes equations system, the principle of maximum for Navier-Stokes equations, uniqueness of weak generalized solutions of Navier-Stokes equations, existence of strong solutions of Navier-Stokes equations.

1. Introduction. The main unsolved problems of the Navier-Stokes equation (NSE) theory for homogeneous fluids are given in [1–3], etc.

In [3] Ch. Fefferman poses two problems for the Navier-Stokes equations:

1[st] Cauchy problem and 2[nd] problem with periodic in space variables boundary values and he considers that «only those solutions which are infinitely smooth functions make physical sense».

And in [2] O. A. Ladyzhenskaya has posed the following:

Problem 1. *«Do the Navier-Stokes equations with initial and boundary values give us a deterministic description of the dynamics of an incompressible fluid, or not?*

When solving Problem 1 a choice of the phase space and the class of generalized solutions must be given to the researcher, without assigning it beforehand infinite smoothness or any kind of smoothness of solutions. Only one thing must be demanded: that the accentuated class of generalized solutions has the uniqueness theorem. It makes sense to begin the study of any initial-boundary problem (and the Cauchy problem) by finding uniqueness classes».

The aim of this work is the study of problem 1 and, as a result, showing the uniqueness and existence of a solution to the Navier-Stokes equations from the following function classes respectively

$$C\left(0,T;\, C(\Omega)\cap W_2^1(\Omega)\right) \text{ и } C\left(0,T;\, C(\Omega)\cap W_{2,0}^{2,1}(\Omega)\right).$$

The results of exploratory research aimed at proving the maximum principle for the NSE are given in a number of works written by the author [4–6], etc. In order to do that a nonlinear parabolic type equation for kinetic energy density was deduced from the system of NSE and an important property was shown — the maximum principle. With the help of the latter it was shown that the maximum principle is true for NSE, which is key from the mathematical point of view. However the proof of the last statement is given with the help of a very hard and complicated technique. These obstacles were overcome in [7] by directly studying the initial system of NSE for the application of the maximum principle (a simplified version of a proof of the maximum principle for a system of NSE). Based on this we prove the uniqueness of weak and existence of strong solution of the NSE in our chosen space in time $t \in [0,T]$, $\forall T < \infty$.

Here the main results are summed up and proven with strict mathematical rigor.

2. A statement of the problem

Consider the initial-boundary problem for NSE [1] relative to the speed vector $U = (U_1, U_2, U_3)$ and pressure P in region $Q = (0,T] \times \Omega$:

$$\frac{\partial U}{\partial t} - \mu \Delta U + (U,\nabla)U + \nabla P = f(t,x); \tag{1a}$$

$$\mathrm{div} U = 0; \tag{1b}$$

$$U(0,x) = \Phi(x); \tag{1c}$$

$$U(t,x)\big|_{\partial\Omega} = 0, \quad x \in \partial\Omega, \tag{1d}$$

where $x \in \Omega \subset R_3$; Ω — convex domain, filled with homogeneous fluid, and $\partial\Omega$ — boundary of the domain Ω, $t \in [0,T]$, $\forall T < \infty$; $0 < \mu$ — dynamic viscosity; Δ, ∇ — Laplace and Hamilton operators respectively. Let $\overset{0}{J}(\Omega)$ — the space of solenoidal vectors, and $G(\Omega)$ consists of $\nabla\eta$, where η is a single-valued function in Ω. It is known from [1, 8], that the orthogonal resolution, $L_2(Q)=G(Q)\oplus\overset{0}{J}(Q)$, where elements of $\overset{0}{J}(Q)$ in $\forall t$ belong to $\overset{0}{J}(\Omega)$; *f* and *Φ* — vectors of functions of external forces and initial data respectively, that satisfy the following conditions:

i) $f(t,x)\in C(\bar{Q})\bigcap\overset{0}{J}(Q)$; ii) $\Phi(x)\in C(\bar{\Omega})\bigcap W_{2,0}^{1}(\Omega)\bigcap\overset{0}{J}(\Omega)$.

3. The Principle of maximum

Rewrite the vector equation (1a) in the form of a system of scalar equations:

$$\frac{\partial U_\alpha}{\partial t}-\mu\Delta U_\alpha+(U,\nabla U_\alpha)+\frac{\partial P}{\partial x_\alpha}=f_\alpha,\quad \alpha=\overline{1,3},\tag{2}$$

where (,) — is the scalar product of vectors.

