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Periodic solutions of boundary value problems for hyperbolic equations with a mixed derivative

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The monograph is devoted to the study of periodic boundary value problems for systems of hyperbolic equations with a mixed derivative. The work proposes methods for finding approximate solutions of periodic boundary value problems for systems of linear hyperbolic equations. The coefficient characteristics of the unique solvability of the periodic boundary value problem for a system of linear hyperbolic equations are established. The definition of the isolated solution of a semi-periodic boundary value problem for the nonlinear system of the hyperbolic equations, generalizing definition of the isolated solution of nonlinear boundary value problems for ordinary differential equations are introduced. The criterion for the existence of a solution of a semi-periodic boundary value problem for systems of nonlinear hyperbolic equations in terms of the problem data is established. The main theorems of the work are illustrated by examples. The monograph is of interest to teachers, undergraduates and university students.

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INTRODUCTION

One of the main and most studied problems of the theory of the hyperbolic equations of the second order is a periodic boundary value problem. A systematic study of periodic boundary value problems for hyperbolic equations with mixed partial derivatives started in 60s with the work of L. Cesari [1-4], J.K. Hale [5], G. Hecquet [6-8], A.K. Aziz [9-12], V. Lakshmikantham [13], S.V. Zhestkov [14], A.M. Samoylenko [15, 16], T.I. Kiguradze [17-27], B.I. Ptashnik [28], Yu.A. Mitropolskiy, G.P. Homa, M.I. Gromyak [29] and others dealt with further investigations of the solvability of periodic boundary value problems. To solve periodic boundary value problems of second order hyperbolic equations, were applied the Fourier method, the method of successive approximations, the methods of functional analysis, the variational method [30-51], etc. Despite the presence of numerous methods for study of periodic boundary value problems, interest in them continues to this day. The application of different approaches, ideas and methods leads to results formulated in different terms. The development of information technologies and its comprehensive application in applied problems imposes new requirements of the developed methods. Particular attention got to be paid to the methods that are different from others in their constructiveness at the stage of approximate construction of solutions and in the study of such qualitative issues as the establishment of the existence of a solution, the rationale for the convergence of approximate solutions to the exact one, an estimate of the inaccuracy of the approximate solution.

One of such constructive methods is the method of a parametrization that was proposed by D.S. Dzhumabaev [52], for solving two-point boundary problems of ordinary differential equations. The point of using this method is to enter additional parameters and bring the original problem to multipoint boundary value problem with a parameter. It allows in terms of initial data to set conditions for the solvability of the boundary value problem for ordinary differential equations and to propose a family of algorithms for finding its approximate solution.

A modification of the parametrization method is the method of adding a functional parameter, devised in the works of

D.S. Dzhumabaev and A.T. Asanova [53-54], which finds its application in the study of nonlocal boundary value problems with data on characteristics for a system of hyperbolic equations with a mixed derivative with two independent variables. There were constructed two-parameter families of algorithms for finding solutions to nonlocal boundary-value problems, at each step of which the Goursat problems are solved. The coefficient criteria for the unique solvability of a boundary value problem with data on characteristics for a system of linear hyperbolic equations and sufficient conditions for the existence of a solution of a boundary value problem for a system of nonlinear hyperbolic equations with a mixed derivative are established on the basis of this algorithm. However, issues related to finding the necessary and sufficient conditions for non-local boundary-value problems for systems of nonlinear hyperbolic equations with a mixed derivative, and building systems of equations that allow one to find initial approximations of solutions have not been studied.

In this work we consider semi-periodic boundary value problems for systems of hyperbolic equations with mixed derivatives. Application of a method of a parametrization allowed us to obtain necessary and sufficient conditions to obtain necessary and sufficient conditions for the existence of an "isolated" solution of the semi-periodic boundary value problem for systems of nonlinear hyperbolic equations with mixed derivative, to construct systems of equations that allow to determine the initial approximations of solutions, to establish new criteria of the correct solvability of the semi-periodic boundary value problem for systems of linear hyperbolic equations with mixed derivative and to propose constructive algorithms for finding its approximate solution. The advantage of the new algorithms from the previously proposed ones is that there is no need to find a solution to the Goursat problem at each step.

1 THE ALGORITHM FOR FINDING SOLUTIONS OF A SEMI-PERIODIC BOUNDARY VALUE PROBLEM FOR SYSTEMS OF LINEAR HYPERBOLIC EQUATIONS WITH MIXED DERIVATIVE

In this section, we consider a linear semi-periodic boundary value problem for systems of hyperbolic equations with a mixed derivative.

Hyperbolic equations with mixed derivatives of the second order from two independent variables are applied in the dynamics and kinetics of gas sorption with a linear and nonlinear isotherm, when describing the kinetics of filtration clarification of low concentrated aqueous suspensions on water purification filters, when considering the processes of drying air flow and isentropic one-dimensional plane flow in gas dynamics [55].

In subsection 1.1, the scheme of a method of a parametrization applied to a semi-periodic boundary value problem for systems of linear hyperbolic equations with a mixed derivative of the form is provided

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + f(x,t), \quad (0.1)$$

$$(x,t) \in \overline{\Omega} = [0,\omega] \times [0,T],$$

$$u(0,t) = \psi(t), \qquad t \in [0,T], \quad (0.2)$$

$$u(x,0) = u(x,T), \qquad x \in [0,\omega], \quad (0.3)$$

where $(n \times n)$ - matrix $A(x,t)$, $B(x,t)$, $C(x,t)$, n - a vector function $f(x,t)$ are continuous on $\overline{\Omega}$, n - vector function $\psi(t)$ is continuously differentiable on $[0,T]$, and satisfies the condition $\psi(0) = \psi(T)$.

In subsection 1.2, on the basis of the equivalence of the semi-periodic boundary value problem for systems of linear hyperbolic equations and the periodic boundary value problem for a family of systems of ordinary differential equations, a criterion for the correct solvability of the investigated problem (0.1)-(0.3) is established.

In subsection 1.3, we consider a semi-periodic boundary value problem of the form (0.1)-(0.3) for $B(x,t) \equiv 0$, i.e.

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + C(x,t)u + f(x,t), \qquad (0.4)$$
$$(x,t) \in \overline{\Omega} = [0,\omega] \times [0,T],$$
with conditions (0.2), (0.3). Similar to subsection 1.1, problem (0.4), (0.2), (0.3) is investigated by the method of parametrization. On the basis of this method, to find an approximate solution of the boundary value problem (0.4), (0.2), (0.3), proposed an efficient algorithm that differs in its constructiveness from the algorithm proposed in subsection 1.1. In terms of the initial data, the coefficient criteria of the unique solvability of the semi-periodic boundary value problem (0.4), (0.2), (0.3) are established.

1.1 Statement of a linear semi-periodic boundary value problem and sufficient conditions for the convergence of algorithms finding its solution

Mathematical modeling of phenomena and processes repeated after a certain period of time leads to the need to study periodic boundary value problems of hyperbolic type. Periodic boundary value problems for hyperbolic equations with mixed partial derivatives are considered in the works of L. Cesari, J.K. Hale, G. Hecquet, A.K. Aziz, V. Lakshmikantham, S.V. Zhestkov, A.M. Samoylenko, T.I. Kiguradze, B.I. Ptashnik, Yu.A. Mitropolskiy, G.P. Homa, M.I. Gromyak and others.

Despite existence of a large number of the works devoted to a research of periodic boundary value problems interest in them doesn't weaken to this day. The application of various approaches, ideas and methods leads to the results formulated in various terms. The development of information technologies and its comprehensive application in solving applied problems imposes new requirements on the proposed methods, paying particular attention to their constructiveness. The main characteristic of constructive methods is the effective verification of the conditions of their applicability and the possibility of using them to find solutions with a given accuracy.

In this regard, the development of new approaches to expand the classes of solvable periodic problems and the creation of constructive algorithms for finding their solutions is relevant both for the development of the theory of boundary value problems with partial derivatives, and for its application in practice.

The subject of research in this subsection 1.1 is the construction of algorithms for finding solutions and determining the conditions of convergence of the proposed algorithms considered on $\overline{\Omega} = [0,\omega]\times[0,T]$ of a linear semi-periodic boundary value problem for a system of hyperbolic equations of the form

$$\frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + f(x,t), (x,t)\in\overline{\Omega}, \tag{1.1}$$

$$u(0,t) = \psi(t), \qquad t\in[0,T], \tag{1.2}$$

$$u(x,0) = u(x,T), \qquad x\in[0,\omega], \tag{1.3}$$

where $(n\times n)$ matrices $A(x,t), B(x,t), C(x,t),$ n - vector-function $f(x,t)$ are continuous on $\overline{\Omega}$, n - vector-function $\psi(t)$ continuously differentiable on $[0,T]$, and satisfies the condition $\psi(0)=\psi(T),$ here $\|u(x,t)\| = \max\limits_{i=\overline{1,n}} |u_i(x,t)|,$ $\|A(x,t)\| = \max\limits_{i=\overline{1,n}} \sum\limits_{j=1}^{n} |a_{ij}(x,t)|.$

Let $C(\overline{\Omega}, R^n)$ - be the space of functions $u:\overline{\Omega}\to R^n$ continuous on $\overline{\Omega},$ with norm $\|u\|_0 = \max\limits_{(x,t)\in\overline{\Omega}} \|u(x,t)\|.$

The function $u(x,t)\in C(\overline{\Omega}, R^n)$, which has partial derivatives

$$\frac{\partial u(x,t)}{\partial x}\in C(\overline{\Omega}, R^n), \quad \frac{\partial u(x,t)}{\partial t}\in C(\overline{\Omega}, R^n), \quad \frac{\partial^2 u(x,t)}{\partial x \partial t}\in C(\overline{\Omega}, R^n)$$

is called a solution to problem (1.1)-(1.3), if it satisfies system (1.1) for all $(x,t)\in\overline{\Omega}$, on the characteristic $x=0$ takes the obtain value $\psi(t), t\in[0,T],$ on the characteristics $t=0, t=T$ has equal values for $x\in[0,\omega]$.

To find a solution to this problem, we introduce new unknown functions

$$v(x,t) = \frac{\partial u(x,t)}{\partial x}, \quad w(x,t) = \frac{\partial u(x,t)}{\partial t}$$

and write problem (1.1)-(1.3) in the form

$$\frac{\partial v}{\partial t} = A(x,t)v + B(x,t)w(x,t) + C(x,t)u(x,t) + f(x,t), \tag{1.4}$$

$$v(x,0) = v(x,T), \quad x \in [0,\omega], \tag{1.5}$$

$$w(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial v(\xi,t)}{\partial t} d\xi, \quad t \in [0,T], \quad x \in [0,\omega], \tag{1.6}$$

$$u(x,t) = \psi(t) + \int_0^x v(\xi,t) d\xi, \quad t \in [0,T], \quad x \in [0,\omega]. \tag{1.7}$$

Here, the problem of finding a solution of a semi-periodic boundary value problem for a system of hyperbolic equations (1.1)-(1.3) is reduced to a family of periodic boundary value problems for ordinary differential equations (1.4), (1.5) and functional relations (1.6), (1.7). Problems (1.1)-(1.3) and (1.4)-(1.7) are equivalent in the sense that if the function $u(x,t)$, is a solution to problem (1.1)-(1.3), then the triple $(v(x,t), w(x,t), u(x,t))$ will be a solution to problem (1.4)-(1.7) and, conversely, if a triple $(\tilde{v}(x,t), \tilde{w}(x,t), \tilde{u}(x,t))$ is a solution to problem (1.4)-(1.7), then $\tilde{u}(x,t)$ solution of problem (1.1)-(1.3).

The method of a parametrization [56-58] is applied to the solution of a problem (1.4)-(1.7). By the step $h > 0 : Nh = T$ we partition the

$$[0,T) = \bigcup_{r=1}^{N} [(r-1)h, rh), \; N = 1,2,\dots.$$

In this case, the range of Ω is divided into N parts.

Let $v_r(x,t)$, $w_r(x,t)$, $u_r(x,t)$ denote, respectively, the restrictions of the functions $v(x,t)$, $w(x,t)$, $u(x,t)$ to $\Omega_r = [0,\omega] \times [(r-1)h, rh), \quad r = \overline{1,N}.$

Then problem (1.4)-(1.7) will be equivalent to the boundary value problem

$$\frac{\partial v_r}{\partial t} = A(x,t)v_r + B(x,t)w_r(x,t) + C(x,t)u_r(x,t) + f(x,t), \tag{1.8}$$

$$(x,t) \in \Omega_r,$$

$$v_1(x,0) - \lim_{t \to T-0} v_N(x,t) = 0, \quad x \in [0,\omega], \tag{1.9}$$

$$\lim_{t \to sh-0} v_s(x,t) = v_{s+1}(x,sh), \qquad s = \overline{1, N-1}, \tag{1.10}$$

$$w_r(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial v_r(\xi,t)}{\partial t} d\xi, \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}, \tag{1.11}$$

$$u_r(x,t) = \psi(t) + \int_0^x v_r(\xi,t)d\xi, \quad (x,t)\in\Omega_r, \quad r=\overline{1,N}, \tag{1.12}$$

where (1.10) - is the condition for gluing the functions $v(x,t)$ in the inner lines of the partition. Let $\lambda_r(x)$ denote the value of the function $v_r(x,t)$ with $t=(r-1)h,$ i.e. $\lambda_r(x) = v_r(x,(r-1)h)$ and make a replacement for $\tilde{v}_r(x,t) = v_r(x,t) - \lambda_r(x),$ $r=\overline{1,N}$. We obtain an equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial\tilde{v}_r}{\partial t} = A(x,t)\tilde{v}_r + A(x,t)\lambda_r(x) + B(x,t)w_r(x,t) +$$

$$+ C(x,t)u_r(x,t) + f(x,t), \tag{1.13}$$

$$\tilde{v}_r(x,(r-1)h) = 0, \quad x\in[0,\omega], \quad r=\overline{1,N}, \tag{1.14}$$

$$\lambda_1(x) - \lambda_N(x) - \lim_{t\to T-0}\tilde{v}_N(x,t) = 0, \quad x\in[0,\omega], \tag{1.15}$$

$$\lambda_s(x) + \lim_{t\to sh-0}\tilde{v}_s(x,t) - \lambda_{s+1}(x) = 0,\ x\in[0,\omega],\ s=\overline{1,N-1}, \tag{1.16}$$

$$w_r(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial\tilde{v}_r(\xi,t)}{\partial t}d\xi, \quad (x,t)\in\Omega_r, \quad r=\overline{1,N}, \tag{1.17}$$

$$u_r(x,t) = \psi(t) + \int_0^x \tilde{v}_r(\xi,t)d\xi + \int_0^x \lambda_r(\xi)d\xi, \quad r=\overline{1,N}. \tag{1.18}$$

Problems (1.8)-(1.12) and (1.13)-(1.18) are equivalent in the sense that if the system of triples $\{v_r(x,t), w_r(x,t),\ u_r(x,t)\}$, $r=\overline{1,N},$ is a solution to problem (1.8)-(1.12), then the system $\{\lambda_r(x)=v_r(x,(r-1)h),$ $\tilde{v}_r(x,t) = v_r(x,t) - v_r(x,(r-1)h),$ $w_r(x,t), u_r(x,t)\},$ $r=\overline{1,N},$ is a solution to problem (1.13)-(1.18) and, conversely, if $\{\lambda_r(x),$ $\tilde{v}_r(x,t),$ $w_r(x,t),$ $u_r(x,t)\},$ $r=\overline{1,N}$ - is a solution to problem (1.13)-(1.18), then triples $\{\lambda_r(x)+\tilde{v}_r(x,t), w_r(x,t), u_r(x,t)\},$ $r=\overline{1,N},$ will be the solution of problem (1.8)-(1.12).

Problem (1.13), (1.14) with fixed $\lambda_r(x), u_r(x,t), w_r(x,t),$ $r=\overline{1,N},$ is a family of Cauchy problems for ordinary differential equations, where $x\in[0,\omega],$ and is equivalent to the integral equation

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} A(x,\tau)\tilde{v}_r(x,\tau)d\tau + \int_{(r-1)h}^{t} A(x,\tau)d\tau \cdot \lambda_r(x) +$$

$$+ \int_{(r-1)h}^{t} [B(x,\tau)w_r(x,\tau) + C(x,\tau)u_r(x,\tau) + f(x,\tau)]d\tau. \qquad (1.19)$$

Replace $\tilde{v}_r(x,\tau)$ with the corresponding right part (1.19) and repeating this process ν $(\nu = 1,2,...)$ times we obtain

$$\tilde{v}_r(x,t) = D_{\nu r}(x,t)\lambda_r(x) + F_{\nu r}(x,t,w_r,u_r) + G_{\nu r}(x,t,\tilde{v}_r), \qquad (1.20)$$

where

$$D_{\nu r}(x,t) = \sum_{j=0}^{\nu-1} \int_{(r-1)h}^{t} A(x,\tau_1)d\tau_1 ... \int_{(r-1)h}^{\tau_j} A(x,\tau_{j+1})d\tau_{j+1}...d\tau_1,$$

$$F_{\nu r}(x,t,w_r,u_r) =$$

$$= \int_{(r-1)h}^{t} [B(x,\tau_1)w_r(x,\tau_1) + C(x,\tau_1)u_r(x,\tau_1) + f(x,\tau_1)]d\tau_1 +$$

$$+ \sum_{j=1}^{\nu-1} \int_{(r-1)h}^{t} A(x,\tau_1) ... \int_{(r-1)h}^{\tau_{j-1}} A(x,\tau_j) \int_{(r-1)h}^{\tau_j} [B(x,\tau_{j+1})w_r(x,\tau_{j+1}) +$$

$$+ C(x,\tau_{j+1})u_r(x,\tau_{j+1}) + f(x,\tau_{j+1})]d\tau_{j+1}d\tau_j...d\tau_1,$$

$$G_{\nu r}(x,t,\tilde{v}_r) =$$

$$= \int_{(r-1)h}^{t} A(x,\tau_1) ... \int_{(r-1)h}^{\tau_{\nu-2}} A(x,\tau_{\nu-1}) \int_{(r-1)h}^{\tau_{\nu-1}} A(x,\tau_\nu)\tilde{v}_r(x,\tau_\nu)d\tau_\nu d\tau_{\nu-1}...d\tau_1, \ \tau_0 = t, r = \overline{1,N}.$$

Passing to the limit as $t \to rh - 0$ (1.20) we have

$$\lim_{t \to rh-0} \tilde{v}_r(x,t) = D_{\nu r}(x,rh)\lambda_r(x) + F_{\nu r}(x,rh,w_r,u_r) + G_{\nu r}(x,rh,\tilde{v}_r),$$

$x \in [0,\omega]$, $r = \overline{1,N}$. Substituting in (1.15), (1.16) instead of $\lim_{t \to rh-0} \tilde{v}_r(x,t)$, $r = \overline{1,N}$, the corresponding right-hand sides for the unknown functions $\lambda_r(x)$, $r = \overline{1,N}$, we obtain a system of functional equations:

$$Q_\nu(x,h)\lambda(x) = -F_\nu(x,h,w,u) - G_\nu(x,h,\tilde{v}), \qquad (1.21)$$

$$Q_\nu(x,h) = \begin{vmatrix} -I & 0 & ... & -[I + D_{\nu N}(x,Nh)] \\ I + D_{\nu 1}(x,h) & -I & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & -I \end{vmatrix},$$

$$F_\nu(x,h,w,u) =$$

$$= (-F_{\nu N}(x,Nh,w_N,u_N), F_{\nu 1}(x,h,w_1,u_1), ..., F_{\nu,N-1}(x,(N-1)h,w_{N-1},$$

$$G_\nu(x,h,\tilde{v}) =$$

$$= (-G_{\nu N}(x,Nh,\tilde{v}_N), G_{\nu 1}(x,h,\tilde{v}_1), ..., G_{\nu,N-1}(x,(N-1)h,\tilde{v}_{N-1})),$$

I - unit matrix of dimension of n.

To find a system of four functions $\{\lambda_r(x), \tilde{v}_r(x,t), w_r(x,t),$ $u_r(x,t)\}, r = \overline{1, N},$ we have a closed system consisting of equations (1.21), (1.20), (1.18) and (1.17).

