# DESCRIPTION OF "SMALL" DEFINABLE SETS

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The main result of this abstract is related to the problem identified in the work of J. Baldwin and D. Kueker on algebraically prime models [1]. By "small" sets we mean sets whose closures give a countable simple or countably atomic model within the framework of the definitions of the given thesis. We have proved the equivalence of atomic and simple models obtained with the help of some closure operator given on definable subsets of the semantic model of some fixed cosemanticness class of the perfect Jonsson spectrum of the class of existentially closed models of some complete inductive theory.

Definition 1. [2] A model $\mathfrak{A}$ of a theory $T$is called atomic if each tuple of its elements implementssome complete formula in this theory.

Definition 2. [3] A model $\mathfrak{A}$ of a theory $T$ is called an algebraically prime model ofa theory $T$ if it can be isomorphically embedded in every model of $T$.

Definition 3.[4] A model $\mathfrak{A}$ of a theory $T$ is called a nice almost-weak $(\Sigma_1, \Sigma_2) - cl$ atomic model of theory $T$ if every $\omega$-sequence of elements of $\mathfrak{A}$ realizes a $\Sigma_1$-principal $\Sigma_2$-$\omega$-type.

Definition 4. [5] A Jonsson theory $T$ is called perfect if every semantic model of $T$ is a saturated model of $T^*$.

Let $\sigma$be some signature, $L$is set of all formulas of the signature $\sigma$. Let $T$ is some complete inductive theory, $K \subseteq E_T$ , where $E_T$ is the class of existentially closed models of the theory T.Let's call the following set a perfect Jonsson spectrum of class $K$:

$$PJSp(K) = \{\tilde{T} \mid \tilde{T}\ is \text{ perfect Jonsson theory in language } \sigma \text{ and } K \subseteq Mod\tilde{T}\}$$

The cosemanticness relation on a set of theories is an equivalence relation.Then $PJSp(K)/_{\bowtie}$is the factor set of a perfect Jonsson spectrum of class $K$ with respect to$\bowtie$.

Let $[\tilde{T}] \in PJSp(K)/_{\bowtie}$.Since for each theory$\Delta \in [\tilde{T}]$ we have$C_\Delta = C_{\tilde{T}}$, then we will call the semantic model of the class $[\tilde{T}]$ the semantic model of the theory $\tilde{T}$:$C_{[\tilde{T}]} = C_{\tilde{T}}$. The center of the Jonsson class $[\tilde{T}]$ we call the elementary theory $[\tilde{T}]^*$of its semantic model $C_{[\tilde{T}]}$, i.e. $[\tilde{T}]^* = Th(C_{[\tilde{T}]})$and$[\tilde{T}]^* = Th(C_\Delta)$for each $\Delta \in [\tilde{T}]$.

Let $R$ is some theoretical-model property of theory, $[\tilde{T}] \in PJSp(K)/_{\bowtie}$. We say that a class $[\tilde{T}]$ has property $R$ if every theory $\Delta \in [\tilde{T}]$has property $R$.

Let$T$ besome complete inductive theory,$K \subseteq E_T$, $[\tilde{T}] \in PJSp(K)/_{\bowtie}$is a complete class for $\Pi_1$-sentences.

Definition 5. Let $A \subseteq C_{[\tilde{T}]}, [\tilde{T}] \in PJSp(K)/_{\bowtie}$. A set $A$ is called $(\Gamma_1, \Gamma_2) - cl$-algebraically prime in class $[\tilde{T}]$ if $cl(A) = M$, $M$ is a $(\Gamma_1, \Gamma_2)$-$cl$-atomic model of the theory $\tilde{T}$, $M \in E_{\tilde{T}} \cap AP_{\tilde{T}}$, where $E_{\tilde{T}} \cap AP_{\tilde{T}} \neq \emptyset$, and the resulting model $M$ is called a $(\Gamma_1, \Gamma_2)$-$cl$ -algebraically prime model of the class $[\tilde{T}]$ .

Let $X \subseteq C_{[\tilde{T}]}$be a Jonsson set defined by some existential strong minimal formula $\varphi(x)$and $cl(X) = M \in E_{[\tilde{T}]}$, where $[\tilde{T}] \in PJSp(K)/_{\bowtie}$

Further, since to the fact that the class under consideration is perfect, it follows that its center is a model complete theory. Therefore, $\Sigma_1$-completeness of the class under consideration will be equivalent to $\Pi_1$-completeness of this class.

Theorem 1.Let $T$ be a some inductive complete theory, $K \subseteq E_T$, $[\tilde{T}] \in PJSp(K)/_{\bowtie}$ complete class for $\Pi_1$-sentences, then $[\tilde{T}]$ has a nice almost-weakly $(\Sigma_1, \Sigma_1)$ -$cl$-atomic model.

Theorem 2. Let $T$ be a some inductive complete theory, $K \subseteq E_T$, $[\tilde{T}] \in PJSp(K)/_{\bowtie}$ complete class for $\Pi_1$-sentences, Then the following conditions are equivalent:

1) $\mathfrak{A}$ is $(\Sigma_1, \Sigma_1)$-$cl$-algebraically prime model of class $[\tilde{T}]$.

2) $\mathfrak{A}$ is a nice almost-weak $(\Sigma_1, \Sigma_1)$-$cl$ -atomic model of the theory $[\tilde{T}]^*$.

The proof follows from Theorem 2 of [6].

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## AFFINE AUTOMORPHISMS OF THE UNIVERSAL MULTIPLICATIVE ENVELOPING ALGEBRA OF THE TWO-DIMENSIONAL LEFT-SYMMETRIC ALGEBRA WITH ZERO MULTIPLICATION

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In this work we rewrite for the left-symmetric algebra the result of D. Kozybaev, U. Umirbaev [1] on the basis of the universal multiplicative enveloping algebra of the right-symmetric algebra. Also we describe the affine automorphisms of the universal multiplicative enveloping algebra of the two-dimensional left-symmetric algebra with zero multiplication. The question of describing all automorphisms of this algebra remains open. Although it was easy to notice that the automorphism groups of the left and the right universal multiplicative enveloping algebras of the two-dimensional left-symmetric algebra with zero multiplication are tame.

An algebra $A$ over an arbitrary field $k$ with a bilinear product $x \cdot y$ is called *a left-symmetric algebra*, if the identity

$$(xy)z - x(yz) = (yx)z - y(xz) \qquad (1)$$

is satisfied for any $x, y, z \in A$.

Recall that $U(A)$ is an associative algebra with 1 generated by the operators of left multiplication $l_x$ and right multiplication $r_x$, where $x \in A$. The identity (1) directly implies the defining relations of the algebra $U(A)$:

$$l_x l_y - l_y l_x = l_{[x,y]}, \quad r_x l_y - l_y r_x - r_x r_y + r_{yx} = 0, \quad x, y \in A.$$

The linear basis of the algebra $U(A)$ is described by the following

Theorem 1. Let $A$ be a left-symmetric algebra with linear basis $x_1, x_2, \ldots, x_k, \ldots$. Then the basis of the universal multiplicative enveloping algebra $U(A)$ of $A$ consists of words of the form

$$l_{x_{j_1}} l_{x_{j_2}} \ldots l_{x_{j_t}} r_{x_{i_1}} r_{x_{i_2}} \ldots r_{x_{i_s}},$$

where $j_1 \le j_2 \le \ldots \le j_t, \quad s, t \ge 0$.