Theorem 1. Let $\bar{\Omega}$ *be the closed and bounded domain in* R_3 *with boundary* $\partial\Omega$, *and* $Q=[0,T]\times\bar{\Omega}$ *— be the cylindrical domain in the variable space of* t,x. *Suppose that functions* $U\in C(\bar{Q})\bigcap C^2(Q)\wedge P\in C^1(Q)$ *and they satisfy equations (1a), (1b). Then, if in some* α *function* $f_\alpha(t,x)\le 0$ $(f_\alpha(t,x)\ge 0)$ *is in Q, then function* U_α *takes on its positive maximum (negative minimum) in the cylinder* Q *on the bottom base* $\{0\}\times\bar{\Omega}$ *or on the side surface* $[0,T]\times\partial\Omega$, *i.e.*

$$U_\alpha(t,x)\le\max\{\sup_{t=0\wedge x\in\bar{\Omega}}U_\alpha(t,x),\ \sup_{t\in[0,T]\wedge x\in\partial\Omega}U_\alpha(t,x)\},\quad (t,x)\in\bar{Q};\tag{3a}$$

$$\Big(U_\alpha(t,x)\ge\min\{\inf_{t=0\wedge x\in\bar{\Omega}}U_\alpha(t,x),\ \inf_{t\in[0,T]\wedge x\in\partial\Omega}U_\alpha(t,x)\},\quad (t,x)\in\bar{Q}\Big).\quad \alpha=\overline{1,3}.\tag{3b}$$

To prove theorem 1 we need to introduce some statements first.

Let us introduce the notation $R=\dfrac{\partial U}{\partial t}-\mu\Delta U-f(t,x)$.

Apply the div operation to the vector equation (1a). Then considering (1b), we get the Neumann problem for Poisson equation, which binds the pressure *P* to the speed vector *U*:

$$-\Delta P=\operatorname{div}I,\ \text{ where }\ I=(U,\nabla)U\tag{4a}$$

and

$$\left.\frac{\partial P}{\partial n}\right|_{\partial\Omega}=0,\ t\in[0,T],\tag{4b}$$

since $Rn|_{\partial\Omega}=0$ and $In|_{\partial\Omega}=0$ respectively, due to $R\in\overset{0}{J}(\Omega)$ and (1d), where *n* is the unit vector of the outward normal in point *x* of the boundary $\partial\Omega$.

By the condition of theorem 1 function div *I* is continuous in a bounded domain Ω when $\forall t\in[0,T]$, thus

$$\operatorname{div}I\in L_p(\Omega),\ p\ge 1\ \forall t\in[0,T].\tag{5}$$

Moreover $\int_\Omega\operatorname{div}I dx=\int_{\partial\Omega}In dx=0$. So the necessary and sufficient conditions of the Neumann problem are satisfied (4). And the solution consists of the sum $P=P_1+P_h$, where P_h — harmonic in Ω.

Let's consider volume potential

$$P_1(x)=\frac{1}{4\pi}\int_\Omega\frac{\operatorname{div}I(\xi)}{r(x,\xi)}d\xi,\ \forall t\in[0,T].\tag{6}$$

On a condition of the theorem 1 vector function $I=(U,\nabla)U$ is continuous and has continuous derivatives of the first order inside Ω then function $P_1(x)$ satisfies inside Ω to the equation (4a).

Following [9], we note one known property of the volume potential (6).

Lemma 1. If (5) is true for density $\operatorname{div}I$, *then the function* $P_1(x)$ *is defined by formula (6), it is harmonic with every domain, complementing* Ω *and thus function* P *in the same place is harmonic.*

Proof. It is clear that within the domain Ω there is a finite or countable number of domains Ω_j, $j=1,2,...$, complementing Ω. Let Ω_j — one of such domains. Take an arbitrary sub domain Ω_j^1 relative to Ω_j and let $x\in\Omega_j^1$. Then in the integral (6) the distance r is bounded below by a positive number δ equal to the least distance between boundary points of domains Ω_j and Ω_j^1. It follows from the theorem on the properties of potential type integrals, that $P_1(x)\in C^\infty(\Omega_j^1)$; since Ω_j^1 is an arbitrary sub domain of Ω_j, then $P_1(x)\in C^\infty(\Omega_j)$ and, particularly,

$$\Delta P_1(x)=\frac{1}{4\pi}\int_\Omega \operatorname{div}\big(I(\xi)\big)\,\Delta_x\left\{\frac{1}{r(x,\xi)}\right\}d\xi=0,\quad x\in\Omega_j, \tag{7}$$

the function P_1 is harmonic in domain Ω_j and thus function P in the same place is harmonic, since P_h harmonic in Ω. Lemma 1 is proved.