Assuming the invertibility of the matrix $Q_\nu(x,h)$ for all $x \in [0,\omega]$, from equation (1.18), where

$$\tilde{v}_r(x,t) = 0, w_r(x,t) = \dot{\psi}(t), u_r(x,t) = \psi(t),$$

we find $\lambda^{(0)}(x) = (\lambda_1^{(0)}(x), \lambda_2^{(0)}(x), \ldots, \lambda_N^{(0)}(x))'$:

$$\lambda^{(0)}(x) = -[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,\dot{\psi},\psi) + G_\nu(x,h,0)\}.$$

Using equation (1.20), with $\lambda_r(x) = \lambda_r^{(0)}(x)$ find the functions $\{\tilde{v}_r^{(0)}(x,t)\},\ r = \overline{1, N},$ i.e.

$$\tilde{v}_r^{(0)}(x,t) = D_{\nu r}(x,t)\lambda_r^{(0)}(x) + F_{\nu r}(x,t,\dot{\psi},\psi) + G_{\nu r}(x,t,0).$$

Functions $u_r^{(0)}(x,t), w_r^{(0)}(x,t), r = \overline{1, N},$ are determined from the relations

$$u_r^{(0)}(x,t) = \psi(t) + \int_0^x \tilde{v}_r^{(0)}(\xi,t)d\xi + \int_0^x \lambda_r^{(0)}(\xi)d\xi,$$

$$w_r^{(0)}(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial \tilde{v}_r^{(0)}(\xi,t)}{\partial t}d\xi,$$

$(x,t) \in \Omega_r$. For the initial approximation of the problem (1.13)-(1.18) we take the system $(\lambda_r^{(0)}(x), \tilde{v}_r^{(0)}(x,t)), w_r^{(0)}(x,t), u_r^{(0)}(x,t)), r = \overline{1, N}$ and successive approximations are constructed according to the following algorithm:

Step 1. A) Assuming that

$$w_r(x,t) = w_r^{(0)}(x,t), \quad u_r(x,t) = u_r^{(0)}(x,t), \quad r = \overline{1, N},$$

first approximations in $\lambda_r(x), \tilde{v}_r(x,t), r = \overline{1, N},$ find solving the problem (1.13)-(1.16). Taking

$$\lambda_r^{(1,0)}(x) = \lambda_r^{(0)}(x), \quad \tilde{v}_r^{(1,0)}(x,t) = \tilde{v}_r^{(0)}(x,t),$$

system couple $\{\lambda_r^{(1)}(x), \tilde{v}_r^{(1)}(x,t)\}, r = \overline{1, N},$ find the limit of the sequence $\lambda_r^{(1,m)}(x),\ \tilde{v}_r^{(1,m)}(x,t),$ defined the next way:

Step 1.1. Assuming the invertibility of the matrix $Q_\nu(x,h),$

$x \in [0, \omega]$, from equation (1.21), where $\tilde{v}_r(x,t) = \tilde{v}_r^{(1,0)}(x,t)$, we find $\lambda^{(1,1)}(x) = (\lambda_1^{(1,1)}(x), \lambda_2^{(1,1)}(x), \ldots, \lambda_N^{(1,1)}(x))'$:

$$\lambda^{(1,1)}(x) = -[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,w^{(0)},u^{(0)}) + G_\nu(x,h,\tilde{v}^{(1,0)})\}.$$

Substituting the found $\lambda_r^{(1,1)}(x)$, $r = \overline{1,N}$, in (1.20) we find

$$\tilde{v}_r^{(1,1)}(x,t) = D_{\nu r}(x,t)\lambda_r^{(1,1)}(x) + F_{\nu r}(x,t,w^{(0)},u^{(0)}) + G_{\nu r}(x,t,\tilde{v}^{(1,0)}).$$

Step 1.2. From equation (1.21), where $\tilde{v}_r(x,t) = \tilde{v}_r^{(1,1)}(x,t)$, we define

$$\lambda^{(1,2)}(x) = -[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,w^{(0)},u^{(0)}) + G_\nu(x,h,\tilde{v}^{(1,1)})\}.$$

Again using expression (1.20), we find the functions $\{\tilde{v}_r^{(1,2)}(x,t)\}$, $r = \overline{1,N}$,

$$\tilde{v}_r^{(1,2)}(x,t) = D_{\nu r}(x,t)\lambda_r^{(1,2)}(x) + F_{\nu r}(x,t,w^{(0)},u^{(0)}) + G_{\nu r}(x,t,\tilde{v}^{(1,1)}).$$

On the step $(1,m)$ we get system couple $\{\lambda_r^{(1,m)}(x), \tilde{v}_r^{(1,m)}(x,t)\}$, $r = \overline{1,N}$.

Suppose that the solution of problem (1.13)-(1.16) the sequence of systems of couples $\{\lambda_r^{(1,m)}(x),\ \tilde{v}_r^{(1,m)}(x,t)\}$ defined and for $m \to \infty$ converges to continuous, respectively, on $x \in [0,\omega]$, $(x,t) \in \Omega_r$ functions $\lambda_r^{(1)}(x)$, $\tilde{v}_r^{(1)}(x,t)$, $r = \overline{1,N}$.

B) Functions $w_r^{(1)}(x,t), u_r^{(1)}(x,t)$, $r = \overline{1,N}$, are determined from the relations

$$w_r^{(1)}(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial \tilde{v}_r^{(1)}(\xi,t)}{\partial t} d\xi,$$

$$u_r^{(1)}(x,t) = \psi(t) + \int_0^x \tilde{v}_r^{(1)}(\xi,t)d\xi + \int_0^x \lambda_r^{(1)}(\xi)d\xi, \qquad (x,t) \in \Omega_r.$$

Step 2. A) Assuming that

$$w_r(x,t) = w_r^{(1)}(x,t), \quad u_r(x,t) = u_r^{(1)}(x,t), \quad r = \overline{1,N},$$

the second approximations on $\lambda_r(x), \tilde{v}_r(x,t), r = \overline{1,N}$, find solving the problem (1.13)-(1.16). Taking the

$$\lambda_r^{(2,0)}(x) = \lambda_r^{(1)}(x), \quad \tilde{v}_r^{(2,0)}(x,t) = \tilde{v}_r^{(1)}(x,t),$$

systems of couples $\{\lambda_r^{(2)}(x), \tilde{v}_r^{(2)}(x,t)\}, r = \overline{1,N}$, we find the limit of the sequence $\lambda_r^{(2,m)}(x), \tilde{v}_r^{(2,m)}(x,t),$ defined in the following way:

Step 2.1. Assuming the invertibility of the matrix $Q_\nu(x,h), \quad x \in [0,\omega]$, from equation (1.21), where $\tilde{v}_r(x,t) = \tilde{v}_r^{(2,0)}(x,t),$ we find

$$\lambda^{(2,1)}(x) = (\lambda_1^{(2,1)}(x), \lambda_2^{(2,1)}(x), ..., \lambda_N^{(2,1)}(x))':$$

$$\lambda^{(2,1)}(x) = -[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,w^{(1)},u^{(1)}) + G_\nu(x,h,\tilde{v}^{(2,0)})\}.$$

Substituting the found $\lambda_r^{(2,1)}(x), \quad r = \overline{1,N},$ at (1.20) we find

$$\tilde{v}_r^{(2,1)}(x,t) = D_{\nu r}(x,t)\lambda_r^{(2,1)}(x) + F_{\nu r}(x,t,w^{(1)},u^{(1)}) + G_{\nu r}(x,t,\tilde{v}^{(2,0)}).$$

Step 2.2. From equation (1.21), where $\tilde{v}_r(x,t) = \tilde{v}_r^{(2,1)}(x,t),$ we define

$$\lambda^{(2,2)}(x) = -[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,w^{(1)},u^{(1)}) + G_\nu(x,h,\tilde{v}^{(2,1)})\}.$$

Again using expression (1.20), we find the functions $\{\tilde{v}_r^{(2,2)}(x,t)\}, \quad r = \overline{1,N}$:

$$\tilde{v}_r^{(2,2)}(x,t) = D_{\nu r}(x,t)\lambda_r^{(2,2)}(x) + F_{\nu r}(x,t,w^{(1)},u^{(1)}) + G_{\nu r}(x,t,\tilde{v}^{(2,1)}).$$

At step $(2,m)$ we obtain a system of couples $\{\lambda_r^{(2,m)}(x), \tilde{v}_r^{(2,m)}(x,t)\}, r = \overline{1,N}.$

Suppose that the solution of problem (1.13)-(1.16) is a sequence of systems of couples $\{\lambda_r^{(2,m)}(x), \quad \tilde{v}_r^{(2,m)}(x,t)\}$ is defined and at $m \to \infty$ converges to $\{\lambda_r^{(2)}(x), \quad \tilde{v}_r^{(2)}(x,t)\}$, $r = \overline{1,N}.$

B) The functions $w_r^{(2)}(x,t), u_r^{(2)}(x,t), r = \overline{1,N},$ are defined from the relations

$$w_r^{(2)}(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial \tilde{v}_r^{(2)}(\xi,t)}{\partial t} d\xi,$$

$$u_r^{(2)}(x,t) = \psi(t) + \int_0^x \tilde{v}_r^{(2)}(\xi,t)d\xi + \int_0^x \lambda_r^{(2)}(\xi)d\xi,$$

where $(x,t) \in \Omega_r.$

Continuing the process, at the step k we obtain the system $\{\lambda_r^{(k)}(x), \tilde{v}_r^{(k)}(x,t), \quad w_r^{(k)}(x,t), u_r^{(k)}(x,t)\}, \quad r = \overline{1,N}.$

The conditions of the following statement provide the feasibility and convergence of the proposed algorithm [59-62], and also the unique solvability of the problem (1.13)-(1.18) .

Theorem 1. *Let for some* $h > 0 : Nh = T,\ N = 1,2,...,$ *and* $\nu, \nu \in N, (nN \times nN)$ *matrix* $Q_\nu(x,h)$ *reversible for all* $x \in [0,\omega]$ *and inequalities are carried out*

1) $\|[Q_\nu(x,h)]^{-1}\| \le \gamma_\nu(x,h);$

2) $q_\nu(x,h) = \dfrac{(\alpha(x)h)^\nu}{\nu!}[1+\gamma_\nu(x,h)\sum\limits_{j=1}^{\nu}\dfrac{(\alpha(x)h)^j}{j!}] \le \mu < 1.$

Then there is a unique solution of problem (1.13)-(1.18) and estimates are valid

$$a)\max\Big\{\max_{r=1,N}\|\lambda_r^*(x) - \lambda_r^{(k)}(x)\| + \max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r^*(x,t) - \tilde{v}_r^{(k)}(x,t)\|,$$

$$\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^*(x,t)}{\partial t} - \frac{\partial\tilde{v}_r^{(k)}(x,t)}{\partial t}\right\|\Big\} \le a_0(x)\sum_{j=k-1}^{\infty}\frac{1}{j!}\left(\int_0^x a_0(\xi)d\xi\right)^j \times$$

$$\times\int_0^x \max\{a_1(\xi), a_2(\xi)\}d\xi \max\{\max_{t\in[0,T]}\|\psi(t)\|, \max_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\}.$$

$$b)\ \max\Big\{\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\|u_r^*(x,t) - u_r^{(k)}(x,t)\|,$$

$$\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\|w_r^*(x,t) - w_r^{(k)}(x,t)\|\Big\} \le$$

$$\le\int_0^x \max\Big\{\max_{r=1,N}\|\lambda_r^*(x) - \lambda_r^{(k)}(x)\| + \max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r^*(x,t) - \tilde{v}_r^{(k)}(x,t)\|,$$

$$\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^*(x,t)}{\partial t} - \frac{\partial\tilde{v}_r^{(k)}(x,t)}{\partial t}\right\|\Big\}, k = 1,2,...,$$

where $\alpha(x) = \max\limits_{t\in[0,T]}\|A(x,t)\|,\ \beta(x) = \max\limits_{t\in[0,T]}\|B(x,t)\|,$

$\sigma(x) = \max\limits_{t\in[0,T]}\|C(x,t)\|$,

$$a_0(x) =$$

$$= \max\left\{\frac{[b_1(x)+b_3(x)][\beta(x)+\sigma(x)]}{1-q_\nu(x,h)}, \int_0^x \alpha(\xi)b_3(\xi)+1][\beta(\xi)+\sigma(\xi)]d\xi\right\},$$

$$a_1(x) = \frac{\gamma_\nu(x,h)}{1-q_\nu(x,h)}[1+\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}]([b_1(x)+b_3(x)][\beta(x)+\sigma(x$$

$$\times\int_0^x \max\{\alpha(\xi)b_3(\xi)+1, b_1(\xi)+b_3(\xi)\}b_2(\xi)d\xi + b_3(x)b_2(x)[q_\nu(x,h) +$$

$$+\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}]),$$

$$a_2(x)=\int_0^x[\alpha(\xi)b_3(\xi)+1][\beta(\xi)+\sigma(\xi)]\int_0^\xi \max\{\alpha(\xi_1)b_3(\xi_1)+$$

$$+1, b_1(\xi_1)+b_3(\xi_1)\}b_2(\xi_1)d\xi_1 d\xi, \quad b_1(x)=\gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}, \ b_2(x)=\beta(x)+\sigma(x)+1, \ b_3(x)=[1+\gamma_\nu(x,h)h\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}]h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}.$$

Proof. The following inequalities take place

$$\|F_\nu(x,h,w,u)\|\le h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\times$$

$$\times\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}[\beta(x)\|w_r(x,t)\|+\sigma(x)\|u_r(x,t)\|+\|f(x,t)\|],$$

$$\|G_\nu(x,h,v)\|\le\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r(x,t)\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|D_{\nu r}(x,t)\|\le\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}.$$

From the zero step of the algorithm, the following estimates follow:

$$\max_{r=\overline{1,N}}\|\lambda_r^{(0)}(x)\|\le$$

$$\le\gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sup_{t\in[0,T]}[\beta(x)\|\dot\psi(t)\|+\sigma(x)\|\psi(t)\|+\|f(x,t)\|]\le$$

$$\le b_1(x)b_2(x)\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\},$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r^{(0)}(x,t)\|\le h\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\|\lambda_r^{(0)}(x)\|+$$

$$+\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}b_2(x)\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\}\le$$

$$\le b_3(x)b_2(x)\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\},$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|w_r^{(0)}(x,t)-\dot\psi(t)\|\le$$

$$\le\int_0^x[\alpha(\xi)b_3(\xi)+1]b_2(\xi)d\xi\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\},$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|u_r^{(0)}(x,t)-\psi(t)\|\le$$

$$\le\int_0^x[b_1(\xi)+b_3(\xi)]b_2(\xi)d\xi\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\}.$$

The following estimates are valid:

$$\max_{r=1,N}\left\|\lambda_r^{(1,1)}(x)-\lambda_r^{(1,0)}(x)\right\|\leq$$

$$\leq\gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\beta(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(0)}(x,t)-\dot{\psi}(t)\right\|+$$

$$+\gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sigma(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|,$$

$$\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(1,1)}(x,t)-\widetilde{v}_r^{(1,0)}(x,t)\right\|\leq$$

$$\leq\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=1,N}\left\|\lambda_r^{(1,1)}(x)-\lambda_r^{(1,0)}(x)\right\|+$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\beta(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(0)}(x,t)-\dot{\psi}(t)\right\|+$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sigma(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|\leq$$

$$\leq b_3(x)\beta(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(0)}(x,t)-\dot{\psi}(t)\right\|+$$

$$+b_3(x)\sigma(x)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left|u_r^{(0)}(x,t)-\psi(t)\right|+$$

$$+q_\nu(x,h)\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|.$$

Let's establish inequality

$$\Delta^{(1,1)}(x)=$$

$$=\max_{r=1,N}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(1,1)}(x,t)-\widetilde{v}_r^{(1,0)}(x,t)\right\|+\max_{r=1,N}\left\|\lambda_r^{(1,1)}(x)-\lambda_r^{(1,0)}(x)\right\|\leq$$

$$\leq[b_1(x)+b_3(x)][\beta(x)+\sigma(x)]\int_0^x\max\{\alpha(\xi)b_3(\xi)+1,b_1(\xi)+b_3(x)\}b_2(\xi)d\xi\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\|f\|_0\}+$$

$$+[q_\nu(x,h)+\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}]b_3(x)b_2(x)\times$$

$$\times\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot{\psi}(t)\|,\|f\|_0\}.$$

Thus

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{(1,m+1)}(x)-\lambda_r^{(1,m)}(x)\right\|\le$$

$$\le\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,m)}(x,t)-\tilde{v}_r^{(1,m-1)}(x,t)\right\|, \tag{1.22}$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,m+1)}(x,t)-\tilde{v}_r^{(1,m)}(x,t)\right\|\le$$

$$\le\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(1,m+1)}(x)-\lambda_r^{(1,m)}(x)\right\|+$$

$$+\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,m)}(x,t)-\tilde{v}_r^{(1,m-1)}(x,t)\right\|\le$$

$$\le q_\nu(x,h)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,m)}(x,t)-\tilde{v}_r^{(1,m-1)}(x,t)\right\|. \tag{1.23}$$

Owing to inequality of $q_\nu(x,h)<1$ follows the uniform convergence of the sequence $\tilde{v}_r^{(1,m+1)}(x,t)$, for $(x,t)\in\Omega_r$, to $\tilde{v}_r^{(1)}(x,t)$ and convergence of a sequence of systems of functions $\lambda_r^{(1,m+1)}(x)$ to continuous on $x\in[0,\omega]$ functions $\lambda_r^{(1)}(x)$ for all $r=\overline{1,N}$:

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,m+1)}(x,t)-\tilde{v}_r^{(1,0)}(x,t)\right\|\le$$

$$\le\sum_{j=0}^{m}[q_\nu(x,h)]^j\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,1)}(x,t)-\tilde{v}_r^{(1,0)}(x,t)\right\|.$$

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{(1,m+1)}(x)-\lambda_r^{(1,0)}(x)\right\|\le$$

$$\le\sum_{j=0}^{m}[q_\nu(x,h)]^j\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1,1)}(x,t)-\tilde{v}_r^{(1,0)}(x,t)\right\|$$

$$+\max_{r=\overline{1,N}}\left\|\lambda_r^{(1,1)}(x)-\lambda_r^{(1,0)}(x)\right\|.$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1,m+1)}(x,t) - \tilde{v}_r^{(1,0)}(x,t) \right\| + \max_{r=\overline{1,N}} \left\| \lambda_r^{(1,m+1)}(x) - \lambda_r^{(1,0)}(x)$$

$$\le \sum_{j=0}^{m} [q_\nu(x,h)]^j \{1 + \gamma_\nu(x,h) \frac{(\alpha(x)h)^\nu}{\nu!}\} \times$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1,1)}(x,t) - \tilde{v}_r^{(1,0)}(x,t) \right\| +$$

$$+ \max_{r=\overline{1,N}} \left\| \lambda_r^{(1,1)}(x) - \lambda_r^{(1,0)}(x) \right\|.$$

Passing to the limit at $m \to \infty$, obtain estimates:

$$\Delta^{(1)}(x) =$$

$$= \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\| + \max_{r=\overline{1,N}} \left\| \lambda_r^{(1)}(x) - \lambda_r^{(0)}(x) \right\| \le$$

$$\le \frac{1 + \gamma_\nu(x,h) \dfrac{(\alpha(x)h)^\nu}{\nu!}}{1 - q_\nu(x,h)} \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1,1)}(x,t) - \tilde{v}_r^{(1,0)}(x,t) \right\| +$$

$$+ \max_{r=\overline{1,N}} \left\| \lambda_r^{(1,1)}(x) - \lambda_r^{(1,0)}(x) \right\| \le a_1(x) \max\{\max_{t\in[0,T]} \|\psi(t)\|, \max_{t\in[0,T]} \|\dot{\psi}(t)\|, \| f \|_0$$

$$\tilde{\Delta}^{(1)}(x) = \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \frac{\partial \tilde{v}_r^{(1)}(x,t)}{\partial t} - \frac{\partial \tilde{v}_r^{(0)}(x,t)}{\partial t} \right\| \le \int_0^x \alpha(\xi) b_3(\xi) +$$

$$+1][\beta(\xi) + \sigma(\xi)] \int_0^\xi \max\{\alpha(\xi_1) b_3(\xi_1) + 1, b_1(\xi_1) + b_3(\xi_1)\} b_2(\xi_1) d\xi_1 d\xi \times$$

$$\times \max\{\max_{t\in[0,T]} \|\psi(t)\|, \max_{t\in[0,T]} \|\dot{\psi}(t)\|, \| f \|_0\} =$$

$$= a_2(x) \max\{\max_{t\in[0,T]} \|\psi(t)\|, \max_{t\in[0,T]} \|\dot{\psi}(t)\|, \| f \|_0\},$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| w_r^{(1)}(x,t) - w_r^{(0)}(x,t) \right\| \le \int_0^x \tilde{\Delta}^{(1)}(\xi) d\xi,$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(1)}(x,t) - u_r^{(0)}(x,t) \right\| \le \int_0^x \Delta^{(1)}(\xi) d\xi.$$

$$\max\{\Delta^{(1)}(x), \tilde{\Delta}^{(1)}(x)\} \le$$

$$\le \max\{a_1(x), a_2(x)\} \max\{\max_{t\in[0,T]} \|\psi(t)\|, \max_{t\in[0,T]} \|\dot{\psi}(t)\|, \| f \|_0\}.$$

