HOLOGRAPHICNESS IN PERFECT JONSSON THEORIES

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When defining the concept of holographicness [1, Definition 1], the authors of [1] used the semantic definition (holographic structure), and at the same time proved the criterion for the holographic structure of structure through syntactic tools (formulas, types, theorems 2,3 [1]). Because the original definition of the concept of holographic structure turned out to be quite interesting for studying the theoretical and model properties of this structure, we would like, firstly, to determine the holographic nature of the theory in the framework of the study of Jonsson theories. Note that the notion of holographicness for the theory is still assumed in an implicit form if only because the elementary (complete) theory of some holographic structure is considered in [1]. An example of this is the notion of - the categoricity of a holographic structure [1, Corollary 1].

Based on the above criteria for the holographic structure from [1], it seems to us quite correct to define the following syntactic concept related to the concept of a holographic structure, namely the definition of a holographic Jonsson theory.

The Jonsson theory T is called holographic if $S_n^J(T)$ is finite where $n = h(\sigma)$ and $S_n^J(T)$ are the set of all complete ∀∃ types in free variables.

And accordingly, we can define the concept of a holographic model of the Jonsson theory through the following definition. A model A of a holographic Jonsson theory T is holographic if

1. $Th_{\forall\exists}(A)$ is the Jonsson theory,
2. $Th_{\forall\exists}(A)$ is holographic.

Fact 1. If the Jonsson theory T is holographic, then $S_m^J(T)$ is finite for any $m < n$, where $n = h(\sigma)$.

Fact 2. Let T be a perfect ∀∃ complete Jonsson theory, then the following conditions are equivalent:

1. T is holographic theory
2. its center $T^* = Th(C_T)$ is the holographic theory.
3. the number of orbits of the action of the group of automorphisms $Aut(C_T)$ on the set $C_T^{h(\sigma)}$ is finite.

Note that we have given an example of a holographic Jonsson theory T for which $Hol_T \neq \emptyset$, the theory T is not ω-categorical.

The following concept generalizes the concept of both the holographic theory and the holographic model in the framework of the study of Jonsson theories and their models in the case of considering classes of models.

Let K be the class of structures of some fixed signature. Then this class is called a Jonsson-holographic class if $Th_{\forall\exists}(K)$ is a Jonsson holographic theory. Let $T = Th_{\forall\exists}(K)$ be the theory of a class of finite cyclic groups. Let's take an example of such a class. It is easy to see that T is categorical in all finite cardinalities, but uncountable categorical.

Thus, taking into account that the holographicness of Abelian groups is related to the finiteness due to the results of [1], all models of this theory are holographic.

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References

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