Proof of theorem 1. To do this we will apply a known method from [10]. Suppose the contrary, i.e. The function $U_\alpha(t,x)$ reaches its maximal value in some point $M_0(t^0,x^0)$ inside the domain $\bar{Q}=[0,T]\times\bar{\Omega}$.

$$U_\alpha(M_0)>\max\left\{\sup_{t=0\wedge x\in\bar{\Omega}}U_\alpha(t,x),\ \sup_{t\in[0,T]\wedge x\in\partial\Omega}U_\alpha(t,x)\right\}=C\ge 0. \tag{8}$$

Denote by $m=U_\alpha(M_0)-C>0$ and introduce a function

$$H_\alpha(t,x)=U_\alpha(t,x)+\frac{m}{2}\left(1-\frac{t}{T}\right).$$

Then for all (t,x) from $\partial\Omega\times[0,T]$ or $\{0\}\times\bar{\Omega}$ we have

$$H_\alpha(t^0,x^0)\ge H_\alpha(t,x)+\frac{m}{2}.$$

In other words the function $H_\alpha(t,x)$ also takes on its maximal value in some point $M_1(t^1,x^1)\in Q$, and $H_\alpha(M_1)\ge H_\alpha(M_0)\ge m$. Let at that moment of time t^1 a function $P(t,x)$ reaches its extremum in some other point $x^e\in\Omega$.

Now construct the inner domain Ω_e, complementing Ω in such a way, that points $M_1(t^1,x^1)\in\Omega_e$ and $M(t^1,x^e)\in\Omega_e$. Then, using the statement of Lemma 1, we obtain $\nabla P\big|_{\Omega_e}=0$.

We can write all the necessary conditions for function H_α to reach its maximum in M_1:

$$\frac{\partial H_\alpha}{\partial t}\ge 0;\ \Delta H_\alpha\le 0;\ \nabla H_\alpha=0;\ \nabla P=0. \tag{9}$$

From equation (2) with properties (9) we can find a chain of inequalities for point M_1

$$LH_\alpha\equiv\frac{\partial H_\alpha}{\partial t}-\mu\Delta H_\alpha+(H,\nabla H_\alpha)+\frac{\partial P}{\partial x_\alpha}(M_1)-f_\alpha+\frac{m}{2T}\ge\frac{m}{2T}>0.$$

It means that inequality (8) is wrong. Hence (3a) is true. Theorem 1 is proved.

Following [10], it is easy to obtain from Theorem 1 the proof of the following statement:

Corollary. If vector-functions f,Φ satisfy conditions i) and ii), then the solutions $U(t,x)$ of problem (1) can be estimated

$$\|U\|_{C(Q)}\le\|\Phi\|_{C(Q)}+T\|f\|_{C(Q)}=A_1,\ \forall T<\infty, \tag{10}$$

where $\|U\|_{C(\bar{Q})}=\max\limits_{1\le\alpha\le 3}\sup\limits_{\bar{Q}}|U_\alpha(t,x)|$.

4. Weak generalized solutions

Multiply the equation (1a) by an arbitrary vector-function $Z(t,x)\in C(\bar{Q})\cap W_2^1(Q)\cap \overset{0}{J}(Q)$, equal to zero when $(t=T)\wedge(x\in\partial\Omega)$. Integrate the product on the region Q and integrating by parts we obtain the derivatives from the first two and fourth summands and take them from U to Z. As a result we get

$$\int_Q \left(-UZ_t + \mu \sum_{k=1}^{3} \nabla U_k \nabla Z_k + (U,\nabla)UZ \right) dxdt = \int_\Omega \Phi Z(0,x)dx + \int_Q fZdxdt. \tag{11}$$