For systems of differences

$\lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x)$, $\tilde{v}_r^{(k+1)}(x,t) - \tilde{v}_r^{(k)}(x,t)$,

$w_r^{(k+1)}(x,t) - w_r^{(k)}(x,t)$, $u_r^{(k+1)}(x,t) - u_r^{(k)}(x,t)$, $\quad r = \overline{1,N}$,

$k = 1,2,\ldots$ the estimates are valid:

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1,1)}(x) - \lambda_r^{(k+1,0)}(x) \right\| \le$$

$$\le \beta(x) b_1(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| w_r^{(k)}(x,t) - w_r^{(k-1)}(x,t) \right\| +$$

$$+ \sigma(x) b_1(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\|,$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,1)}(x,t) - \tilde{v}_r^{(k+1,0)}(x,t) \right\| \le$$

$$\le b_3(x) \beta(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| w_r^{(k)}(x,t) - w_r^{(k-1)}(x,t) \right\| +$$

$$+ b_3(x) \sigma(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\|,$$

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1,m+1)}(x) - \lambda_r^{(k+1,m)}(x) \right\| \le$$

$$\le \gamma_\nu(x,h) \frac{(\alpha(x)h)^\nu}{\nu!} \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,m)}(x,t) - \tilde{v}_r^{(k+1,m-1)}(x,t) \right\|,$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,m+1)}(x,t) - \tilde{v}_r^{(k+1,m)}(x,t) \right\| \le$$

$$\le q_\nu(x,h) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,m)}(x,t) - \tilde{v}_r^{(k+1,m-1)}(x,t) \right\|.$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,m+1)}(x,t) - \tilde{v}_r^{(k+1,0)}(x,t) \right\| \le$$

$$\le \sum_{j=0}^{m} [q_\nu(x,h)]^j \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,1)}(x,t) - \tilde{v}_r^{(k+1,0)}(x,t) \right\|.$$

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1,m+1)}(x) - \lambda_r^{(k+1,0)}(x) \right\| \le$$

$$\le \sum_{j=0}^{m-1} [q_\nu(x,h)]^j \gamma_\nu(x,h) \frac{(\alpha(x)h)^\nu}{\nu!} \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1,1)}(x,t) - \tilde{v}_r^{(k+1,0)}(x,t) \right\| +$$

$$+ \max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1,1)}(x) - \lambda_r^{(k+1,0)}(x) \right\|.$$

Passing to the limit at $m \to \infty$, we obtain estimates:

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1)}(x,t) - \tilde{v}_r^{(k)}(x,t) \right\| \le$$

$$\le \frac{b_3(x)\beta(x)}{1 - q_\nu(x,h)} \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| w_r^{(k)}(x,t) - w_r^{(k-1)}(x,t) \right\| +$$

$$+\frac{b_3(x)\sigma(x)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k)}(x,t)-u_r^{(k-1)}(x,t)\right\|, \qquad (1.24)$$

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{(k+1)}(x)-\lambda_r^{(k)}(x)\right\|\le$$

$$\le\frac{b_1(x)\beta(x)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k)}(x,t)-w_r^{(k-1)}(x,t)\right\|+$$

$$+\frac{b_1(x)\sigma(x)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k)}(x,t)-u_r^{(k-1)}(x,t)\right\|, \qquad (1.25)$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k+1)}(x,t)-w_r^{(k)}(x,t)\right\|\le$$

$$\le\int_0^x\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\widetilde{v}_r^{(k+1)}(\xi,t)}{\partial t}-\frac{\partial\widetilde{v}_r^{(k)}(\xi,t)}{\partial t}\right\|d\xi,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k+1)}(x,t)-u_r^{(k)}(x,t)\right\|\le$$

$$\le\int_0^x\max_{r=\overline{1,N}}\left\|\lambda_r^{(k+1)}(\xi)-\lambda_r^{(k)}(\xi)\right\|d\xi+$$

$$+\int_0^x\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(k+1)}(\xi,t)-\widetilde{v}_r^{(k)}(\xi,t)\right\|d\xi.$$

Summing, respectively, the left and right parts of the inequalities (1.24), (1.25) we have

$$\Delta^{(k+1)}(x)=$$

$$=\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(k+1)}(x,t)-\widetilde{v}_r^{(k)}(x,t)\right\|+\left\|\max_{r=\overline{1,N}}\lambda_r^{(k+1)}(x)-\lambda_r^{(k)}(x)\right\|\le$$

$$\le\frac{[b_1(x)+b_3(x)]\beta(x)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k)}(x,t)-w_r^{(k-1)}(x,t)\right\|+$$

$$+\frac{[b_1(x)+b_3(x)]\sigma(x)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k)}(x,t)-u_r^{(k-1)}(x,t)\right\|, \qquad (1.26)$$

$$\widetilde{\Delta}^{(k+1)}(x)=\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\widetilde{v}_r^{(k+1)}(x,t)}{\partial t}-\frac{\partial\widetilde{v}_r^{(k)}(x,t)}{\partial t}\right\|\le$$

$$\le\int_0^x[\alpha(\xi)b_3(\xi)+1][\beta(\xi)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k)}(x,t)-w_r^{(k-1)}(x,t)\right\|+$$

$$+\sigma(\xi)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k)}(x,t)-u_r^{(k-1)}(x,t)\right\|]d\xi, \qquad (1.27)$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k+1)}(x,t)-w_r^{(k)}(x,t)\right\|\le\int_0^x\widetilde{\Delta}^{(k+1)}(\xi)d\xi,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k+1)}(x,t)-u_r^{(k)}(x,t)\right\|\le\int_0^x\Delta^{(k+1)}(\xi)d\xi.$$

For function $\max\{\Delta^{(k+1)}(x),\widetilde{\Delta}^{(k+1)}(x)\}$ on the basis of (1.26), (1.27) we establish inequality

$$\max\{\Delta^{(k+1)}(x),\widetilde{\Delta}^{(k+1)}(x)\}\le a_0(x)\int_0^x\max\{\Delta^{(k)}(\xi),\widetilde{\Delta}^{(k)}(\xi)\}d\xi, \qquad (1.28)$$

$$\max\{\Delta^{(k+1)}(x),\widetilde{\Delta}^{(k+1)}(x)\}\le$$

$$\le\frac{a_0(x)}{(k-1)!}\left(\int_0^x a_0(\xi)d\xi\right)^{k-1}\int_0^x\max\{\Delta^{(1)}(\xi),\widetilde{\Delta}^{(1)}(\xi)\}d\xi.$$

Establish inequalities

$$\max\left\{\max_{r=\overline{1,N}}\left\|\lambda_r^{(k+p)}(x)-\lambda_r^{(k)}(x)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(k+p)}(x,t)-\widetilde{v}_r^{(k)}(x,t)\right\|\right.$$

$$\left.\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\widetilde{v}_r^{(k+p)}(x,t)}{\partial t}-\frac{\partial\widetilde{v}_r^{(k)}(x,t)}{\partial t}\right\|\right\}\le\max\{\Delta^{(k+p)}(x),\widetilde{\Delta}^{(k+p)}(x)\}$$

$$+\max\{\Delta^{(k+p-1)}(x),\widetilde{\Delta}^{(k+p-1)}(x)\}+\ldots+\max\{\Delta^{(1)}(x),\widetilde{\Delta}^{(1)}(x)\}\le$$

$$\le a_0(x)\sum_{j=k-1}^{k+p-2}\frac{1}{j!}\left(\int_0^x a_0(\xi)d\xi\right)^j\int_0^x\max\{\Delta^{(1)}(\xi),\widetilde{\Delta}^{(1)}(\xi)\}d\xi\le$$

$$\le a_0(x)\sum_{j=k-1}^{k+p-2}\frac{1}{j!}\left(\int_0^x a_0(\xi)d\xi\right)^j\int_0^x\max\{a_1(\xi),a_2(\xi)\}d\xi\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\|f\|_0\},$$

$$\max\left\{\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k+p)}(x,t)-u_r^{(k)}(x,t)\right\|,\right.$$

$$\left.\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(k+p)}(x,t)-w_r^{(k)}(x,t)\right\|\right\}\le$$

$$\le\int_0^x\max\left\{\max_{r=\overline{1,N}}\left\|\lambda_r^{(k+p)}(x)-\lambda_r^{(k)}(x)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(k+p)}(x,t)-\widetilde{v}_r^{(k)}(x\right.\right.$$

$$\left.\left.\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\widetilde{v}_r^{(k+p)}(x,t)}{\partial t}-\frac{\partial\widetilde{v}_r^{(k)}(x,t)}{\partial t}\right\|\right\},$$

passing to a limit at $p\to\infty$, at all $(x,t)\in\Omega_r$, $r=\overline{1,N}$, we obtain the estimates of Theorem 1.

Let's prove uniqueness. Let exists $\left(\lambda_r^{**}(x),\widetilde{v}_r^{**}(x,t),w_r^{**}(x,t),\right.$

$u_r^{**}(x,t))$, $r=\overline{1,N}$, another solution of the boundary value problem (1.13)-(1.18). Similar to relation (1.28) for differences $\lambda_r^{*}(x)-\lambda_r^{**}(x)$, $\tilde{v}_r^{*}(x,t)-\tilde{v}_r^{**}(x,t)$, $\dfrac{\partial\tilde{v}_r^{*}(x,t)}{\partial t}-\dfrac{\partial\tilde{v}_r^{**}(x,t)}{\partial t}$ for all $(x,t)\in\Omega_r$, $r=\overline{1,N}$, we obtain:

$$\max\left\{\max_{r=\overline{1,N}}\left\|\lambda_r^{*}(x)-\lambda_r^{**}(x)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(x,t)-\tilde{v}_r^{**}(x,t)\right\|,\right.$$
$$\left.\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^{*}(x,t)}{\partial t}-\frac{\partial\tilde{v}_r^{**}(x,t)}{\partial t}\right\|\right\}\le$$
$$\le a_0(x)\int_0^x\max\left\{\max_{r=\overline{1,N}}\left\|\lambda_r^{*}(\xi)-\lambda_r^{*}(\xi)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(\xi,t)-\tilde{v}_r^{**}(\xi,t)\right\|,\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^{*}(\xi,t)}{\partial t}-\frac{\partial\tilde{v}_r^{**}(\xi,t)}{\partial t}\right\|\right\}d\xi.$$

Using Bellman-Gronwall inequality [63] we have

$$\max\left\{\max_{r=\overline{1,N}}\left\|\lambda_r^{*}(x)-\lambda_r^{**}(x)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(x,t)-\tilde{v}_r^{**}(x,t)\right\|,\right.$$
$$\left.\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^{*}(x,t)}{\partial t}-\frac{\partial\tilde{v}_r^{**}(x,t)}{\partial t}\right\|\right\}=0.$$

From where follows that $\tilde{v}_r^{*}(x,t)=\tilde{v}_r^{**}(x,t)$, $\lambda_r^{*}(x)=\lambda_r^{**}(x)$, $\dfrac{\partial\tilde{v}_r^{*}(x,t)}{\partial t}=\dfrac{\partial\tilde{v}_r^{**}(x,t)}{\partial t}$, $r=\overline{1,N}$. From inequalities

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{*}(x,t)-w_r^{**}(x,t)\right\|\le$$
$$\le\int_0^x\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\frac{\partial\tilde{v}_r^{*}(\xi,t)}{\partial t}-\frac{\partial\tilde{v}_r^{**}(\xi,t)}{\partial t}\right\|d\xi,$$
$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{*}(x,t)-u_r^{**}(x,t)\right\|\le$$
$$\le\int_0^x\left\{\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(\xi,t)-\tilde{v}_r^{**}(\xi,t)\right\|+\max_{r=\overline{1,N}}\left\|\lambda_r^{*}(\xi)-\lambda_r^{**}(\xi)\right\|\right\}d\xi$$

have $w_r^{*}(x,t)=w_r^{**}(x,t), u_r^{*}(x,t)=u_r^{**}(x,t), r=\overline{1,N}$, for all $(x,t)\in\Omega_r$. Theorem 1 is proved.

1.2 Necessary and sufficient conditions for the unique solvability

The equivalence of problems (1.1)-(1.3) and (1.13)-(1.18) implies that the conditions of Theorem 1 are sufficient conditions for the unique solvability of the linear semi-periodic boundary value problem for a system of hyperbolic equations with a mixed variable.

In this subsection, the coefficient characteristics of the correct solvability of the boundary value problem (1.1)-(1.3) are established.

In addition, a relationship has been established between the constant of the correct solvability of the boundary value problem (1.1)-(1.3) and the function $\gamma_\nu(x,h)$, which bounds the norm of the matrix $Q_\nu(x,h)$ from above.

The theorem is valid.

Theorem 2. *Let the conditions of theorem 1 are fulfilled. Then the problem (1.1)-(1.3) has the single solution* $u^*(x,t)$ *and estimate are valid*

$$\max\left\{\|u^*\|_0, \left\|\frac{\partial u^*}{\partial x}\right\|_0, \left\|\frac{\partial u^*}{\partial t}\right\|_0\right\} \leq$$

$$\leq M_\nu(x,h)\max\{\max_{t\in[0,T]}\|\psi(t)\|, \max_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\}, \qquad (1.29)$$

where $M_\nu(x,h) = a_0(x)e^{\int_0^x a_0(\xi)d\xi}\int_0^x \max\{a_1(\xi), a_2(\xi)\}d\xi +$

$+\max\{a_1(x), a_2(x)\} + \max\{b_1(x)+b_3(x), \alpha(x)[b_1(x)+b_3(x)]+1\}b_2(x).$

Definition 1. *The boundary value problem (1.1)-(1.3) is called correctly solvable if for any* $f(x,t)\in C(\overline{\Omega}, R^n)$, *the continuous for* $[0,T]$ *functions* $\psi(t)$, $\dot{\psi}(t)$ *has the single solution of* $u(x,t)\in C(\overline{\Omega}, R^n)$ *and the inequality is valid*

$$\max\left\{\|u\|_0, \left\|\frac{\partial u}{\partial x}\right\|_0, \left\|\frac{\partial u}{\partial t}\right\|_0\right\} \leq$$

$$\leq K\max\{\max_{t\in[0,T]}\|\psi(t)\|, \max_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\}, \qquad (1.30)$$

where K *- constant, does not depend on* $f(x,t), \psi(t)$.

From Theorem 3 [64, p.342] follows that the boundary value problem (1.1)-(1.3) is correctly solvable if and only if the periodic boundary value problem is correctly solvable

$$\frac{\partial v}{\partial t} = A(x,t)v + F(x,t), \qquad (x,t)\in\overline{\Omega}, \qquad (1.31)$$

$$v(x,0) = v(x,T), \qquad x\in[0,\omega]. \qquad (1.32)$$

Function $v(x,t)\in C(\overline{\Omega}, R^n)$, with continuous on $\overline{\Omega}$ the derivative at argument t, is called the solution of the problem (1.31), (1.32), if it satisfies system (1.31) and periodic condition (1.32).

Definition 2. *The boundary value problem (1.31), (1.32) is called correctly solvable if, for any* $F(x,t)\in C(\overline{\Omega}, R^n)$, *it has the* *single solution and the inequality is valid*

$$\|v\|_0 \le K_1 \|F\|_0, \qquad (1.33)$$

where K_1 *- constant, independent of* $F(x,t)$.

Let $v^*(x,t)$ - be the solution of problem (1.31), (1.32). Then

a) a couple $(\lambda^*(x), \tilde{v}^*(x,[t]))$ with components

$$\lambda_r^*(x) = \tilde{v}^*(x,(r-1)h), \tilde{v}_r^*(x,t) = v^*(x,t) - v^*(x,(r-1)h),$$

where $(x,t) \in \Omega_r, \quad r = \overline{1,N},$ is the solution to the problem

$$\frac{\partial \tilde{v}_r}{\partial t} = A(x,t)\tilde{v}_r + A(x,t)\lambda_r(x) + F(x,t),$$

$$\tilde{v}_r(x,(r-1)h) = 0, \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}, \qquad (1.34)$$

$$\lambda_1(x) - \lambda_N(x) - \lim_{t \to T-0} \tilde{v}_N(x,t) = 0, \quad x \in [0,\omega], \qquad (1.35)$$

$$\lambda_s(x) + \lim_{t \to sh-0} \tilde{v}_s(x,t) - \lambda_{s+1}(x) = 0, \; x \in [0,\omega], \; s = \overline{1,N-1}, \qquad (1.36)$$

b) there are such numbers $\zeta_1, \zeta_2 > 0,$ that

$$\|\lambda_r^*(x)\| \le \zeta_1, \quad \|\tilde{v}_r^*(x,t)\| \le \zeta_2, \quad (x,t) \in \Omega_r, \quad r = \overline{1,N},$$

c) for any $\nu \in \mathbb{N}$ equalities are performed

$$\tilde{v}_r^*(x,t) = D_{\nu r}(x,t)\lambda_r^*(x) + \tilde{F}_{\nu r}(x,t) + G_{\nu r}(x,t,\tilde{v}_r^*), \; r = \overline{1,N}, \qquad (1.37)$$

$$Q_\nu(x,h)\lambda^*(x) = -\tilde{F}_\nu(x,h) - G_\nu(x,h,\tilde{v}^*). \qquad (1.38)$$

Because $\left\|G_{\nu r}(x,h,\tilde{v}^*)\right\| \le \dfrac{(\alpha(x)h)^\nu}{\nu!}\zeta_2,$ and

$D_{\nu r}(x,t), \tilde{F}_{\nu r}(x,t)$ at $\nu \to \infty$ Ω_r uniformly converge to

$$D_{*r}(x,t) = \sum_{j=0}^{\infty} \int_{(r-1)h}^{t} A(x,\tau_1)d\tau_1 \ldots \int_{(r-1)h}^{\tau_j} A(x,\tau_{j+1})d\tau_{j+1} \ldots d\tau_1,$$

$$\tilde{F}_{*r}(x,t) = \int_{(r-1)h}^{t} F(x,\tau_1)d\tau_1 +$$

$$+\sum_{j=0}^{\infty} \int_{(r-1)h}^{t} A(x,\tau_1) \ldots \int_{(r-1)h}^{\tau_{j-1}} A(x,\tau_j) \int_{(r-1)h}^{\tau_j} F(x,\tau_{j+1})d\tau_{j+1}d\tau_j \ldots d\tau_1,$$

$\tau_0 = t, \quad r = \overline{1,N},$ then going to the limit at $\nu \to \infty$ in (1.37), (1.38) and dividing both parts (1.38) into $h > 0,$ we obtain

$$\tilde{v}_r^*(x,t) = D_{*r}(x,t)\lambda_r^*(x) + \tilde{F}_{*r}(x,t), \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}, \qquad (1.39)$$

$$\frac{1}{h}Q_*(x,h)\lambda^*(x) = -\tilde{F}_*(A,F,x,h), \quad x \in [0,\omega], \qquad (1.40)$$

where

$$\tilde{F}_*(A,F,x,h) = (-\tilde{F}_{*N}(x,Nh), \frac{1}{h}\tilde{F}_{*1}(x,h), \ldots, \frac{1}{h}\tilde{F}_{*,N-1}(x,(N-1)h)).$$

Thus, if $v^*(x,t)$ is a solution to problem (1.31), (1.32) at $F(x,t) \in C(\overline{\Omega}, R^n)$, then

$$\lambda^*(x) = (\lambda_1^*(x), \lambda_2^*(x), \ldots, \lambda_N^*(x))' \in C([0,\omega], R^{nN})$$

is the solution to equation (1.40).