Once again we multiply the equation (1a) by the gradient of an arbitrary single-valued function $\eta \in L_2(0,T;W_2^1(\Omega))$. Then integrate again by the region Q, using the orthogonality of subspaces $G(Q)$, $\overset{0}{J}(Q)$ from [8]. As a result we obtain the equation

$$\int_Q \nabla P \nabla \eta \, dxdt = -\int_Q (U,\nabla)U \nabla \eta \, dxdt. \tag{12}$$

Definition[*]*1. A weak generalized solution of an initial-boundary value Navier-Stokes problem (1) is the vector-function U and function P from the subspaces*

$$U \in C\left(\bar{Q}\right) \cap L_\infty(0,T;L_2(\Omega)) \cap L_2(0,T;W_{2,0}^1(\Omega)) \cap \overset{0}{J}(Q);$$

$$P \in L_2\left(0,T;W_2^1(\Omega)\right) \wedge \left(\int_\Omega P(t,x)dx = 0, t \in [0,T] \right) \tag{13}$$

satisfying equalities (11), (12) *for any*

$$Z(t,x) \in C\left(\bar{Q}\right) \cap W_2^1(Q) \cap \overset{0}{J}(Q) \wedge \left(Z\big|_{(t=T)\wedge(x\in\partial\Omega)} = 0 \right); \ \eta(t,x) \in L_2(0,T;W_2^1(\Omega)).$$

For this definition to be true all integrals in (11), (12) must be finite for any Z, η from the denoted classes.

Lemma 2 [4–6]. If the input data of problem (1) satisfy conditions i), ii), then the following estimations are true for weak generalized solutions of problem (1):

$$\| U \|^2_{L_\infty((0,T];L_2(\Omega))} \le 2 \| \Phi \|^2_{L_2(\Omega)} + 4T^2 \| f \|^2_{L_\infty((0,T];\ L_2(\Omega))} = A; \tag{14}$$

$$\sum_{k=1}^{3} \int_0^t \left\| \nabla U_k(\tau) \right\|^2_{L_2(\Omega)} d\tau \le \frac{1}{\mu} \left\| \Phi \right\|^2_{L_2(\Omega)} + \frac{2T^2}{\mu} \left\| f \right\|^2_{L_\infty(0,T;\ L_2(\Omega))} = A_2; \tag{15}$$

$$\| \nabla P \|^2_{L_2(Q)} \le \| (U,\nabla)U \|^2_{L_2(Q)} \le 9A_1^2 A_2 \equiv A_3. \tag{16}$$

Inequalities analogous to (14), (15) were known for a long time, for example from [1, 184]. To prove (16) we use the following chain

$$\left\| (U,\nabla)U \right\|^2_{L_2(Q)} = \int_Q \left((U,\nabla)U \right)^2 dxdt \le 3 \int_Q \left| U \right|^2 \sum_{k=1}^{3} \left| \nabla U_k \right|^2 dxdt \le 9 \max_k \left\| U_k \right\|^2_{L_\infty(Q)} \sum_{k=1}^{3} \int_0^T \left\| \nabla U_k \right\|^2_{L_2(\Omega)} dt. \tag{17}$$

Here we consequently apply the Cauchy-Bunyakovsky inequality for vector product and Hölder's inequality for $p = \infty \wedge q = 1$. From (12) and (17), based on estimations (10) and (15), follows (16). Lemma 2 is proved.

The uniqueness of a solution for problem (1) follows from the maximum principle and the acquired a priori estimates:

Theorem 2 [4–6]. If the input data f and Φ satisfy conditions i), ii) respectively, then problem (1) has a unique weak generalized solution U and P satisfying equalities (11), (12) for any Z and η *from definition 1.*

Proof. Let the pairs of functions $\{U,P\}$ and $\{U^*,P^*\}$ — be two solutions of problem (1). Suppose $V = U - U^*$, $R = P - P^*$, then applying the usual method from problem (1) we can move to equation

$$\int_{Q_t} \left(\frac{\partial V}{\partial t} V - \mu \Delta V V + (V,\nabla)UV + (U^*,\nabla)VV + \nabla R V \right) dxd\tau = 0, \ \forall t \in (0,T]. \tag{18}$$

Due to the orthogonality [1, 8] of subspaces $\overset{0}{J}(Q)$ and $G(Q)$ the fourth and fifth members disappear (18). All others are transformed by integrating by part. Hence obtaining from (18)

$$0.5 \left\| V(t) \right\|^2_{L_2(\Omega)} + \mu \sum_{k=1}^{3} \int_0^t \left\| \nabla V_k(\tau) \right\|^2_{L_2(\Omega)} d\tau = -\int_{Q_t} \sum_{k,\beta=1}^{3} V_\beta \frac{\partial V_k}{\partial x_\beta} U_k dxd\tau. \tag{19}$$

[*] Because of the maximum principle, here the weak solutions are considered in a more narrow class of functions, than in [1].