Theorem 3. [65] *The boundary value problem (1.1)-(1.3) is correctly solvable if and only if for any* $h > 0 : Nh = T, N = 1,2,\ldots,$ *there is a* $\nu, \nu \in N$, $(nN \times nN)$ *- matrix* $Q_\nu(x,h)$ *reversible for all* $x \in [0,\omega]$ *and the inequalities 1), 2) of Theorem 1 are satisfied.*

Proof. If the conditions are satisfied, the correct solvability of problem (1.1)-(1.3) follows from Theorem 1 [66].

Let the problem (1.1)-(1.3) be correctly solvable. Then by Theorem 3 of [64, p.342] the problem (1.31), (1.32) is correctly solvable with a constant K_1. Let us prove the invertibility of the matrix $Q_*(x,h)$ at each fixed $x \in [0,\omega]$. Consider the equation

$$\frac{1}{h} Q_*(x,h)\lambda(x) = b(x), \quad b(x), \lambda(x) \in C([0,\omega], R^{nN}). \quad (1.41)$$

Take $\varepsilon > 0$ and choose $h_0 = h_0(\varepsilon)$ to satisfy the inequality

$$\frac{1}{(\tilde{\alpha}h)}[e^{\tilde{\alpha}h} - 1 - \tilde{\alpha}h] \leq \frac{\varepsilon}{2(1+\frac{\varepsilon}{2})(1+\varepsilon)},$$

$\tilde{\alpha} = \max_{x\in[0,\omega]} \|\alpha(x)\|$. Now for all $h \in (0, h_0]$, $b(x) \in C([0,\omega], R^{nN})$ on the basis of a Lemma from [52, p.54] it is possible to construct a function $F_b(x,t) \in C(\overline{\Omega}, R^n)$, with the properties:

$$\max_{(x,t)\in\overline{\Omega}} \|F_b(x,t)\| \leq (1+\varepsilon) \max_{x\in[0,\omega]} \|b(x)\|,$$

$$\tilde{F}_*(A,F_b,x,h) = \frac{1}{h}\int_0^h F_b(x,t)dt + \frac{1}{h}\int_0^h A(x,t)\int_0^t F_b(x,\tau)d\tau dt +$$

$$+ \frac{1}{h}\int_0^h A(x,t)\int_0^t A(x,\tau)\int_0^\tau F(x,\tau_1)d\tau_1 d\tau dt + \ldots = b(x), \quad x \in [0,\omega].$$

By assumption, problem (1.31), (1.32) has a solution for any $F(x,t)$. Therefore, the equation

$$\frac{1}{h}Q_*(x,h)\lambda(x) = -\tilde{F}_*(A,F_b,x,h), \tag{1.42}$$

as shown above, for any F_b has a solution $\lambda_b(x)\in C([0,\omega],R^{nN})$:

$$\frac{1}{h}Q_*(x,h)\lambda_b(x) = -\tilde{F}_*(A,F_b,x,h). \tag{1.43}$$

From here, considering equality

$$-\tilde{F}_*(A,F_b,x,h) = b(x)$$

we obtain that equation (1.41) has a solution $\lambda_b(x)\in C([0,\omega],R^{nN})$ for all $b(x)\in C([0,\omega],R^{nN})$ and estimate is valid

$$\max_{x\in[0,\omega]}\|\lambda_b(x)\| \le \max_{(x,t)\in\underline{\Omega}}\|v_b(x,t)\| \le$$

$$\le K_1 \max_{(x,t)\in\underline{\Omega}}\|F_b(x,t)\| \le (1+\varepsilon)K_1 \max_{x\in[0,\omega]}\|b(x)\|$$

i.e. operator $\frac{1}{h}Q_*(\,;h): C([0,\omega],R^{nN}) \to C([0,\omega],R^{nN})$ has a right inverse and

$$\left\|\left[\frac{1}{h}Q_*(\,;h)\right]_r^{-1}\right\|_{L(C([0,\omega],R^{nN}))} \le (1+\varepsilon)K_1,$$

where $L(C([0,\omega],R^{nN}))$ - is the space of linearly bounded operators $L: C([0,\omega],R^{nN}) \to C([0,\omega],R^{nN})$. Then by owing of the relations

$$\left[\frac{1}{h}Q_*(x,h)\right]_r^{-1} = h[Q_*(x,h)]_r^{-1},$$

we have

$$\|[Q_*(\,;h)]_r^{-1}\|_{L(C([0,\omega],R^{nN}))} \le \frac{(1+\varepsilon)K_1}{h}.$$

As at any fixed $\bar{x}\in[0,\omega]$ the equation

$$\frac{1}{h}Q_*(\bar{x},h)\lambda = b,$$

has a solution

$$\lambda_b = \lambda_b(\bar{x})$$

for any $b \in R^{nN}$, then the matrix $Q_*(\overline{x}, h)$ is invertible and

$$[Q_*(x,h)]_r^{-1} = [Q_*(x,h)]^{-1},$$

for any $x \in [0, \omega]$ and

$$\left\| [Q_*(x,h)]^{-1} \right\| = \left\| [Q_*(x,h)]_r^{-1} \right\| \leq \frac{(1+\varepsilon)K_1}{h}.$$

Taking into account the inequality

$$\left\| Q_*(x,h) - Q_\nu(x,h) \right\| \leq e^{\tilde{\alpha}h} - \sum_{j=0}^{\nu} \frac{(\tilde{\alpha}h)^j}{j!},$$

and using the theorem on small perturbations of limitedly invertible operators [63, p.142], we find $\overline{\nu}$ such that

$$\frac{(1+\varepsilon)K_1}{h}\left(e^{\tilde{\alpha}h} - \sum_{j=0}^{\overline{\nu}} \frac{(\tilde{\alpha}h)^j}{j!} \right) \leq \frac{\varepsilon}{1+2\varepsilon}.$$

Then the matrix $Q_\nu(x,h)$ be invertible and estimates are performed

$$\left\| [Q_\nu(x,h)]^{-1} \right\| \leq \frac{\dfrac{(1+\varepsilon)K_1}{h}}{1 - \dfrac{\varepsilon}{1+2\varepsilon}} = \frac{(1+2\varepsilon)K_1}{h},$$

$$q_\nu(x,h) \leq \frac{(\tilde{\alpha}h)^\nu}{\nu!}\left[1 + \frac{(1+2\varepsilon)K_1}{h} \sum_{j=1}^{\nu-1} \frac{(\tilde{\alpha}h)^j}{j!}\right], \quad \forall \nu \geq \overline{\nu}.$$

Hence the existence of $\overline{\overline{\nu}} \geq \overline{\nu}$ such that

a) $\left\| [Q_{\overline{\overline{\nu}}}(x,h)]^{-1} \right\| \leq \dfrac{(1+2\varepsilon)K_1}{h}$, b) $q_{\overline{\overline{\nu}}}(h) < 1$.

Theorem 3 is proved.

The following theorem is valid

Theorem 4. *The boundary value problem (1.1)-(1.3) is correctly solvable if and only if for any* $\nu, \nu \in N$, *exists* $h = h(\nu) > 0 : Nh = T, N = 1,2,\ldots,$ *at which* $(nN \times nN)$ *- matrix* $Q_\nu(x,h)$ *is invertible for all* $x \in [0, \omega]$ *and inequalities of the Theorem 1 are performed.*

Proof. The sufficiency of the conditions of the theorem for the correct solvability of problem (1.1)-(1.3) follows from Theorem 1.

Need. Let the problem (1.1)-(1.3) is correctly solvable. Then according to Theorem 3 [64, p.342] the problem (1.31), (1.32) will be correctly solvable. Let us denote by K_1 the constant of the correct solvability of the problem (1.31), (1.32). As it was established in Theorem 3, there exists $h_0 > 0$, at which for all $h = h(\varepsilon,\nu) \in (0, h_0] : Nh = T$ estimate is valid:

$$\left\| [Q_*(x,h)]^{-1} \right\| \le \frac{(1+\varepsilon)K_1}{h}.$$

Because

$$\left\| Q_*(x,h) - Q_\nu(x,h) \right\| \le e^{\tilde{\alpha}h} - \sum_{j=0}^{\nu} \frac{(\tilde{\alpha}h)^j}{j!},$$

then choosing $h_1 = h_1(\varepsilon,\nu) \in (0, h_0] : Nh_1 = T$ satisfying inequalities

$$\frac{(1+\varepsilon)K_1}{h}\left(e^{\tilde{\alpha}h} - \sum_{j=0}^{\nu} \frac{(\tilde{\alpha}h)^j}{j!}\right) \le \frac{\varepsilon}{1+2\varepsilon} < 1,$$

$$\frac{(\tilde{\alpha}h)^\nu}{\nu!}\left[1 + \frac{(1+2\varepsilon)K_1}{h} \sum_{j=1}^{\nu-1} \frac{(\tilde{\alpha}h)^j}{j!}\right] < 1,$$

according to the theorem on small perturbations of limitedly invertible operators [63, p.142] for any $\nu \in N$ we have

$$\left\| [Q_\nu(x,h_1)]^{-1} \right\| \le \frac{(1+2\varepsilon)K_1}{h_1},$$

i.e. conditions of the Theorem 1 are satisfied.

Theorem 4 is proved.

The following statements establish the relationship between the constant of the correct solvability of the problem (1.1)-(1.3) and function $\gamma_\nu(x,h)$, limiting the matrix norm from above. $[Q_\nu(x,h)]^{-1}$.

Theorem 5. *If the boundary value problem (1.1)-(1.3) is correctly solvable, then for any* $\nu, \nu \in N$, *exists* $h_0 = h_0(\nu)$ *such that for all* $h \in (0, h_0]$: $Nh = T$ *and* $x \in [0, \omega]$ $(nN \times nN)$ *- matrix* $Q_\nu(x,h)$ *will be invertible and*

$$\left\| [Q_\nu(x,h)]^{-1} \right\| \le \frac{\gamma}{h}, \qquad (1.44)$$

where γ *- constant, independent of* h*. Moreover, if the* K *- constant of the correct solvability of problem (1.1)-(1.3) is known,* *then for any* $\varepsilon > 0$ *exists* $h_1 = h_1(\varepsilon, \nu)$, *at which the matrix* $Q_\nu(x,h)$ *is invertible for all* $h \in (0, h_1]$: $Nh = T$, $x \in [0, \omega]$ *and the estimate (1.44) will be performed with constant*

$$\gamma = (1+\varepsilon)KK_2,$$

where $K_2 = e^{K\omega(\tilde{\beta}+\tilde{\sigma})}$, $\tilde{\beta} = \|B\|_0$, $\tilde{\sigma} = \|C\|_0$.

Proof. Let the problem (1.1)-(1.3) be correctly solvable with a constant K. Then for any

$$f(x,t)(\overline{\Omega}, R^n), \quad \psi(t) = 0, \quad t \in [0,T],$$

there exists a unique solution to problem (1.1)-(1.3) and for it inequality of

$$\max(\|u\|_0, \left\|\frac{\partial u}{\partial x}\right\|_0, \left\|\frac{\partial u}{\partial t}\right\|_0) \le K\|f\|_0$$

is valid. For a given $F(x,t)$ we define the function $f(x,t)$ from the functional equation

$$f(x,t) + B(x,t)w(x,t) + C(x,t)u(x,t) = F(x,t), \qquad (1.45)$$

where $u(x,t)$ - the solution of problem (1.1)-(1.3) for this function $f(x,t)$ and $\psi(t) = 0$. From the correct solvability of the problem (1.1)-(1.3) and assumptions about the matrices $A(x,t), B(x,t), C(x,t)$, follows the existence of the operator $U \in L(C(\overline{\Omega}, R^n))$, defining the solution of the problem (1.1)-(1.3) function $u(x,t) = Uf(x,t)$ and the belonging of the operators

$$U_1 = \frac{\partial}{\partial x} \circ U, \quad U_2 = \frac{\partial^2}{\partial x \partial t} \circ U$$

to the space $L(C(\overline{\Omega}, R^n))$. Using the initial condition $u(0,t) = 0$ and expressing $u(x,t)$ through the integral of $U_1 f(\xi,t)$ on the interval $[0,x] \subseteq [0,\omega]$, substituting them into (1.45), we obtain a one-parameter family of Volterra second-kind integral systems regarding function $f(x,t)$,

$$f(x,t) + B(x,t)\int_0^x U_2 f(\xi,t)d\xi +$$

$$+ C(x,t)\int_0^x U_1 f(\xi,t)d\xi = F(x,t), (x,t) \in \overline{\Omega}. \qquad (1.46)$$

Taking the function $F(x,t)$, as the initial approximation the next approximation is determined from the system of equations

$$f^{(k)}(x,t) = F(x,t) - B(x,t)\int_0^x U_2 f^{(k-1)}(\xi,t)d\xi -$$
$$- C(x,t)\int_0^x U_1 f^{(k-1)}(\xi,t)d\xi.$$

The sequence $f^{(k)}(x,t)$ at $k \to \infty$ converges to the only solution of the system (1.46) - function $f(x,t) \in C(\overline{\Omega}, R^n)$ and estimate is valid

$$\| f \|_0 \le K_2 \| F \|_0. \tag{1.47}$$

Let $u(x,t)$ - be the solution of problem (1.1)-(1.3) for the found function $f(x,t)$ and $\psi(t) = 0$. Then the function $v(x,t) = \dfrac{\partial u(x,t)}{\partial x}$ will be the solution of problem (1.31), (1.32) for the selected function $F(x,t)$. Indeed, by owing of (1.2) and (1.45)

$$\frac{\partial v}{\partial t} = \frac{\partial^2 u}{\partial x \partial t} = A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u + f(x,t) =$$
$$= A(x,t)v + F(x,t), (x,t) \in \overline{\Omega},$$
$$v(x,0) - v(x,T) = \frac{\partial u(x,0)}{\partial x} - \frac{\partial u(x,T)}{\partial x} = 0, \qquad x \in [0,\omega].$$

And assessment of

$$\| v \|_0 \le \left\| \frac{\partial u}{\partial x} \right\|_0 \le K \| f \|_0 \le K K_2 \| F \|_0,$$

from a correct solubility of a task (1.1)-(1.3) and inequalities (1.47), i.e. family of periodic boundary value problems for the system of ordinary differential equations (1.31), (1.32) is correctly solvable with $K_1 = KK_2$. At each fixed $x \in [0,\omega]$ according to the Theorem 4 [52, p.61] $(nN \times nN)$ - matrix $Q_\nu(x,h)$ will be invertible and the estimate (1.44) is realized with the constant $\gamma = (1+\varepsilon)K_1$.

Theorem 5 is proved.

Theorem 6. *Let for some* $\nu = 1,2,...,$ *there exists* $h_0 = h_0(\nu)$ *such that for all* $h \in (0,h_0]: Nh = T$ *and* $x \in [0,\omega]$ $(nN \times nN)$ *- matrix* $Q_\nu(x,h)$ *is invertible and its inverse satisfies (1.44). Then problem (1.1)-(1.3) is correctly solvable with a constant*

$$K = \max\{(1+\omega(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega}),$$
$$(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega}\}\max\{\max_{t\in[0,T]}\|\psi(t)\|,$$

$\max\limits_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\}$, *where* $\widetilde{\alpha} = \|A\|_0$.

Proof. Let for some $\nu = 1,2,...$ $(nN \times nN)$ - matrix $Q_\nu(x,h)$ is invertible for all $h \in (0,h_0]: Nh = T$, $x \in [0,\omega]$ and we have the estimate (1.44). By owing of the equivalence of problems (1.1)-(1.3) and (1.4)-(1.7) it is sufficient to show the correct solvability of the problem (1.4)-(1.7). The solution of problem (1.4)-(1.7) - the triple of functions $(v(x,t), u(x,t), w(x,t))$ we find by the method of successive approximations. For the zero approximation, we take $u(x,t) = \psi(t)$, $w(x,t) = \dot{\psi}(t)$, and $v^{(0)}(x,t)$ find the solution of the problem

$$\frac{\partial v}{\partial t} = A(x,t)v + B(x,t)\dot{\psi}(t) + C(x,t)\psi(t) + f(x,t), (x,t) \in \overline{\Omega}, \tag{1.48}$$

$$v(x,0) = v(x,T), \qquad x \in [0,\omega]. \tag{1.49}$$

Under the conditions of the theorem, the problem (1.48), (1.49) has a unique solution $v^{(0)}(x,t)$ and is correctly solvable with a constant $K_1 = \gamma$

$$\|v^{(0)}\|_0 \le K_1 \|F\|_0 \le$$

$$\le \gamma\{\|B\|_0 \max_{t\in[0,T]}\|\dot{\psi}(t)\| + \|C\|_0 \max_{t\in[0,T]}\|\psi(t)\| + \|f\|_0\} \le$$

$$\le \gamma(1 + \widetilde{\beta} + \widetilde{\sigma})\max\{\max_{t\in[0,T]}\|\psi(t)\|, \max_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\},$$

$$\left\|\frac{\partial v^{(0)}}{\partial t}\right\|_0 \le (\widetilde{\alpha}\gamma + 1)(1 + \widetilde{\beta} + \widetilde{\sigma})\max\{\max_{t\in[0,T]}\|\psi(t)\|, \max_{t\in[0,T]}\|\dot{\psi}(t)\|, \|f\|_0\}.$$

The functions $u^{(0)}(x,t), w^{(0)}(x,t)$ are determined from the relations

$u^{(0)}(x,t) = \psi(t) + \int_0^x v^{(0)}(\xi,t)d\xi$, $w^{(0)}(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial v^{(0)}(\xi,t)}{\partial t}d\xi$. If $u^{(k-1)}(x,t)$, is known, then $v^{(k)}(x,t)$ we will find solving a problem (1.4), (1.5), where in the right sides of the equation are

$$u(x,t) = u^{(k-1)}(x,t), w(x,t) = w^{(k-1)}(x,t), k = 0,1,....$$

At the found $v^{(k)}(x,t)$ the following approximation for $u(x,t), w(x,t)$ is determined from the relations

$$u^{(k)}(x,t)=\psi(t)+\int_0^x v^{(k)}(\xi,t)d\xi,$$

$$w^{(k)}(x,t)=\dot{\psi}(t)+\int_0^x \frac{\partial v^{(k)}(\xi,t)}{\partial t}d\xi.$$

Then $v^{(k+1)}(x,t)-v^{(k)}(x,t)$ is a solution to the problem

$$\frac{\partial[v^{(k+1)}(x,t)-v^{(k)}(x,t)]}{\partial t}=A(x,t)[v^{(k+1)}(x,t)-v^{(k)}(x,t)]+$$

$$+B(x,t)[w^{(k)}(x,t)-w^{(k-1)}(x,t)]+C(x,t)[u^{(k)}(x,t)-u^{(k-1)}(x,t)],$$

$$v^{(k+1)}(x,0)-v^{(k)}(x,0)=v^{(k+1)}(x,T)-v^{(k)}(x,T),\qquad x\in[0,\omega],$$

and for the differences of

$v^{(k+1)}(x,t)-v^{(k)}(x,t), u^{(k)}(x,t)-u^{(k-1)}(x,t), w^{(k)}(x,t)-w^{(k-1)}(x,t)$

estimates are valid

$$\max_{t\in[0,T]}\left\|v^{(k+1)}(x,t)-v^{(k)}(x,t)\right\|\leq\gamma\beta(x)\max_{t\in[0,T]}\left\|w^{(k)}(x,t)-w^{(k-1)}(x,t)\right\|+$$

$$+\gamma\sigma(x)\max_{t\in[0,T]}\left\|u^{(k)}(x,t)-u^{(k-1)}(x,t)\right\|\leq\gamma(\beta(x)+\sigma(x))\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|u^{(k)}(x,t)-u^{(k-1)}(x,t)\right\|,\max_{t\in[0,T]}\left\|w^{(k)}(x,t)-w^{(k-1)}(x,t)\right\|\},$$

$$\max_{t\in[0,T]}\left\|\frac{\partial v^{(k+1)}(x,t)}{\partial t}-\frac{\partial v^{(k)}(x,t)}{\partial t}\right\|\leq$$

$$\leq(\alpha(x)\gamma+1)(\beta(x)+\sigma(x)\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|u^{(k)}(x,t)-u^{(k-1)}(x,t)\right\|,\max_{t\in[0,T]}\left\|w^{(k)}(x,t)-w^{(k-1)}(x,t)\right\|\},$$

$$\max\left\{\max_{t\in[0,T]}\left\|v^{(k+1)}(x,t)-v^{(k)}(x,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(k+1)}(x,t)}{\partial t}-\frac{\partial v^{(k)}(x,t)}{\partial t}\right\|\right\}\leq$$

$$\leq\max\{\gamma,\alpha(x)\gamma+1\}(\beta(x)+\sigma(x))\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|u^{(k)}(x,t)-u^{(k-1)}(x,t)\right\|,\max_{t\in[0,T]}\left\|w^{(k)}(x,t)-w^{(k-1)}(x,t)\right\|\},$$

$$\max_{t\in[0,T]}\left\|u^{(k+1)}(x,t)-u^{(k)}(x,t)\right\|\leq\int_0^x\max_{t\in[0,T]}\left\|v^{(k+1)}(\xi,t)-v^{(k)}(\xi,t)\right\|d\xi,$$

$$\max_{t\in[0,T]}\left\|w^{(k+1)}(x,t)-w^{(k)}(x,t)\right\|\leq\int_0^x\max_{t\in[0,T]}\left\|\frac{\partial v^{(k+1)}(\xi,t)}{\partial t}-\frac{\partial v^{(k)}(\xi,t)}{\partial t}\right\|d\xi.$$

$$\max\left\{\max_{t\in[0,T]}\left\|u^{(k+1)}(x,t)-u^{(k)}(x,t)\right\|,\max_{t\in[0,T]}\left\|w^{(k+1)}(x,t)-w^{(k)}(x,t)\right\|\right\}\le$$

$$\le\int_0^x\max\left\{\max_{t\in[0,T]}\left\|v^{(k+1)}(\xi,t)-v^{(k)}(\xi,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(k+1)}(\xi,t)}{\partial t}-\frac{\partial v^{(k)}(\xi,t)}{\partial t}\right\|\right.$$

(1.50)

From here the main inequality follows

$$\max\left\{\max_{t\in[0,T]}\left\|v^{(k+1)}(x,t)-v^{(k)}(x,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(k+1)}(x,t)}{\partial t}-\frac{\partial v^{(k)}(x,t)}{\partial t}\right\|\right\}\le$$

$$\le(\alpha(x)\gamma+1)(\beta(x)+\sigma(x))\times$$

$$\times\int_0^x\max\left\{\max_{t\in[0,T]}\left\|v^{(k)}(\xi,t)-v^{(k-1)}(\xi,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^{(k)}(\xi,t)}{\partial t}-\frac{\partial v^{(k-1)}(\xi,t)}{\partial t}\right\|\right.$$

$$\le(\tilde\alpha\gamma+1)^k(\tilde\beta+\tilde\sigma)^k\frac{x^k}{k!}\max\left\{\left\|v^{(0)}\right\|_0,\left\|\frac{\partial v^{(0)}}{\partial t}\right\|_0\right\}\le(\tilde\alpha\gamma+1)^k(\tilde\beta+$$

$$+\tilde\sigma)^k\frac{x^k}{k!}\max\{\tilde\alpha\gamma+1,\gamma\}(1+\tilde\beta+\tilde\sigma)\max\{\max_{t\in[0,T]}\|\psi(t)\|,\max_{t\in[0,T]}\|\dot\psi(t)\|,\|f\|_0\}.$$

From here the uniform convergence of sequences $\{v^{(k)}(x,t)\}$ in the norm of space $C(\overline{\Omega},R^n)$ at $k\to\infty$. Then the uniform convergence of the sequences $\{u^{(k)}(x,t)\}$, $\{w^{(k)}(x,t)\}$ follows from the estimates (1.50). In this case, the limit functions $v^*(x,t),u^*(x,t),w^*(x,t)$ are continuous on $\overline{\Omega}$ and are the solution of problem (1.4)-(1.7). For the solution of a problem (1.4)-(1.7) estimates are valid

$$\max_{t\in[0,T]}\left\|v^*(x,t)\right\|\le\gamma\max_{t\in[0,T]}\left\|F(x,t)\right\|\le$$

$$\le\gamma\{\beta(x)\max_{t\in[0,T]}\left\|w^*(x,t)\right\|+\sigma(x)\max_{t\in[0,T]}\left\|u^*(x,t)\right\|+\max_{t\in[0,T]}\left\|f(x,t)\right\|\},$$

$$\max_{t\in[0,T]}\left\|\frac{\partial v^*(x,t)}{\partial t}\right\|\le\alpha(x)\max_{t\in[0,T]}\left\|v^*(x,t)\right\|+$$

$$+\beta(x)\max_{t\in[0,T]}\left\|w^*(x,t)\right\|+\sigma(x)\max_{t\in[0,T]}\left\|u^*(x,t)\right\|+\max_{t\in[0,T]}\left\|f(x,t)\right\|\le$$

$$\le(\alpha(x)\gamma+1)(\beta(x)\max_{t\in[0,T]}\left\|w^*(x,t)\right\|+\sigma(x)\max_{t\in[0,T]}\left\|u^*(x,t)\right\|+\max_{t\in[0,T]}\left\|f(x,t)\right\|$$

$$\max\left\{\max_{t\in[0,T]}\left\|v^*(x,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^*(x,t)}{\partial t}\right\|\right\}\le$$

$$\le(\alpha(x)\gamma+\gamma+1)(\beta(x)\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|+$$

$$\sigma(x)\max_{t\in[0,T]}\left\|\psi(t)\right\|+\max_{t\in[0,T]}\left\|f(x,t)\right\|)+(\alpha(x)\gamma+\gamma+1)\times$$

$$\times(\beta(x)+\sigma(x))\int_0^x\max\left\{\max_{t\in[0,T]}\left\|\frac{\partial v^*(\xi,t)}{\partial t}\right\|,\max_{t\in[0,T]}\left\|v^*(\xi,t)\right\|\right\}d\xi.$$

Using the Bellman-Gronwall inequality, we obtain

$$\max\left\{\max_{t\in[0,T]}\left\|v^*(x,t)\right\|,\max_{t\in[0,T]}\left\|\frac{\partial v^*(x,t)}{\partial t}\right\|\right\}\le$$

$$\le(\alpha(x)\gamma+\gamma+1)(\beta(x)\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|+$$

$$+\sigma(x)\max_{t\in[0,T]}\left\|\psi(t)\right\|+\max_{t\in[0,T]}\left\|f(x,t)\right\|)e^{(\alpha(x)\gamma+\gamma+1)(\beta(x)+\sigma(x))x}.$$

$$\max_{t\in[0,T]}\left\|u^*(x,t)\right\|\le\max_{t\in[0,T]}\left\|\psi(t)\right\|+\int_0^x\max_{t\in[0,T]}\left\|v^*(\xi,t)\right\|d\xi,$$

$$\max_{t\in[0,T]}\left\|w^*(x,t)\right\|\le\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|+\int_0^x\max_{t\in[0,T]}\left[\frac{\partial v^*(\xi,t)}{\partial t}\right]d\xi,$$

$$\max\left\{\left\|v^*\right\|_0,\left\|\frac{\partial v^*}{\partial t}\right\|_0\right\}\le$$

$$\le[\tilde{\alpha}\gamma+\gamma+1](\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega}\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\left\|f\right\|_0$$

$$\max\{\left\|u^*\right\|_0,\left\|w^*\right\|_0\}\le(1+\omega(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega})\times$$

$$\times\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\left\|f\right\|_0\},$$

$$\max\{\left\|u^*\right\|_0,\left\|v^*\right\|_0,\left\|w^*\right\|_0\}\le$$

$$\le\max\{(1+\omega(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega}),$$

$$\omega(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma}+1)e^{(\tilde{\alpha}\gamma+\gamma+1)(\tilde{\beta}+\tilde{\sigma})\omega}\}\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\|$$

$$=K\max\{\max_{t\in[0,T]}\left\|\psi(t)\right\|,\max_{t\in[0,T]}\left\|\dot{\psi}(t)\right\|,\left\|f\right\|_0\}.$$

Theorem 6 is proved.

Example 1. Consider for $[0,\frac{\pi}{4}]\times[0,\frac{\pi}{4}]$ semi-periodic boundary value problem

$$\frac{\partial^2 u_1}{\partial x\partial t} = \frac{1}{3}\frac{\partial u_2}{\partial x} - 1, \quad \frac{\partial^2 u_2}{\partial x\partial t} = \frac{1}{2}\frac{\partial u_1}{\partial x} + u_1(x,t) - x - t(t-\frac{\pi}{4}) - \frac{1}{2},$$

$$u_1(x,0) = u_1(x,\frac{\pi}{4}), \quad u_2(x,0) = u_2(x,\frac{\pi}{4}),$$

$$u_1(0,t) = t(t-\frac{\pi}{4}), \quad u_2(0,t) = \sin 4t.$$

$\alpha(x) = \frac{1}{2}.$ Here h is taken equal to $\frac{\pi}{4}$ (i.e. $N=1$), $\nu = 1.$ Let us verify the conditions of Theorem 1:

$$1)\left\|\left[Q_1(x,\frac{\pi}{4})\right]^{-1}\right\| \le \frac{8}{\pi}, \quad 2) q_1(x,\frac{\pi}{4}) = \frac{1}{2}\cdot\frac{\pi}{4}(1+\frac{8}{\pi}\cdot\frac{1}{2}\cdot\frac{\pi}{4}) = \frac{\pi}{4} < 1.$$

Thus, the problem under consideration is uniquely solvable.

1.3 About one modification of the algorithms for finding a solution of a semi-periodic boundary value problem

In subsection 1.3, we consider a semi-periodic boundary value problem of the form (1.1)-(1.3) at $B(x,t)\equiv 0.$ Based on the method of parametrization to find an approximate solution of the boundary value problem (1.1)-(1.3) at $B(x,t)\equiv 0,$ an efficient algorithm is proposed, which differs in its simplicity from the algorithm proposed in subsection 1.1. In terms of the initial data, the coefficient characteristics of the unique solvability of the boundary value problem

$$\frac{\partial^2 u}{\partial x\partial t} = A(x,t)\frac{\partial u}{\partial x} + C(x,t)u + f(x,t), \qquad (x,t)\in\overline{\Omega}, \qquad (1.51)$$

with the conditions (1.2), (1.3) are established. Analogically to subsection 1.1 entering function

$$v(x,t) = \frac{\partial u(x,t)}{\partial x}$$

and applying the scheme of a method of a parametrization [67-69], we reduce the problem (1.51), (1.2), (1.3) to an equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial \tilde{v}_r}{\partial t} = A(x,t)\tilde{v}_r + A(x,t)\lambda_r(x) + C(x,t)u_r(x,t) + f(x,t), \quad (1.52)$$

$$\tilde{v}_r(x,(r-1)h) = 0, \quad x \in [0,\omega], \quad r = \overline{1,N}, \quad (1.53)$$

$$u_r(x,t) = \psi(t) + \int_0^x \tilde{v}_r(\xi,t)d\xi + \int_0^x \lambda_r(\xi)d\xi, \quad r = \overline{1,N}, \quad (1.54)$$

$$\lambda_1(x) - \lambda_N(x) - \lim_{t\to T-0} \tilde{v}_N(x,t) = 0, \quad x \in [0,\omega], \quad (1.55)$$

$$\lambda_s(x) + \lim_{t\to sh-0} \tilde{v}_s(x,t) - \lambda_{s+1}(x) = 0, \; x \in [0,\omega], \; s = \overline{1,N-1}. \quad (1.56)$$

The problem (1.52), (1.53) for fixed $\lambda(x), u_r(x,t)$ is a family of Cauchy problems for ordinary differential equations [70-71], where $x \in [0,\omega]$, and is equivalent to the integral equation

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} (A(x,\tau)[\tilde{v}_r(x,\tau) + \lambda_r(x)] + $$
$$+ C(x,\tau)u_r(x,\tau) + f(x,\tau))d\tau. \quad (1.57)$$

Instead of $\tilde{v}_r(x,\tau)$ we substitute the appropriate right side of (1.57) and by repeating this process $\nu \;\; (\nu = 1,2,...)$ times, we obtain

$$\tilde{v}_r(x,t) = D_{\nu r}(x,t)\lambda_r(x) + F_{\nu r}(x,t,u_r) + G_{\nu r}(x,t,\tilde{v}_r), r = \overline{1,N}, \quad (1.58)$$

where $F_{\nu r}(x,t,u_r) = \int_{(r-1)h}^{t} [C(x,\tau_1)u_r(x,\tau_1) + f(x,\tau_1)]d\tau_1 + \sum_{j=1}^{\nu-1} \int_{(r-1)h}^{t} A(x,\tau_1)...$

$$... \int_{(r-1)h}^{\tau_{j-1}} A(x,\tau_j) \int_{(r-1)h}^{\tau_j} [C(x,\tau_{j+1})u_r(x,\tau_{j+1}) + f(x,\tau_{j+1})]d\tau_{j+1}d\tau_j...d\tau_1,$$

$\tau_0 = t, r = \overline{1,N}.$ By moving to the limit at $t \to rh-0$ in (1.58) we have

$$\lim_{t\to rh-0} \tilde{v}_r(x,t) = D_{\nu r}(x,rh)\lambda_r(x) + F_{\nu r}(x,rh,u_r) + G_{\nu r}(x,rh,\tilde{v}_r),$$

$x \in [0,\omega], r = \overline{1,N}.$ Substituting in (1.55), (1.56) instead of $\lim_{t\to rh-0} \tilde{v}_r(x,t), r = \overline{1,N},$ corresponding right sides for unknown functions $\lambda_r(x), r = \overline{1,N},$ we obtain the system of functional equations:

$$Q_\nu(x,h)\lambda(x) = -F_\nu(x,h,u) - G_\nu(x,h,\tilde{v}), \quad (1.59)$$

where

$$F_\nu(x,h,u) =$$
$$= (-F_{\nu N}(x,Nh,u_N), F_{\nu 1}(x,h,u_1),\dots,F_{\nu,N-1}(x,(N-1)h,u_{N-1})).$$

To find a system of three functions $\{\lambda_r(x),\tilde{v}_r(x,t),u_r(x,t)\}, r=\overline{1,N},$ we have a closed system consisting of equations (1.59), (1.58) and (1.54).

Functions $\lambda_r(x),\ \tilde{v}_r(x,t),\ u_r(x,t), r=\overline{1,N},$ we find as the limits of the corresponding sequences $\{\lambda_r^{(k)}(x)\},\{\tilde{v}_r^{(k)}(x,t)\},\{u_r^{(k)}(x,t)\},\quad r=\overline{1,N},$ at $k\to\infty$ determined by the following algorithm:

Step 0. Assuming the invertibility of the matrix $Q_\nu(x,h)$ for all $x\in[0,\omega]$, from equation (1.59), where

$$u_r(x,t)=\psi(t), \tilde{v}_r(x,t)=0,$$

we find $\lambda^{(0)}(x)=(\lambda_1^{(0)}(x),\lambda_2^{(0)}(x),\dots,\lambda_N^{(0)}(x))'$:

$$\lambda^{(0)}(x)=-[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,\psi)+G_\nu(x,h,0)\}.$$

Using equation (1.58), at $\lambda_r(x)=\lambda_r^{(0)}(x)$ find the functions $\{\tilde{v}_r^{(0)}(x,t)\},\ r=\overline{1,N},$ i.e.

$$\tilde{v}_r^{(0)}(x,t)=D_{\nu r}(x,t)\lambda_r^{(0)}(x)+F_{\nu r}(x,t,\psi)+G_{\nu r}(x,t,0).$$

Functions $u_r^{(0)}(x,t), r=\overline{1,N},$ determined from correlation

$$u_r^{(0)}(x,t)=\psi(t)+\int_0^x \tilde{v}_r^{(0)}(\xi,t)d\xi+\int_0^x \lambda_r^{(0)}(\xi)d\xi,\quad (x,t)\in\Omega_r.$$

Step 1. Considering the invertibility of the matrix $Q_\nu(x,h)$ at $x\in[0,\omega]$, we obtain the function $\lambda^{(1)}(x)=(\lambda_1^{(1)}(x),\lambda_2^{(1)}(x),\dots,\lambda_N^{(1)}(x))'$ as a solution of the system of equations (1.59):

$$\lambda^{(1)}(x)=-[Q_\nu(x,h)]^{-1}\{F_\nu(x,h,u^{(0)})+G_\nu(x,h,\tilde{v}^{(0)})\},$$

where

$$u_r(x,t)=u_r^{(0)}(x,t),\quad \tilde{v}_r(x,t)=\tilde{v}_r^{(0)}(x,t),\quad r=\overline{1,N}.$$

By using equation (1.58) again at $\lambda_r(x)=\lambda_r^{(1)}(x)$ we find $\{\tilde{v}_r^{(1)}(x,t)\},\ r=\overline{1,N},$ i.e.

$$\tilde{v}_r^{(1)}(x,t)=D_{\nu r}(x,t)\lambda_r^{(1)}(x)+F_{\nu r}(x,t,u_r^{(0)})+G_{\nu r}(x,t,\tilde{v}_r^{(0)}).$$

Functions $u_r^{(1)}(x,t), r=\overline{1,N},$ determine from correlations

$$u_r^{(1)}(x,t)=\psi(t)+\int_0^x \tilde{v}_r^{(1)}(\xi,t)d\xi+\int_0^x \lambda_r^{(1)}(\xi)d\xi, \quad (x,t)\in\Omega_r.$$

By continuing the process, on the k step we obtain a system of triples $\{\lambda_r^{(k)}(x), \tilde{v}_r^{(k)}(x,t),$ $u_r^{(k)}(x,t)\}$, $r=\overline{1,N}$.

The conditions of the following statement provide a uniform relative to $(x,t)\in\Omega_r, r=\overline{1,N},$ the convergence of the proposed algorithm [72, 73] to the solution of a boundary value problem with unknown functions (1.52)-(1.56).