The integral in the right-hand side can be estimated step by step using Hölder's inequality for $p=\infty \wedge q=1$ and Jung's inequality for $p=2$, obtaining the inequality

$$\int_{Q_t} \sum_{k,\beta=1}^{3} V_k \frac{\partial V_k}{\partial x_\beta} U_k dxd\tau \le A_1\varepsilon/2\sum_{k=1}^{3}\int_0^t \left\|\nabla V_k(\tau)\right\|^2_{L_2(\Omega)} d\tau + A_4\int_0^t \left\|V(\tau)\right\|^2_{L_2(\Omega)} d\tau,\ A_4=\frac{3A_1}{2\varepsilon}.$$

Considering estimates (10), (15) and, applying the last inequality for $\varepsilon=2\mu/A_1$ from (19), we find

$$\frac{d}{dt}\left(\exp(-A_4t)\left\|V(t)\right\|^2_{L_2(\Omega)}\right)\le 0,\ \forall t\in(0,T].$$

Then it follows that $V\equiv 0,\ \forall t\in(0,T]$, i.e., the solutions U and U^* coincide.

Now using the functional equation (12), and taking into account the uniqueness of *U*, that we've just proved, we get an integral correlation for $\nabla R \int_Q \nabla R \nabla\eta dxdt = 0,\ \forall t\in[0,T]$.

It follows that $\forall\nabla\eta$, we get $\nabla R\equiv 0$, i.e. the gradient of pressure *P* from definition 1 is unique and obtained from the vector-function *U*. Theorem 2 is proved.

5. Strong solutions

Definition 2. If the solution of the initial-boundary Navier-Stokes problem (1) in domain Q has all possible generalized derivatives of the same order as the equations themselves, then this generalized solution is called strong.

Theorem 3 [4–6]. If the initial data for problem (1) satisfy conditions i), ii) *and the boundary of domain* $\partial\Omega\in C^2$, *then the problem (1) has a unique strong generalized solution U and P from spaces*

$$U\in W^{2,1}_{2,0}(Q)\cap \overset{0}{J}_\infty(Q);\ P\in L_2\left(0,T;W_2^2(\Omega)\right)\wedge\left(\int_\Omega Pdx=0,\ \forall t\in[0,T]\right),$$

satisfying equations (1a) and almost everywhere in Q and they possess the following estimation:

$$\left\|\frac{\partial U}{\partial t}\right\|^2_{L_2(Q)} \le \mu\sum_{k=1}^{3}\left\|\nabla \Phi_k\right\|^2_{L_2(\Omega)} + 5A_3 + 2T\left\|f\right\|^2_{L_\infty(0,T;L_2(\Omega))} \equiv A_5; \tag{20}$$

$$\left\|\Delta U\right\|^2_{L_2(Q)} \le A_5/\mu^2 \equiv A_6; \tag{21}$$

$$\left\|\nabla U_k\right\|^2_{L_\infty(0,T;L_2(\Omega))} \le A_5/\mu \equiv A_7,\ k=\overline{1,3}; \tag{22}$$

$$\left\|\nabla P\right\|^2_{L_\infty(0,T;L_2(\Omega))} \le 3A_1^2A_7 \equiv A_{10}; \tag{23}$$

$$\left\|U\right\|_{L_2(0,T;W_2^2(\Omega))} \le A_8\left\|\Delta U\right\|_{L_2(Q)},\ A_8 \text{— } const; \tag{24}$$

$$\left\|P\right\|_{L_2\left(0,T;W_2^2(\Omega)\right)} \le A_p\left\|\Delta P\right\|_{L_2(Q)} \le A_C\left\|U\right\|_{L_2\left(0,T;W_2^2(\Omega)\right)},\ A_p, A_C \text{— } const. \tag{25}$$