Theorem 7. *Let for some* $h>0: \quad Nh=T, \quad N=1,2,...,$ *and* $\nu, \nu\in\mathbb{N}, (nN\times nN)$ *matrix* $Q_\nu(x,h)$ *invertible for all* $x\in[0,\omega]$ *and inequalities are satisfied*

1) $\|[Q_\nu(x,h)]^{-1}\|\le\gamma_\nu(x,h);$

2) $q_\nu(x,h)=\dfrac{(\alpha(x)h)^\nu}{\nu!}[1+\gamma_\nu(x,h)\sum_{j=0}^{\nu}\dfrac{(\alpha(x)h)^j}{j!}]\le\mu<\dfrac{1}{2}.$

the algorithm is realizable, converges to the unique solution of the problem (1.52)-(1.56) and estimates are valid.

$$\|\lambda^*(x)-\lambda^{(k)}(x)\|+\|\tilde{v}^*(x,t)-\tilde{v}^{(k)}(x,t)\|\le$$

$$\le\frac{1}{1-2\mu}\frac{\left(\dfrac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right]}}{\left[\dfrac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right]!}(2\mu)^{k+1}\tilde{\chi}\max_{(x,t)\in\underline{\Omega}}[\sigma(x)\|\psi(t)\|+\|f(x,t)\|],$$

$\|u^*(x,t)-u^{(k)}(x,t)\|\le\int_0^x(\|\lambda^*(\xi)-\lambda^{(k)}(\xi)\|+\|\tilde{v}^*(\xi,t)-\tilde{v}^{(k)}(\xi,t)\|)d\xi,$ *where*

$\tilde{\sigma}=\max_{x\in[0,\omega]}\sigma(x), \quad \tilde{\chi}=\max_{x\in[0,\omega]}\chi(x),$

$$\chi(x)=[1+\gamma_\nu(x,h)h\sum_{j=0}^{\nu}\frac{(\alpha(x)h)^j}{j!}]h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}.$$

Proof. Concerning the task's data there is an inequality

$\|F_\nu(x,h,u)\|\le h\sum_{j=0}^{\nu-1}\dfrac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}[\sigma(x)\|u_r(x,t)\|+\|f(x,t)\|].$ Th

e following estimates follow from the zero and first steps of the algorithm:

$$\left\|\lambda^{(0)}(x)\right\| \le \gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sup_{t\in[0,T]}\left[\sigma(x)\|\psi(t)\| + \|f(x,t)\|\right],$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\| \le \max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|D_{\nu r}(x,t)\right\|\max_{r=\overline{1,N}}\left\|\lambda_r^{(0)}(x)\right\| +$$

$$+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|F_{\nu r}(x,t,\psi(t))\right\| + \max_{r=\overline{1,N}}\left\|G_{\nu r}(x,t,0)\right\| \le$$

$$\le \sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(0)}(x)\right\| +$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sup_{t\in[0,T]}\left[\sigma(x)\|\psi(t)\| + \|f(x,t)\|\right],$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\| \le \int_0^x \Delta^{(0)}(\xi)d\xi,$$

where $\Delta^{(0)}(x) = \max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\| + \max_{r=\overline{1,N}}\left\|\lambda_r^{(0)}(x)\right\| \le$

$$\gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sup_{t\in[0,T]}\left[\sigma(x)\|\psi(t)\| + \|f(x,t)\|\right] +$$

$$\sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(0)}(x)\right\| +$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sup_{t\in[0,T]}\left[\sigma(x)\|\psi(t)\| + \|f(x,t)\|\right] \le$$

$$\le \chi(x)\sup_{t\in[0,T]}\left[\sigma(x)\|\psi(t)\| + \|f(x,t)\|\right],$$

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{(1)}(x)-\lambda_r^{(0)}(x)\right\| \le$$

$$\le \gamma_\nu(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\| +$$

$$+\gamma_\nu(x,h)\frac{(\alpha(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(1)}(x,t)-\widetilde{v}_r^{(0)}(x,t)\right\| \le$$

$$\le \sum_{j=1}^{\nu}\frac{(\alpha(x)h)^j}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(1)}(x)-\lambda_r^{(0)}(x)\right\| +$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^j}{j!}\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\| +$$

$$+\frac{(\alpha(x)h)^{\nu}}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|.$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(1)}(x,t)-u_r^{(0)}(x,t)\right\|\le\int_0^x\Delta^{(1)}(\xi)d\xi.$$

Let's set the inequality

$$\Delta^{(1)}(x)\le\sum_{j=0}^{\nu}\frac{(\alpha(x)h)^{j}}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(1)}(x)-\lambda_r^{(0)}(x)\right\|+$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^{j}}{j!}\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+\frac{(\alpha(x)h)^{\nu}}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|\le$$

$$\le\sum_{j=0}^{\nu}\frac{(\alpha(x)h)^{j}}{j!}\gamma_{\nu}(x,h)h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^{j}}{j!}\times$$

$$\times\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+\sum_{j=0}^{\nu}\frac{(\alpha(x)h)^{j}}{j!}\gamma_{\nu}(x,h)\frac{(\alpha(x)h)^{\nu}}{(\nu!)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|+$$

$$+h\sum_{j=0}^{\nu-1}\frac{(\alpha(x)h)^{j}}{j!}\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+\frac{(\alpha(x)h)^{\nu}}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|\le$$

$$\le\chi(x)\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+q_{\nu}(x,h)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|.$$

Thus

$$\Delta^{(1)}(x)\le\chi(x)\sigma(x)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)\right\|+$$

$$+q_{\nu}(x,h)\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|.$$

For systems of differences $\lambda_r^{(k+1)}(x)-\lambda_r^{(k)}(x)$, $\widetilde{v}_r^{(k+1)}(x,t)-\widetilde{v}_r^{(k)}(x,t)$, $u_r^{(k+1)}(x,t)-u_r^{(k)}(x,t)$, $r=\overline{1,N}$, $k=0,1,2,\ldots,$ the following estimates are valid:

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x) \right\| \le$$

$$\le \gamma_\nu(x,h) h \sum_{j=0}^{\nu-1} \frac{(\alpha(x)h)^j}{j!} \sigma(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| +$$

$$+ \gamma_\nu(x,h) \frac{(\alpha(x)h)^\nu}{\nu!} \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k)}(x,t) - \tilde{v}_r^{(k-1)}(x,t) \right\|, \tag{1.60}$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1)}(x,t) - \tilde{v}_r^{(k)}(x,t) \right\| \le$$

$$\le \{1 + \gamma_\nu(x,h) \sum_{j=1}^{\nu} \frac{(\alpha(x)h)^j}{j!}\} h \sum_{j=0}^{\nu-1} \frac{(\alpha(x)h)^j}{j!} \sigma(x) \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| +$$

$$+ \{1 + \gamma_\nu(x,h) \sum_{j=1}^{\nu} \frac{(\alpha(x)h)^j}{j!}\} \frac{(\alpha(x)h)^\nu}{\nu!} \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k)}(x,t) - \tilde{v}_r^{(k-1)}(x,t) \right\|. \tag{1.61}$$

Summing, respectively, the left and right sides of the inequalities (1.60), (1.61) we have

$$\Delta^{(k+1)}(x) \le \{\gamma_\nu(x,h) h \sum_{j=0}^{\nu-1} \frac{(\alpha(x)h)^j}{j!} \sigma(x) +$$

$$+ [1 + \gamma_\nu(x,h) \sum_{j=1}^{\nu} \frac{(\alpha(x)h)^j}{j!}] h \sum_{j=0}^{\nu-1} \frac{(\alpha(x)h)^j}{j!} \sigma(x)\} \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| +$$

$$+ \{\gamma_\nu(x,h) \frac{(\alpha(x)h)^\nu}{\nu!} + [1 + \gamma_\nu(x,h) \sum_{j=1}^{\nu} \frac{(\alpha(x)h)^j}{j!}] \frac{(\alpha(x)h)^\nu}{\nu!}\} \times$$

$$\times \{\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k)}(x,t) - \tilde{v}_r^{(k-1)}(x,t) \right\| + \max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x) \right\|\} \le$$

$$\le \chi(x)\sigma(x) \int_0^x \Delta^{(k)}(\xi) d\xi + q_\nu(x,h) \Delta^{(k)}(x), \tag{1.62}$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k+1)}(x,t) - u_r^{(k)}(x,t) \right\| \le \int_0^x \Delta^{(k+1)}(\xi) d\xi.$$

For function $\Delta^{(k+1)}(x)$ based on (1.62) we establish the inequality

$$\Delta^{(k+1)}(x) \le \sum_{j=0}^{k+1} \frac{(k+1)!}{(k+1-j)!\,j!} \cdot \mu^{k+1-j} \cdot \frac{1}{j!}(\tilde{\sigma}\tilde{\chi}\omega)^{j} \cdot \max_{x\in[0,\omega]} \Delta^{(0)}(x) \le$$

$$\le \mu^{k+1} \sum_{j=0}^{k+1} \frac{(k+1)!}{(k+1-j)!\,j!} \cdot \max_{j} \frac{1}{j!}\left(\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right)^{j} \cdot \max_{x\in[0,\omega]} \Delta^{(0)}(x),$$

by calculating the maximum term [74, p.7] among the terms of the sequence $\frac{1}{j!}\left(\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right)^{j}$ we obtain

$$\Delta^{(k+1)}(x) \le (2\mu)^{k+1} \frac{\left(\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right)^{[\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}]}}{[\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}]!} \cdot \max_{x\in[0,\omega]} \Delta^{(0)}(x). \qquad (1.63)$$

By owing of the inequality $2\mu < 1$ follows a uniform convergence of the series $\sum_{k=1}^{\infty} \Delta^{(k+1)}(x)$ at $x \in [0,\omega]$, providing uniform sequence of convergence $\lambda_r^{(k)}(x)$, $v_r^{(k)}(x,t)$ to the continuous functions $\lambda_r^{*}(x), v_r^{*}(x,t)$ respectively on $x \in [0,\omega]$ and $(x,t) \in \Omega_r$ for all $r = \overline{1,N}$. On the basis of (1.54) follows uniform sequence convergence $u_r^{(k)}(x,t), r = \overline{1,N}$, relatively $(x,t) \in \Omega_r$ to function $u_r^{*}(x,t)$ that appertain to $\tilde{C}(\Omega_r, R^n)$. Let's set the inequalities

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+p)}(x) - \lambda_r^{(k)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+p)}(x,t) - \tilde{v}_r^{(k)}(x,t) \right\| \le$$

$$\le \max_{r=\overline{1,N}} \left\| \lambda_r^{(k+p)}(x) - \lambda_r^{(k+p-1)}(x) \right\| +$$

$$+ \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+p)}(x,t) - \tilde{v}_r^{(k+p-1)}(x,t) \right\| +$$

$$+ \max_{r=\overline{1,N}} \left\| \lambda_r^{(k+p-1)}(x) - \lambda_r^{(k+p-2)}(x) \right\| +$$

$$+ \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+p-1)}(x,t) - \tilde{v}_r^{(k+p-2)}(x,t) \right\| +$$

$$+ \ldots + \max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1)}(x,t) - \tilde{v}_r^{(k)}(x,t) \right\| \le$$

$$\le \Delta^{(k+p)}(x) + \Delta^{(k+p-1)}(x) + \ldots + \Delta^{(k+1)}(x) \le$$

$$\leq \frac{1}{1-2\mu}\frac{\left(\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}\right)^{[\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}]}}{[\frac{\tilde{\sigma}\tilde{\chi}\omega}{\mu}]!}(2\mu)^{k+1}\tilde{\chi}\max_{(x,t)\in\Omega}[\sigma(x)\|\psi(t)\|+\|f(x,t)\|].$$

$$\|u^{(k+p)}(x,t)-u^{(k)}(x,t)\|\leq$$

$$\leq \int_0^x(\|\lambda^{(k+p)}(\xi)-\lambda^{(k)}(\xi)\|+\|\tilde{v}^{(k+p)}(\xi,t)-\tilde{v}^{(k)}(\xi,t)\|)d\xi$$

by moving to the limit at $p\to\infty$, for all $(x,t)\in\Omega_r$, $r=\overline{1,N}$, we obtain the estimates 1), 2) of Theorem 7. The uniqueness of the solution of problem (1.52)-(1.56) is proved in the same way as in the proof of Theorem 1.

Theorem 7 is proved.

By owing of the equivalence of problems (1.51), (1.2), (1.3) and (1.52)-(1.56), Theorem 7 follows

Theorem 8. *Let's suppose that the conditions of Theorem 7 are satisfied. Then the problem (1.51), (1.2), (1.3) have a unique solution* $u^*(x,t)$ *and the estimate*

$$\max\{\|u^*\|_0,\left\|\frac{\partial u^*}{\partial x}\right\|_0\}\leq M_\nu(h)\max\{\max_{t\in[0,T]}\|\psi(t)\|,\|f\|_0\},$$

where $M_\nu(h)=$

$$=\max\left\{1+(\tilde{\sigma}+1)\omega\tilde{\chi}\left[1+\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{[\frac{\tilde{\chi}\omega}{\mu}]}}{[\frac{\tilde{\chi}\omega}{\mu}]!}\right],(\tilde{\sigma}+1)\tilde{\chi}\left[1+\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{[\frac{\tilde{\chi}\omega}{\mu}]}}{[\frac{\tilde{\chi}\omega}{\mu}]!}\right]\right\}$$

is valid.

The following statements establish that the conditions of Theorem 7 are not only sufficient, but also necessary for correct solvability [75-78].

Theorem 9. *The boundary value problem (1.51), (1.2), (1.3) is correctly solvable only if for any* $h>0: Nh=T, N=1,2,\ldots,$ *exists* ν, $\nu\in\mathbb{N}$, $(nN\times nN)$ - *matrix* $Q_\nu(x,h)$ *is invertible for all* $x\in[0,\omega]$ *and the inequalities 1), 2) of Theorem 7 are fulfilled.*

Theorem 10. *The boundary value problem (1.51), (1.2), (1.3) is* *correctly solvable only if for any* ν, $\nu = 2,3,\ldots,$ *exists* $h = h(\nu) > 0 : Nh = T, N = 1,2,\ldots,$ *at which* $(nN \times nN)$ *- matrix* $Q_\nu(x,h)$ *is invertible for all* $x \in [0,\omega]$ *and the inequalities a), b) of Theorem 7 are fulfilled.*

The proof of Theorems 9, 10 is completely analogous to the proofs of Theorems 3, 4 [77].

Example 2. Consider on $[0,1]\times[0,1]$ a system

$$\frac{\partial^2 u}{\partial x \partial t} = \begin{pmatrix} 0 & \frac{1}{2}(1+t^2) \\ t^3 & 0 \end{pmatrix} \frac{\partial u}{\partial x} + c(x,t)u + f(x,t), \qquad (1.64)$$

$$u(0,t) = 0, \qquad t \in [0,1], \qquad (1.65)$$

$$u(x,0) = u(x,1), \qquad x \in [0,1], \qquad (1.66)$$

where $u(x,t)$ - is a two-dimensional vector-function, $c(x,t)$ - is a continuous two-dimensional matrix, $f(x,t)$ - is a continuous vector-function. Here h is taken equal to $\frac{1}{3}$ (i.e. $N = 3$), $\nu = 3$.

Direct calculations show that (6×6) - matrix $Q_3(x,1/3)$ is invertible and

$$\left\|[Q_3(x,1/3)]^{-1}\right\| \le 21.286,$$

$$q_3(x,1/3) = \frac{(1\cdot\frac{1}{3})^3}{6}(1+21.286(1+1/3+1/18)) = 0.0629 < \frac{1}{2}.$$

Therefore, problem (1.64)-(1.66) by Theorem 7 is uniquely solvable.

Example 3. On $[0.5,1]\times[0,1]$ we consider the problem

$$\frac{\partial^2 u}{\partial x \partial t} = \begin{pmatrix} 0 & \frac{1}{3}(x+t) \\ xt & 0 \end{pmatrix} \frac{\partial u}{\partial x} + \begin{pmatrix} 0 & e^x \\ \frac{1}{2}t & 0 \end{pmatrix} u + \begin{pmatrix} x^2 + t^2 \\ 1 + xt^3 \end{pmatrix}, \qquad (1.67)$$

$$u(0,t) = 0, \qquad t \in [0,1], \qquad (1.68)$$

$$u(x,0) = u(x,1), \qquad x \in [0.5,1], \qquad (1.69)$$

where $u(x,t)$ - is a two-dimensional vector-function. For this task $\alpha(x) = x$ and for $h = 1$ $(N = 1), \nu = 4,$ (2×2) - matrix $Q_4(x,1)$ has the form

$$Q_4(x,1) = \begin{pmatrix} \tilde{q}_{11}(x) & \tilde{q}_{12}(x) \\ \tilde{q}_{21}(x) & \tilde{q}_{22}(x) \end{pmatrix},$$

where

$$\tilde{q}_{11}(x) = \frac{x^4}{1620} + \frac{13x^3}{15120} + \frac{193x^2}{3456} + \frac{x}{24},$$

$$\tilde{q}_{12}(x) = \frac{x^3}{108} + \frac{11x^2}{1080} + \frac{145x}{432} + \frac{1}{6},$$

$$\tilde{q}_{21}(x) = \frac{x^3}{90} + \frac{x^2}{144} + \frac{x}{2},$$

$$\tilde{q}_{22}(x) = \frac{x^4}{648} + \frac{11x^3}{7560} + \frac{x^2}{3456}.$$

Its inverse will be in the form

$$\left\|[Q_4(x,1)]^{-1}\right\| \leq \begin{pmatrix} \tilde{q}_{22}(x)\Delta^{-1}(x) & \tilde{q}_{12}(x)\Delta^{-1}(x) \\ \tilde{q}_{21}(x)\Delta^{-1}(x) & \tilde{q}_{11}(x)\Delta^{-1}(x) \end{pmatrix},$$

$$\Delta(x) = \tilde{q}_{11}(x)\tilde{q}_{22}(x) - \tilde{q}_{12}(x)\tilde{q}_{21}(x).$$

It is obvious that $\left\|[Q_4(x,1)]^{-1}\right\| \leq 4,$ and

$$q_4(x,1) = \frac{1}{24}(1 + 4(1 + 1 + 1/2 + 1/6)) = 0.486 < \frac{1}{2}.$$

By Theorem 7, problem (1.67)-(1.69) is uniquely solvable.

Example 4. Consider on $[0,\frac{\pi}{4}]\times[0,\frac{\pi}{4}]$ problem

$$\frac{\partial^2 u}{\partial x \partial t} = \frac{\partial u}{\partial x} - \frac{1}{3}u - \cos(x+t) - \frac{2}{3}\sin(x+t), \qquad (1.70)$$

$$u(0,t) = \sin t, \qquad t \in [0,\frac{\pi}{4}], \qquad (1.71)$$

$$u(x,0) = u(x,\frac{\pi}{4}), \qquad x \in [0,\frac{\pi}{4}], \qquad (1.72)$$

where $u(x,t)$ - is a two-dimensional vector-function. Here h is equal to $\frac{\pi}{8}$ ($N = 2$), $\nu = 2$. Direct calculations show that

(2×2)- matrix $Q_2(x,\frac{\pi}{8})$ is invertible and

$$\left\| \left[Q_2(x,\frac{\pi}{8}) \right]^{-1} \right\| \leq 2.197,$$

$$q_2(x,\frac{\pi}{4}) = \frac{\pi^2}{128}[1 + 2.197(1 + \frac{\pi}{8})] = 0.312684 < \frac{1}{2}.$$

Therefore, the problem under consideration by Theorem 7 is uniquely solvable.

2 ISOLATED SOLUTIONS OF A SEMI-PERIODIC BOUNDARY VALUE PROBLEM FOR SYSTEMS OF NONLINEAR HYPERBOLIC EQUATIONS WITH A MIXED DERIVATIVE

In recent years, the theory of nonlocal boundary value problems for nonlinear hyperbolic equations with mixed derivatives has been actively developed.

Such problems are used in the study of shock waves in an elastic or viscoplastic environment [61], [69]. In addition, systems of such equations appear in the study of the movement of adsorbed mixtures of substances consisting of many components through a porous medium that is pre-saturated with one or more substances for small or large concentrations of adsorbed substances at a constant or variable filtration rate [79]. The study of boundary value problems with non-local conditions for hyperbolic-type equations was stimulated by various reasons: considerations of the general theory of boundary problems by the theory of plasma physics, moisture transfer. The simplest example of non-local conditions are periodic boundary conditions.

The second section of the paper is devoted to the study of a semi-periodic boundary value problem for systems of nonlinear hyperbolic equations with a mixed derivative of the form

$$\frac{\partial^2 u}{\partial x \partial t} = f\left(x, t, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t}\right), \qquad (x,t) \in \overline{\Omega}, \qquad u \in R^n,$$

with conditions (0.2), (0.3).

In subsection 2.1, the method of a parametrization investigated the family of periodic boundary value problems for systems of ordinary differential equations. In terms of the initial data, the conditions for the isolation of its solution are obtained.

In subsection 2.2, by analogy with the linear part, a semi-periodic boundary value problem for systems of nonlinear hyperbolic equations will be reduced to an equivalent boundary value problem with functional parameters. For the obtained problem, sufficient conditions for the feasibility, convergence of the proposed algorithm and the existence of an isolated solution are established.

In subsection 2.3, the semi-periodic boundary value problem of the type

$$\frac{\partial^2 u}{\partial x \partial t} = f(x,t,u,\frac{\partial u}{\partial x}), \quad (x,t) \in \overline{\Omega} = [0,\omega] \times [0,T], \quad u \in R^n,$$

with conditions (0.2), (0.3) is investigated by the parametrization method, to find the solution of which an effective algorithm is constructed, and sufficient conditions for its convergence are established. These conditions ensure the existence of an isolated solution of the problem under study.

One of the main problems in nonlinear boundary value problems is the choice of the initial approximation. In applications, such a choice can be made on the basis of restrictions on the solutions of the considered boundary value problem. In subsection 2.4 methods for finding the initial approximations are proposed. In subsection 2.5, the necessary and sufficient condition for the existence of an isolated, solution of a nonlinear problem is established. It is shown that an isolated in the sense of a definition solution of a nonlinear problem will be isolated in the usual sense. Thus, the $\overline{\Omega} = [0,\omega] \times [0,T]$ boundary value problem is considered on

$$\frac{\partial^2 u}{\partial x \partial t} = f\left(x,t,u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial t}\right), \qquad (x,t) \in \overline{\Omega}, \qquad u \in R^n, \qquad (2.1)$$

$$u(0,t) = \psi(t), \qquad t \in [0,T], \qquad (2.2)$$

$$u(x,0) = u(x,T), \qquad x \in [0,\omega], \qquad (2.3)$$

where $f : \overline{\Omega} \times R^n \times R^n \times R^n \to R^n$ is continuous, n - vector-function $\psi(t)$ is continuously differentiable on $[0,T]$ and satisfies the condition $\psi(0) = \psi(T)$.

To solve this problem, we introduce new unknown functions. $v(x,t) = \dfrac{\partial u(x,t)}{\partial x}$, $w(x,t) = \dfrac{\partial u(x,t)}{\partial t}$ and write problem (2.1)-(2.3) as

$$\frac{\partial v}{\partial t} = f(x,t,u(x,t),v,w(x,t)), \qquad (x,t) \in \overline{\Omega}, \qquad (2.4)$$

$$v(x,0) = v(x,T), \qquad x \in [0,\omega], \qquad (2.5)$$

$$u(x,t) = \psi(t) + \int_0^x v(\xi,t)d\xi, \qquad t \in [0,T], \quad x \in [0,\omega]. \qquad (2.6)$$

$$w(x,t) = \psi(t) + \int_0^x \frac{\partial v(\xi,t)}{\partial t} d\xi, \qquad t \in [0,T], \quad x \in [0,\omega]. \quad (2.7)$$

Here we have reduced the nonlinear semi-periodic boundary value problem for a system of hyperbolic equations to a family of periodic boundary value problems for ordinary differential equations and functional relations [80-86]. The triple belonging $C(\overline{\Omega}, R^n)$ function $(v(x,t), u(x,t), w(x,t))$ is called a solution to problem (2.4)-(2.7), if the function $v(x,t) \in C(\overline{\Omega}, R^n)$ is a solution to problem (2.4), (2.5), where the function $v(x,t)$ is connected with $u(x,t),\ w(x,t)$ functional relations (2.6), (2.7).

Problems (2.1)-(2.3) and (2.4)-(2.7) are equivalent in the sense that if the function $u^*(x,t),$ is a solution to problem (2.1)-(2.3), then the triple

$$\left\{ v^*(x,t) = \frac{\partial u^*(x,t)}{\partial x}, u^*(x,t), w^*(x,t) = \frac{\partial u^*(x,t)}{\partial t} \right\}$$

will be the solution of the problem (2.4)-(2.7) and vice versa, if the triple $(\tilde{v}(x,t), \tilde{u}(x,t),\ \tilde{w}(x,t))$ - will be a solution to problem (2.4)-(2.7), then $\tilde{u}(x,t)$ - is a solution to problem (2.1)-(2.3).

2.1 Statement of a nonlinear semi-periodic boundary value problem and convergence conditions for algorithms and "isolation" of the solution of a family of periodic boundary value problems for ordinary differential equations

For completeness of the study of boundary value problems for systems of nonlinear hyperbolic equations with a mixed derivative, in this subsection, the parametrization method on $\overline{\Omega}$ investigates a family of periodic boundary value problems for a system of linear ordinary differential equations

$$\frac{\partial v}{\partial t} = F(x,t,v), \qquad (x,t) \in \overline{\Omega}, \qquad (2.8)$$

$$v(x,0) = v(x,T), \qquad x \in [0,\omega]. \qquad (2.9)$$

In terms of the initial data, the conditions for the solvability of the problem and the isolation of its solution are obtained.

By the step $h > 0 : Nh = T$ we perform the partition $[0,T) = \bigcup_{r=1}^{N}[(r-1)h, rh),\ N = 1,2,\ldots$ he area of Ω is divided into N parts. Through $v_r(x,t)$ denote the narrowing of the functions $v(x,t)$ to $\Omega_r = [0,\omega]\times[(r-1)h, rh),\quad r=\overline{1,N}$. Then the problem (2.7)-(2.9) will be equivalent to the boundary value problem

$$\frac{\partial v_r}{\partial t} = F(x,t,v_r),\quad (x,t)\in\Omega_r, \tag{2.10}$$

$$v_1(x,0) - \lim_{t\to T-0} v_N(x,t) = 0,\quad x\in[0,\omega], \tag{2.11}$$

$$\lim_{t\to sh-0} v_s(x,t) = v_{s+1}(x,sh),\quad x\in[0,\omega],\quad s=\overline{1,N-1}, \tag{2.12}$$

where (2.12) - is the condition for gluing the solution in the inner lines of the partition.

Through $\lambda_r(x)$ we denote the value of the function $v_r(x,t)$ at $t=(r-1)h,$ i.e. $\lambda_r(x) = v_r(x,(r-1)h)$ and make a replacement $\tilde{v}_r(x,t) = v_r(x,t) - \lambda_r(x),\ r=\overline{1,N}$. We obtain an equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial \tilde{v}_r}{\partial t} = F(x,t,\tilde{v}_r + \lambda_r(x)),\quad (x,t)\in\Omega_r, \tag{2.13}$$

$$\tilde{v}_r(x,(r-1)h) = 0,\quad x\in[0,\omega],\quad r=\overline{1,N}, \tag{2.14}$$

$$\lambda_1(x) - \lambda_N(x) - \lim_{t\to T-0}\tilde{v}_N(x,t) = 0,\quad x\in[0,\omega], \tag{2.15}$$

$$\lambda_s(x) + \lim_{t\to sh-0}\tilde{v}_s(x,t) - \lambda_{s+1}(x) = 0,\quad x\in[0,\omega], s=\overline{1,N-1}. \tag{2.16}$$

Problem (2.13), (2.14) with fixed $\lambda_r(x),$ is a one-parameter family of Cauchy problems for systems of ordinary differential equations, where $x\in[0,\omega],$ and equivalent to nonlinear integral equation

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} F(x,\tau,\tilde{v}_r(x,\tau) + \lambda_r(x))d\tau. \tag{2.17}$$

Instead of $\tilde{v}_r(x,\tau)$ we substitute the corresponding right side of (2.17) and repeating this process $\nu\ \ (\nu = 1,2,\ldots)$ times we obtain

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} F(x,\tau_1, \int_{(r-1)h}^{\tau_1} F(x,\tau_2,\ldots \int_{(r-1)h}^{\tau_{\nu-1}} F(x,\tau_\nu,\tilde{v}_r(x,\tau_\nu) +$$

$$+\lambda_r(x))d\tau_\nu + \ldots + \lambda_r(x))d\tau_2 + \lambda_r(x))d\tau_1. \tag{2.18}$$

From here, having defined $\lim\limits_{t \to rh-0} \tilde{v}_r(x,t),$ substituting them in (2.15), (2.16) we obtain a system of nonlinear equations with respect to $\lambda_r(x)$:

$$\lambda_1(x) - \lambda_N(x) - \int\limits_{(N-1)h}^{Nh} F(x,\tau_1, \int\limits_{(N-1)h}^{\tau_1} F(x,\tau_2,\ldots \int\limits_{(N-1)h}^{\tau_{\nu-1}} F(x,\tau_\nu,\tilde{v}_N(x,\tau_\nu) +$$

$$+\lambda_N(x))d\tau_\nu + \ldots + \lambda_N(x))d\tau_2 + \lambda_N(x))d\tau_1 = 0, x \in [0,\omega], \quad (2.19)$$

$$\lambda_s(x) + \int\limits_{(s-1)h}^{sh} F(x,\tau_1, \int\limits_{(s-1)h}^{\tau_1} F(x,\tau_2,\ldots \int\limits_{(s-1)h}^{\tau_{\nu-1}} F(x,\tau_\nu,\tilde{v}_s(x,\tau_\nu) +$$

$$+\lambda_s(x))d\tau_\nu + \ldots + \lambda_s(x))d\tau_2 + \lambda_s(x))d\tau_1 - \lambda_{s+1}(x) = 0, \quad (2.20)$$

$$x \in [0,\omega], s = \overline{1, N-1}.$$

which we write in the form

$$Q_{\nu,h}(x, x \in [0,\omega], \tilde{v}(x,[\cdot]), \lambda(x)) = 0. \quad (2.21)$$

To find a system of couples $\{\lambda_r(x), \tilde{v}_r(x,t)\}, r = \overline{1,N},$ we have a closed system consisting of equations (2.21) and (2.18), defined through the function F, partition step $h > 0$ and the number of permutations ν. $C([0,\omega], R^{nN})$ - is the set of functions $\lambda : [0,\omega] \to R^{nN}$ continuous on $[0,\omega]$.

Choose the step $h > 0 : Nh = T (N = 1,2,\ldots),$ vector function $\lambda^{(0)}(x) = (\lambda_1^{(0)}(x), \lambda_2^{(0)}(x), \ldots, \lambda_N^{(0)}(x))' \in C([0,\omega], R^{Nn})$ and suppose that problem (2.13)-(2.16) at $\lambda_r(x) = \lambda_r^{(0)}(x),$ $r = \overline{1,N},$ has a solution $\tilde{v}_r^{(0)}(x,t) \in \tilde{C}(\Omega_r, R^n),$ $r = \overline{1,N}.$ Taking $\lambda^{(0)}(x),$ the corresponding $\tilde{v}^{(0)}(x,[t]),$ functions $\rho(x) > 0,$ $\theta(x) > 0$ construct the sets:

$$S(\lambda^{(0)}(x), \rho(x)) = \{(\lambda_1(x), \lambda_2(x), \ldots, \lambda_N(x))' \in C([0,\omega], R^{nN}):$$

$$\left\|\lambda_r(x) - \lambda_r^{(0)}(x)\right\| < \rho(x), r = \overline{1,N}\},$$

$$S(\tilde{v}^{(0)}(x,[t]), \theta(x)) = \{(\tilde{v}_1(x,t), \tilde{v}_2(x,t), \ldots, \tilde{v}_N(x,t))', \tilde{v}_r(x,t) \in$$

$$\in \tilde{C}(\Omega_r, R^n) : \max_{r=1,N} \sup_{t \in [(r-1)h, rh)} \left\|\tilde{v}_r(x,t) - \tilde{v}_r^{(0)}(x,t)\right\| < \theta(x), r = \overline{1,N}\},$$

$$G^0(\rho(x), \theta(x)) = \{(x,t,v) : (x,t) \in \bar{\Omega}_r, \left\|v - \lambda_r^{(0)}(x) - \tilde{v}_r^{(0)}(x,t)\right\| < \rho(x) + \theta(x), (x,t) \in \Omega_r,$$

$$r = \overline{1,N}, \left\|v - \lambda_N^{(0)}(x) - \lim_{t \to T-0} \tilde{v}_N^{(0)}(x,t)\right\| < \rho(x) + \theta(x), t = T\}.$$

Through $D_0(F, L_1(x), x, h)$ denote the set

$(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]), \rho(x), \theta(x))$ at which the function $F(x,t,v)$ in $G^0(\rho(x),\theta(x))$ has a continuous partial derivative $F_v'(x,t,v)$ and

$$\|F_v'(x,t,v)\| \le L_1(x),$$

where $L_1(x)$ - is continuous on $[0,\omega]$ function. Take a system of couples $(\lambda_r^{(0)}(x),\ \tilde{v}_r^{(0)}(x,t)),\ r=\overline{1,N},$ and we construct the subsequent approximations on the following algorithm:

Step 1.

a) Parameter $\lambda^{(1)}(x) = (\lambda_1^{(1)}(x),...,\lambda_N^{(1)}(x))' \in C([0,\omega], R^{Nn}),$ determined from equations (2.20), where $\tilde{v}_r(x,t) = \tilde{v}_r^{(0)}(x,t), r=\overline{1,N}.$

b) In the right side (2.18) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(0)}(x,t)$, $\lambda_r^{(1)}(x)$, $r=\overline{1,N}$, we obtain $\{\tilde{v}_r^{(1)}(x,t)\}$, $r=\overline{1,N}$.

Step 2.

a) Substituting instead of $\tilde{v}_r(x,t)$ found $\tilde{v}_r^{(1)}(x,t), r=\overline{1,N},$ and solving equation (2.20) is defined by $\lambda^{(2)}(x) \in C([0,\omega], R^{Nn}).$

b) In the right side (2.18) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(1)}(x,t)$, $\lambda_r^{(2)}(x)$, $r=\overline{1,N}$, we obtain $\{\tilde{v}_r^{(2)}(x,t)\}$, $r=\overline{1,N}$.

Continuing the process, at the k step we obtain a system of couples $\{\lambda_r^{(k)}(x),\ \tilde{v}_r^{(k)}(x,t)\}$, $r=\overline{1,N}$.

Sufficient conditions for the existence, convergence of the algorithm, and the existence of a solution to a boundary value problem with the functional parameter (2.13)-(2.16) establish

Theorem 11. *Let there exist* $h>0: Nh=T, (N=1,2,...),$ $\nu \in \mathbb{N}$,
$(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]), \rho(x), \theta(x)) \in D_0(F, L_1(x), x, h),$ *for*

which the Jacobi matrix $\dfrac{\partial Q_{\nu,h}(x,\tilde{v}(x,[\cdot]),\lambda(x))}{\partial\lambda}$ *is invertible for all* $(x,\tilde{v}(x,[\cdot]),\lambda(x)),$ *where*

$x\in[0,\omega],(\lambda(x),\tilde{v}(x,[t]))\in S(\lambda^{(0)}(x),\rho(x))\times S(\tilde{v}^{(0)}(x,[t]),\theta(x))$

and executed the following inequalities:

1) $\left\|\left[\dfrac{\partial Q_{\nu,h}(x,\tilde{v}(x,[\cdot]),\lambda(x))}{\partial\lambda}\right]^{-1}\right\|\le\gamma_\nu(x,h),$

2) $q_\nu(x,h)=\dfrac{(L_1(x)h)^\nu}{\nu!}[1+\gamma_\nu(x,h)\sum\limits_{j=1}^{\nu}\dfrac{(L_1(x)h)^j}{j!}]\le\mu<1,$

3)

$$\frac{(L_1(x)h)^\nu}{\nu!}\frac{\gamma_\nu(x,h)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\|+$$

$$+\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\|<\rho(x),$$

4)

$$\frac{1}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\|<\theta(x).$$

Then the sequence of couples determined by an algorithm $\{\lambda^{(k)}(x),\tilde{v}^{(k)}(x,[t])\},$ $k=1,2,...,$ *is contained in* $S(\lambda^{(0)}(x),\rho(x))\times S(\tilde{v}^{(0)}(x,[t]),\theta(x))$ *converges to* $(\lambda^*(x),\tilde{v}^*(x,[t]))$ - *solving the problem (2.13)-(2.16) and the following estimates are valid:*

a) $\max\limits_{r=\overline{1,N}}\sup\limits_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\|\le$

$$\le\frac{1}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\|,$$

b) $\left\|\lambda^*(x)-\lambda^{(0)}(x)\right\|\le\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\|+$

$$+\frac{(L_1(x)h)^\nu}{\nu!}\frac{\gamma_\nu(x,h)}{1-q_\nu(x,h)}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\|.$$

Moreover, any solution $(\lambda(x),\tilde{v}(x,[t]))$ *of problem (2.13)-(2.16) in* $S(\lambda^{(0)}(x),\rho(x))\times S(\tilde{v}^{(0)}(x,[t])\theta(x))$ *is isolated.*

The function $v^{(k)}(x,t)$, $k = 0,1,2,\ldots$, is defined by the equation:

$$v^{(k)}(x,t) = \begin{cases} \lambda_r^{(k)}(x) + \tilde{v}_r^{(k)}(x,t), & at\ (x,t) \in \Omega_r,\ r = \overline{1,N}, \\ \lambda_N^{(k)}(x) + \lim\limits_{t \to T-0} \tilde{v}_N^{(k)}(x,t), & at\ t = Nh, \end{cases}$$

and by $S_0(v^{(0)}(x,t), \rho(x) + \theta(x))$ denote the set of piecewise-continuously differentiable with respect to t functions $v : \overline{\Omega} \to R^n$, and satisfy the inequalities

$$\left\| v(x,t) - \lambda^{(0)}(x) - \tilde{v}^{(0)}(x,t) \right\| < \rho(x) + \theta(x),$$
$$\left\| v(x,T) - \lambda^{(0)}(x) - \tilde{v}^{(0)}(x,T) \right\| < \rho(x) + \theta(x).$$

In view of the equivalence of problems (2.8), (2.9) and (2.13)-(2.16) from Theorem 11 follows

Theorem 12. *If the conditions of Theorem 11 are satisfied, then the sequence of functions* $\{v^{(k)}(x,t)\}$, $k = 1,2,\ldots$, *contains in* $S_0(v^{(0)}(x,t),\ \rho(x) + \theta(x))$, *converges to* $v^*(x,t)$ *- solution of the problem (2.8), (2.9)* $S_0(v^{(0)}(x,t),\ \rho(x) + \theta(x))$ *and the inequality is valid*

$$\left\| v^*(x,t) - v^{(0)}(x,t) \right\| \leq \gamma_\nu(x,h) \left\| Q_{\nu,h}(x, \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x)) \right\| +$$
$$+ \frac{\dfrac{(L_1(x)h)^\nu}{\nu!} \gamma_\nu(x,h) + 1}{1 - q_\nu(x,h)} \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\|,$$

$(x,t) \in \overline{\Omega}$. *Moreover, any solution of the problem (2.8), (2.9)* $S_0(v^{(0)}(x,t), \rho(x) + \theta(x))$ *is isolated.*

2.2 Sufficient conditions for the existence of an "isolated" solution of a semi-periodic boundary value problem for systems of nonlinear hyperbolic equations

The subject of the study of subdivision 2.2 is a boundary value problem for systems of nonlinear hyperbolic equations (2.1)-(2.3). Similar to the linear case, the boundary value problem for systems of nonlinear hyperbolic equations with functional parameters.

Consider problem (2.1)-(2.3) and the general scheme of parametrization parameters applied to it. Take the step $h > 0 : Nh = T$ and produce a partition $[0,T) = \bigcup_{r=1}^{N}[(r-1)h, rh),\ N = 1,2,...$. In this case, the range of Ω is divided into N parts. Let $v_r(x,t), u_r(x,t), w_r(x,t)$ denote, respectively, the restrictions of the functions $v(x,t)$, $u(x,t)$, $w(x,t)$ to $\Omega_r = [0,\omega] \times [(r-1)h, rh),\ r = \overline{1,N}$. Then problem (2.4)-(2.7) will be equivalent to the boundary value problem

$$\frac{\partial v_r}{\partial t} = f(x,t,u_r(x,t),v_r,w_r(x,t)), \quad (x,t) \in \Omega_r, \tag{2.22}$$

$$v_1(x,0) - \lim_{t \to T-0} v_N(x,t) = 0, \quad x \in [0,\omega], \tag{2.23}$$

$$\lim_{t \to sh-0} v_s(x,t) = v_{s+1}(x,sh), \quad x \in [0,\omega], \quad s = \overline{1,N-1}, \tag{2.24}$$

$$u_r(x,t) = \psi(t) + \int_0^x v_r(\xi,t)d\xi, \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}, \tag{2.25}$$

$$w_r(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial v_r(\xi,t)}{\partial t} d\xi, \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}, \tag{2.26}$$

where (2.24) - is the condition for gluing the functions $v(x,t)$ in the inner lines of the partition. The solution of the problem (2.22)-(2.26) is the systems of functions

$$v(x,[t]) = (v_1(x,t), v_2(x,t), ..., v_N(x,t))',$$
$$u(x,[t]) = (u_1(x,t), u_2(x,t), ..., u_N(x,t))',$$
$$w(x,[t]) = (w_1(x,t), w_2(x,t), ..., w_N(x,t))',$$

where the functions $v_r(x,t)$, $u_r(x,t)$, $w_r(x,t)$, $r = \overline{1,N}$, are continuous and bounded on Ω_r, function $v_r(x,t)$, $r = \overline{1,N}$, having continuous and bounded on Ω_r private derivative $\frac{\partial v_r(x,t)}{\partial t}$ satisfies the system of differential equations (2.22) for all $(x,t) \in \Omega_r$, $r = \overline{1,N}$, (at $t = (r-1)h$ system (2.22) satisfies $\frac{\partial v_{r,np.}(x,t)}{\partial t}$) and for the values

$v_1(x,0), \lim\limits_{t\to T-0} v_N(x,t), \lim\limits_{t\to sh-0} v_s(x,t),$ $v_{s+1}(x,sh),$ $s=\overline{1,N-1},$ equalities is valid (2.22)-(2.24).