To prove inequalities (20) from equation (1a) we find the equation

$$\int_{Q_t}(U_t-\mu\Delta U)^2dxd\tau=\int_{Q_t}(f-(U,\nabla)U-\nabla P)^2dxd\tau.$$

We will square the integrands. Then we transform the product in the left-hand side using integration by part, and strengthen the ones in the right-hand side by applying Jung's inequality for $\varepsilon=1\wedge p=2$. Then we get inequality

$$\int_{Q_t}U_t^2dxd\tau+\mu^2\int_{Q_t}(\Delta U)^2dxd\tau+\mu\sum_{k=1}^{3}\int_\Omega\left|\nabla U_k\right|^2dx\le$$

$$\le\mu\sum_{k=1}^{3}\int_\Omega\left\|\nabla\Phi_k\right\|^2dx+2\int_{Q_t}f^2dxd\tau+5\int_{Q_t}((U,\nabla)U)^2dxd\tau.$$

Taking (16) into consideration we obtain from the last inequality the estimations (20)–(22) for strong generalized solutions for problem (1). Note that (22) is a better estimate than (15).

We multiply equations (1a) by ∇P and integrate the result on region Ω. Thus we will obtain the inequality

$$\int_\Omega |\nabla P|^2 dx \le \int_\Omega |U|^2 \sum_{k=1}^{3} |\nabla U_k|^2 dx.$$

Estimating the right-hand side by Hölder's inequality for $p = 1 \wedge q = \infty$, we get

$$\int_\Omega |\nabla P|^2 dx \le 3\|U(t)\|^2_{L_\infty(\Omega)} \int_\Omega \sum_{k=1}^{3} |\nabla U_k|^2 dx, \ \forall t \in [0,T].$$

From this, using (22), we get (23).

Now let us show that $\Delta P \in L_2(Q)$. Since the boundary of region $\partial\Omega \in C^2$, find the estimation (24), using inequalities from [1; 26], true for any function $U(x) \in W_2^2(\Omega) \cap W_{2,0}^2(\Omega)$:

$$\|U\|_{W_2^2(\Omega)} \le A_8 \|\Delta U\|_{L_2(\Omega)}, \ \forall t \in [0,T], \ A_8 \text{— } const.$$

We can find the estimate of ΔP from the ratio $\Delta P = \sum_{\alpha,k=1}^{3} \frac{\partial U_k}{\partial x_\alpha} \frac{\partial U_\alpha}{\partial x_k}$, found from the vector equation (1a) by applying the div operation and taking (1b) into consideration.

Squaring both parts of the last equation, integrating on the region *Q*, we make an estimation of the right-hand side, acquiring the inequality

$$\int_Q (\Delta P)^2 dxdt \le 9 \sum_{k,\alpha=1}^{3} \int_Q \left| \frac{\partial U_k}{\partial x_\alpha} \right|^4 dxdt, \ \forall t \in [0,T]. \tag{26}$$

So we have $W_2^2(\Omega) \subset W_{6-\varepsilon}^1(\Omega), \ \forall \varepsilon > 0$ from the Sobolev embedding theorem. It follows that when $\varepsilon = 2$, we have the inequality

$$\|U_k\|_{W_4^1(\Omega)} \le A_9 \|U_k\|_{W_2^2(\Omega)}, \ \forall t \in [0,T],$$

where A_9 — is some constant. Based on the last inequality and (21), (24) we obtain from (26) the estimation (25). Vector-function *U* and pressure function *P* subject to estimates (20)–(25), satisfy equations (1a) almost everywhere in the *Q* region. Theorem 3 is proved.

Remark 1. Theorem 2 on the uniqueness of weak generalized solutions of problem (1) is also true for their strong classical solutions.

*Remark 2.*This work was conducted with the support of the Kazakh fund of fundamental research, project № 0112 PK 00832.

References

1 *Ladyzhenskaya O.A.* Mathematical problems of dynamics of viscous incompressible fluids. — Moscow: Nauka, 1970. — 288 p.

2 *Ladyzhenskaya O.A.* Sixth millennium problem: Navier-Stokes equation, existence and smoothness // UMN, 2003, 58:2(350). — P. 45–78.

3 *Fefferman Ch.* Existence and smoothness of the Navier-Stokes equation //http: //claymath.org /Millenium Prize Problems // Navier-Stokes Equations. Cambridge, MA: Clay Mathematics Institute, 2000. — P. 1–5.

4 *Akysh A.Sh.* The maximum principle of the Navier-Stokes equation \\ http://www.ksu.kz // BuketovKarSU Herald. 2012. — № 2 (66). — P. 4–16.

5 *Akysh A.Sh.* The maximum principle of the Navier-Stokes equation // USA, 2012. arXiv.org: 1204.2668v [math-ph]. — 16 p.

6 *Akysh A.Sh.* On extreme properties of the Navier-Stokes equation solution // Differential equations. Functional spaces. Approximation theory. Proceedings of the International conference dedicated to the 105 years since the birth of S.L. Sobolev (Novosibirsk, August 18–24, 2013). — P. 76.

7 *Akysh A.Sh.* On the extreme properties of the solution to the Navier-Stokes equation // Izvestiya NAS RK, phys.-math. ser., 2013. — № 4. — P. 75–80.

8 *Sobolev S.L.* On one new problem of mathematical physics // Izvestiya AS USSR, mathematics ser., 1954. — № 18. — P. 3–50.

9 *Michlin S.G.* Linear equations in partial derivatives. — Moscow: Vishaya shk., 1977. — 431 p.

10 Vladimirov *V.S.* Equations of mathematical physics. — Moscow: Nauka, 1976. — P. 510.

А.Ш.Ақыш

Навье-Стокс теңдеулері үшін ықшамдалған максимум қағидасы

Мақалада үш өлшемді бейсызықты Навье-Стокс теңдеулері (НСТ) үшін максимум принципінің орындалатындығы көрсетілген. Оның негізінде таңдалынған кеңістікте НСТ-ға қойылған есептің барлық уақыт $t \in [0,T], \forall T < \infty$ аралығында әлсіз шешуінің жалқылығы мен қоса әлді шешуінің болатындығы дәлелденген.

А.Ш.Акыш

Упрощённый принцип максимума для уравнений Навье-Стокса

В статье показана справедливость принципа максимума для уравнений Навье-Стокса (УНС). На основе чего в выбранном пространстве доказаны единственность слабых и существование сильных решений задачи для УНС в целом по времени $t \in [0,T], \forall T < \infty$.

УДК 378:658.336.3

Ю.Н.Антипов[1], Б.К.Шаяхметова[2], Г.Т.Омаров[3]

[1]Калининградский технический университет, Россия;
[2]Карагандинский государственный университет им. Е.А.Букетова;
[3]Карагандинский экономический университет Казпотребсоюза (E-mail:shakirova_ainura@mail.ru)

Основы проектирования и строительства программ

В статье рассмотрена новая парадигма информатики о проектировании алгоритмов, структур данных и программных структур. Известно, что инженерия опирается на специфические методы и методики, в том числе эвристические. Основная цель этой статьи в том, что инженерия программирования является формальным процессом, который можно изучать, выражать в методиках и совершенствовать. С этой точки зрения и написана настоящая работа. В ней изложены основополагающие идеи новой для программирования парадигмы. Также определены этапы и дана характеристика каждого из них.

Ключевые слова: проектирование, алгоритм, структура данных, метод, средства и процедуры, методика, технология программирования, инженерия программирования, система автоматизированного проектирования, проект, интерфейс.

Анализ современного состояния и развития производства программных изделий показывает, что в настоящее время программирование трансформировалось в целую индустрию. В рамках дальнейшего образования уже мало знать только язык программирования и операционный подход к составлению алгоритмов. Выпускник, как профессиональный разработчик программных продуктов, должен владеть теорией проектирования (как без оформленного проекта вполне можно построить скворечник, но невозможно строительство высотного здания, так и без проекта можно реализовать лишь небольшую программу, но не автоматизированное рабочее место специалиста и т.д.), методами активизации мышления (на ранних этапах развития программирования, когда программы писались в виде последовательностей машинных команд, какая-либо технология программирования отсутствовала, программирование, как правило, было работой отдельных одаренных людей). Ему (выпускнику) необходимо умение оперирования моделями, методами генерации решения и выбора их оптимальных вариантов. Наблюдается стремительный рост объема и сложности изучаемого материала, что делает необходимым его оперативное обновление и формирование принципиально нового подхода к конструированию содержания и организации учебного материала.