By $\lambda_r(x)$ we denote the value of the function $v_r(x,t)$ at $t=(r-1)h,$ i.e. $\lambda_r(x)=v_r(x,(r-1)h)$ and make the replacement

$$\tilde{v}_r(x,t)=v_r(x,t)-\lambda_r(x), r=\overline{1,N}.$$

We obtain an equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial\tilde{v}_r}{\partial t}=f(x,t,u_r(x,t),\tilde{v}_r+\lambda_r(x),w_r(x,t)),\quad (x,t)\in\Omega_r, \qquad (2.27)$$

$$\tilde{v}_r(x,(r-1)h)=0,\quad x\in[0,\omega],\quad r=\overline{1,N}, \qquad (2.28)$$

$$\lambda_1(x)-\lambda_N(x)-\lim_{t\to T-0}\tilde{v}_N(x,t)=0,\quad x\in[0,\omega], \qquad (2.29)$$

$$\lambda_s(x)+\lim_{t\to sh-0}\tilde{v}_s(x,t)-\lambda_{s+1}(x)=0,\quad x\in[0,\omega], s=\overline{1,N-1}. \qquad (2.30)$$

$$u_r(x,t)=\psi(t)+\int_0^x\tilde{v}_r(\xi,t)d\xi+\int_0^x\lambda_r(\xi)d\xi,\quad r=\overline{1,N}, \qquad (2.31)$$

$$w_r(x,t)=\dot{\psi}(t)+\int_0^x\frac{\partial\tilde{v}_r(\xi,t)}{\partial t}d\xi,\quad (x,t)\in\Omega_r,\quad r=\overline{1,N}, \qquad (2.32)$$

The problem (2.22)-(2.26) and (2.27)-(2.32) are equivalent in the sense that if the system of triples $\{v_r(x,t),u_r(x,t),w_r(x,t)\},$ $r=\overline{1,N},$ is a solution to problem (2.22)-(2.26), then the quadruple system $\{\lambda_r(x)=v_r(x,(r-1)h);$ $\tilde{v}_r(x,t)=v_r(x,t)-v_r(x,(r-1)h),$ $u_r(x,t),w_r(x,t)\},$ $r=\overline{1,N},$ will be the solution of the problem (2.27)-(2.32) and, vice versa, if $\{\lambda_r(x),$ $\tilde{v}_r(x,t),u_r(x,t),w_r(x,t)\},$ $r=\overline{1,N}$ - is a solution to problem (2.27)-(2.32), then the system $\{\lambda_r(x)+\tilde{v}_r(x,t),u_r(x,t),w_r(x,t)\}, r=\overline{1,N},$ will be the solution of the problem (2.22)-(2.26).

Problem (2.27), (2.28) with fixed $\lambda_r(x),u_r(x,t),w_r(x,t),$ is a one-parameter family of Cauchy problems for systems of ordinary differential equations, where $x\in[0,\omega],$ and is equivalent to the nonlinear integral equation

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} f(x,\tau,u_r(x,\tau),\tilde{v}_r(x,\tau)+\lambda_r(x),w_r(x,\tau))d\tau. \quad (2.33)$$

Instead of $\tilde{v}_r(x,\tau)$ we substitute the corresponding right side of (2.33) and repeating this process ν $(\nu = 1,2,...)$ times we obtain

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} f(x,\tau_1,u_r(x,\tau_1), \int_{(r-1)h}^{\tau_1} f(x,\tau_2,u_r(x,\tau_2),...$$

$$... \int_{(r-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_r(x,\tau_\nu),\tilde{v}_r(x,\tau_\nu)+\lambda_r(x),w_r(x,\tau_\nu))d\tau_\nu +$$

$$+ ... + \lambda_r(x),w_r(x,\tau_2))d\tau_2 + \lambda_r(x),w_r(x,\tau_1))d\tau_1. \quad (2.34)$$

Hence, by defining $\lim\limits_{t\to rh-0} \tilde{v}_r(x,t),$ substituting them in (2.29), (2.30) we obtain a system of nonlinear equations with respect to $\lambda_r(x)$:

$$\lambda_1(x) - \lambda_N(x) - \int_{(N-1)h}^{Nh} f(x,\tau_1,u_N(x,\tau_1), \int_{(N-1)h}^{\tau_1} f(x,\tau_2,u_N(x,\tau_2),...$$

$$... \int_{(N-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_N(x,\tau_\nu),\tilde{v}_N(x,\tau_\nu)+\lambda_N(x),w_N(x,\tau_\nu))d\tau_\nu + ...$$

$$... + \lambda_N(x),w_N(x,\tau_2))d\tau_2 + \lambda_N(x),w_N(x,\tau_1))d\tau_1 = 0, \quad x\in[0,\omega],$$

$$\lambda_s(x) + \int_{(s-1)h}^{sh} f(x,\tau_1,u_s(x,\tau_1), \int_{(s-1)h}^{\tau_1} f(x,\tau_2,u_s(x,\tau_2),...$$

$$... \int_{(s-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_s(x,\tau_\nu),\tilde{v}_s(x,\tau_\nu)+\lambda_s(x),w_s(x,\tau_\nu))d\tau_\nu + ...$$

$$... + \lambda_s(x),w_s(x,\tau_2))d\tau_2 + \lambda_s(x),w_s(x,\tau_1))d\tau_1 - \lambda_{s+1}(x) = 0,$$

$x\in[0,\omega]$, $s = \overline{1, N-1}$, which we write in the form

$$Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x),w(x,[\cdot])) = 0. \quad (2.35)$$

In the absence of a partition $(N = 1, h = T)$ the system of equations (2.35) has the form

$$\int_0^T f(x,\tau_1,u(x,\tau_1), \int_0^{\tau_1} f(x,\tau_2,u(x,\tau_2),... \int_0^{\tau_{\nu-1}} f(x,\tau_\nu,u(x,\tau_\nu),\tilde{v}(x,\tau_\nu)+$$

$$+\lambda(x),w(x,\tau_\nu))d\tau_\nu + ... + \lambda(x),w(x,\tau_2))d\tau_2 +$$

$$+\lambda(x),w(x,\tau_1))d\tau_1 = 0, x\in[0,\omega].$$

for finding systems of four functions $\{\lambda_r(x), \tilde{v}_r(x,t), u_r(x,t), w_r(x,t)\}$, $r = \overline{1, N}$, we have a closed system consisting of equations (2.35), (2.34), (2.31) and (2.32), defined through the function f, step partition $h > 0$ and the number of permutations ν.

Choose the step $h > 0:\ Nh = T \quad (N = 1,2,...)$, vector function

$$\lambda^{(0)}(x) = (\lambda_1^{(0)}(x), \lambda_2^{(0)}(x), ..., \lambda_N^{(0)}(x))' \in C([0,\omega], R^{Nn}), \quad \text{and}$$

assume that problem (2.27)-(2.32) at $\lambda_r(x) = \lambda_r^{(0)}(x)$, $r = \overline{1, N}$, has the solution

$$u_r^{(0)}(x,t) \in \widetilde{C}(\Omega_r, R^n), w_r^{(0)}(x,t) \in \widetilde{C}(\Omega_r, R^n),$$
$$\widetilde{v}_r^{(0)}(x,t) \in \widetilde{C}(\Omega_r, R^n), r = \overline{1,N}.$$

The set of such $\lambda^{(0)}(x) \in C([0,\omega], R^{nN})$ is denoted by $G_0(f, x, h),$ and the corresponding $\lambda^{(0)}(x)$ system of solutions of problems (2.27)-(2.32) through

$$\widetilde{v}^{(0)}(x,[t]) = (\widetilde{v}_1^{(0)}(x,t), \widetilde{v}_2^{(0)}(x,t), \ldots, \widetilde{v}_N^{(0)}(x,t))',$$
$$u^{(0)}(x,[t]) = (u_1^{(0)}(x,t), u_2^{(0)}(x,t), \ldots, u_N^{(0)}(x,t))',$$
$$w^{(0)}(x,[t]) = (w_1^{(0)}(x,t), w_2^{(0)}(x,t), \ldots, w_N^{(0)}(x,t))'.$$

Taking $\lambda^{(0)}(x) \in G_0(f, x, h),$ $\widetilde{v}^{(0)}(x,[t]),$ $u^{(0)}(x,[t]), w^{(0)}(x,[t]),$ continuous on $[0,\omega]$ functions $\rho(x) > 0,$ (x)>0, $\phi(x) > 0$ we construct the sets:

$$S(\lambda^{(0)}(x), \rho(x)) = \{(\lambda_1(x), \lambda_2(x), \ldots, \lambda_N(x))' \in C([0,\omega], R^{nN}):$$
$$\|\lambda_r(x) - \lambda_r^{(0)}(x)\| < \rho(x), r = \overline{1,N}\},$$

$$S(\widetilde{v}^{(0)}(x,[t]), \theta(x)) = \{(\widetilde{v}_1(x,t), \widetilde{v}_2(x,t), \ldots, \widetilde{v}_N(x,t))', \widetilde{v}_r(x,t) \in$$
$$\in \widetilde{C}(\Omega_r, R^n): \|\widetilde{v}_r(x,t) - \widetilde{v}_r^{(0)}(x,t)\| < \theta(x), r = \overline{1,N}\},$$

$$S(u^{(0)}(x,[t]), \phi(x)) = \{(u_1(x,t), u_2(x,t), \ldots, u_N(x,t))', u_r(x,t) \in$$
$$\in \widetilde{C}(\Omega_r, R^n): \|u_r(x,t) - u_r^{(0)}(x,t)\| < \phi(x), (x,t) \in \Omega_r, r = \overline{1,N}\},$$

$$S(w^{(0)}(x,[t]), \phi(x)) = \{(w_1(x,t), w_2(x,t), \ldots, w_N(x,t))',$$
$$w_r(x,t) \in \widetilde{C}(\Omega_r, R^n): \|w_r(x,t) - w_r^{(0)}(x,t)\| < \phi(x), (x,t) \in \Omega_r, r = \overline{1,N}\},$$

$$G_1^0(\rho(x), \theta(x), \phi(x)) = \{(x,t,u,v,w): (x,t) \in \overline{\Omega}_r, \|u - u_r^{(0)}(x,t)\| < \phi(x), (x,t) \in \Omega_r,$$
$$r = \overline{1,N}, \quad \|u - \lim_{t \to Nh-0} u_N^{(0)}(x,t)\| < \phi(x), \quad t = T,$$
$$\|v - \lambda_r^{(0)}(x) - \widetilde{v}_r^{(0)}(x,t)\| < \rho(x) + \theta(x), \quad (x,t) \in \Omega_r, \quad r = \overline{1,N},$$
$$\|v - \lambda_N^{(0)}(x) - \lim_{t \to T-0} \widetilde{v}_N^{(0)}(x,t)\| < \rho(x) + \theta(x),$$
$$t = T, \quad \|w - w_r^{(0)}(x,t)\| < \phi(x), \quad (x,t) \in \Omega_r, \quad r = \overline{1,N},$$
$$\|w - \lim_{t \to Nh-0} w_N^{(0)}(x,t)\| < \phi(x), \quad t = T\}.$$

By $U_0(f, L_1(x), L_2(x), L_3(x), x, h)$ we denote the set of

$(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]), u^{(0)}(x,[t]), w^{(0)}(x,[t]), \rho(x), \theta(x), \phi(x)),$ for which the function $f(x,t,u,v,w)$ in $G_1^0(\rho(x), \theta(x), \phi(x))$ has continuous partial derivatives $f_v'(x,t,u,v,w)$, $f_u'(x,t,u,v,w)$, $f_w'(x,t,u,v,w)$ and $\| f_v'(x,t,u,v,w) \| \leq L_1(x)$, $\| f_u'(x,t,u,v,w) \| \leq L_2(x)$, $\| f_w'(x,t,u,v,w) \| \leq L_3(x)$, where $L_1(x), L_2(x), L_3(x)$ - are continuous on $[0,\omega]$ functions.

According to the system

$$\{\lambda_r(x), \tilde{v}_r(x,t), u_r(x,t), w_r(x,t)\}, r = \overline{1,N},$$

we compose a quadruple $\{\lambda(x), \tilde{v}(x,[t]),$ $u(x,[t]), w(x,[t])\}$, where

$$\lambda(x) = (\lambda_1(x), ..., \lambda_N(x))',$$

$$\tilde{v}(x,[t]) = (\tilde{v}_1(x,t), ..., \tilde{v}_N(x,t))',$$

$$u(x,[t]) = (u_1(x,t), ..., u_N(x,t))', w(x,[t]) = (w_1(x,t), ..., w_N(x,t))'.$$

Assuming the existence of $\lambda^{(0)}(x) \in G_0(f,x,h)$, for the initial approximation of the problem (2.27)-(2.32) we take the quadruple $\{\lambda^{(0)}(x),$ $\tilde{v}^{(0)}(x,[t]), u^{(0)}(x,[t]), w^{(0)}(x,[t])\}$ and construct successive approximations following to the following algorithm:

Step 1. A) Assuming that

$$u_r(x,t) = u_r^{(0)}(x,t), \quad w_r(x,t) = w_r^{(0)}(x,t), \quad r = \overline{1,N},$$

first approximations in $\lambda_r(x), \tilde{v}_r(x,t)$ we find by solving the problem (2.22)-(2.23). Taking

$$\lambda^{(1,0)}(x) = \lambda^{(0)}(x), \quad \tilde{v}_r^{(1,0)}(x,t) = \tilde{v}_r^{(0)}(x,t),$$

system of couples $\{\lambda_r^{(1)}(x), \tilde{v}_r^{(1)}(x,t)\}, r = \overline{1,N}$, we find as the limit of the sequence $\lambda_r^{(1,m)}(x)$, $\tilde{v}_r^{(1,m)}(x,t)$, determined by the following algorithm:

Step 1.1. a) Substituting $\tilde{v}_r^{(1,0)}(x,t)$, $r = \overline{1,N}$, to (2.35) from the system of functional equations

$$Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(1,0)}(x,[\cdot]), \lambda(x), w^{(0)}(x,[\cdot])) = 0$$

defined by $\lambda_r^{(1,1)}(x)$, $r = \overline{1,N}$.

b) To the right side of (2.34) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(1,0)}(x,t)$, $\lambda_r^{(1,1)}(x)$, $r=\overline{1,N}$, we define $\{\tilde{v}_r^{(1,1)}(x,t)\}$, $r=\overline{1,N}$.

Step 1.2. a) Substituting $\tilde{v}_r^{(1,1)}(x,t)$, $r=\overline{1,N}$, to (2.35) from the system of functional equations

$$Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(1,1)}(x,[\cdot]),\lambda(x),w^{(0)}(x,[\cdot]))=0$$

defined by $\lambda_r^{(1,2)}(x)$, $r=\overline{1,N}$.

b) To the right side of (2.34) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(1,1)}(x,t)$, $\lambda_r^{(1,2)}(x)$, $r=\overline{1,N}$, we define $\{\tilde{v}_r^{(1,2)}(x,t)\}$, $r=\overline{1,N}$. On $(1,m)$ step we obtain system of couples $\{\lambda_r^{(1,m)}(x),\ \tilde{v}_r^{(1,m)}(x,t)\}$, $r=\overline{1,N}$.

Suppose that the solution of problem (2.27)-(2.30) sequence of system of couples $\{\lambda_r^{(1,m)}(x),\ \tilde{v}_r^{(1,m)}(x,t)\}$ is defined and at $m\to\infty$ converges to $\{\lambda_r^{(1)}(x),\ \tilde{v}_r^{(1)}(x,t)\}$, $r=\overline{1,N}$.

B) The functions $u_r^{(1)}(x,t), w_r^{(1)}(x,t), r=\overline{1,N}$, determined from the relations

$$u_r^{(1)}(x,t)=\psi(t)+\int_0^x \tilde{v}_r^{(1)}(\xi,t)d\xi+\int_0^x \lambda_r^{(1)}(\xi)d\xi,\ w_r^{(1)}(x,t)=\dot{\psi}(t)+\int_0^x \frac{\partial \tilde{v}_r^{(1)}(\xi,t)}{\partial t}d\xi,\ (x,t)\in\Omega_r.$$

Step 2. A) Assuming that

$$u_r(x,t)=u_r^{(1)}(x,t),\quad w_r(x,t)=w_r^{(1)}(x,t),\quad r=\overline{1,N},$$

second approximations in $\lambda_r(x),\tilde{v}_r(x,t)$ we find by solving problem (2.27)-(2.30). Taking

$$\lambda^{(2,0)}(x)=\lambda^{(1)}(x),\quad \tilde{v}_r^{(2,0)}(x,t)=\tilde{v}_r^{(1)}(x,t),$$

system of couples $\{\lambda_r^{(2)}(x),\ \tilde{v}_r^{(2)}(x,t)\}, r=\overline{1,N}$, we found as the limit of the sequence $\lambda_r^{(2,m)}(x)$, $\tilde{v}_r^{(2,m)}(x,t)$, determined by the following algorithm:

Step 2.1. a) Substituting $\tilde{v}_r^{(2,0)}(x,t)$, $r=\overline{1,N}$, in (2.35) from the system of functional equations

$$Q_{\nu,h}(x, u^{(1)}(x,[\cdot]), \tilde{v}^{(2,0)}(x,[\cdot]), \lambda(x), w^{(1)}(x,[\cdot])) = 0$$

we define $\lambda_r^{(2,1)}(x)$, $r=\overline{1,N}$.

b) To the right side of (2.34) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(2,0)}(x,t)$, $\lambda_r^{(2,1)}(x)$, $r=\overline{1,N}$, we define $\{\tilde{v}_r^{(2,1)}(x,t)\}$, $r=\overline{1,N}$.

Step 2.2. a) Substituting $\tilde{v}_r^{(2,1)}(x,t)$, $r=\overline{1,N}$, in (2.35) from the system of functional equations

$$Q_{\nu,h}(x, u^{(1)}(x,[\cdot]), \tilde{v}^{(2,1)}(x,[\cdot]), \lambda(x), w^{(1)}(x,[\cdot])) = 0$$

we define $\lambda_r^{(2,2)}(x)$, $r=\overline{1,N}$.

b) To the right side of (2.34) substituting instead of $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $\tilde{v}_r^{(2,1)}(x,t)$, $\lambda_r^{(2,2)}(x)$, $r=\overline{1,N}$, we define $\{\tilde{v}_r^{(2,2)}(x,t)\}$, $r=\overline{1,N}$.

On the $(2,m)$ step, we obtain the system of couples $\{\lambda_r^{(2,m)}(x), \tilde{v}_r^{(2,m)}(x,t)\}$, $r=\overline{1,N}$.

Suppose that the solution of problem (2.27)-(2.30) is a sequence of system of couples $\{\lambda_r^{(2,m)}(x)$, $\tilde{v}_r^{(2,m)}(x,t)\}$ is defined and at $m\to\infty$ converges to $\{\lambda_r^{(2)}(x)$, $\tilde{v}_r^{(2)}(x,t)\}$, $r=\overline{1,N}$.

B) The functions $u_r^{(2)}(x,t), w_r^{(2)}(x,t), r=\overline{1,N}$, are determined from the relations

$$u_r^{(2)}(x,t) = \psi(t) + \int_0^x \tilde{v}_r^{(2)}(\xi,t)d\xi + \int_0^x \lambda_r^{(2)}(\xi)d\xi,$$

$$w_r^{(2)}(x,t) = \dot{\psi}(t) + \int_0^x \frac{\partial \tilde{v}_r^{(2)}(\xi,t)}{\partial t}d\xi,$$

$(x,t)\in\Omega_r$. Continuing the process, at the k step we obtain a system of triples $\{\lambda_r^{(k)}(x)$, $\tilde{v}_r^{(k)}(x,t), u_r^{(k)}(x,t)\}$, $r=\overline{1,N}$.

Sufficient conditions for the feasibility, convergence of the algorithm, and the existence of a solution to a multi-characteristic boundary value problem with functional parameters (2.27)-(2.32) establish

Theorem 13. *Let there exist* $>0: Nh=T, (N=1,2,...), \nu\in N,$

$(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]), u^{(0)}(x,[t]), w^{(0)}(x,[t]), \rho(x), \theta(x), \phi(x)) \in$ $\in U_0(f, L_1(x), L_2(x), L_3(x), x, h),$ at which the Jacobi matrix $\dfrac{\partial Q_{\nu,h}(x, u(x,[\cdot]), \tilde{v}(x,[\cdot]), \lambda(x), w(x,[\cdot]))}{\partial \lambda}$ is invertible for all $(x, u(x,[\cdot]), \tilde{v}(x,[\cdot]), \lambda(x), w(x,[\cdot])),$ *where*

$x \in [0,\omega], (\lambda(x), \tilde{v}(x,[t]),$

$u(x,[t]), w(x,[t])) \in S(\lambda^{(0)}(x), \rho(x)) \times$

$\times S(\tilde{v}^{(0)}(x,[t]), \theta(x)) \times S(u^{(0)}(x,[t]), \phi(x)) \times S(w^{(0)}(x,[t]), \phi(x))$

and the following inequalities are satisfied:

1)

$$\left\| \left[\frac{\partial Q_{\nu,h}(x, u(x,[\cdot]), \tilde{v}(x,[\cdot]), \lambda(x), w(x,[\cdot]))}{\partial \lambda} \right]^{-1} \right\| \le \gamma_\nu(x,h),$$

2) $q_\nu(x,h) = \dfrac{(L_1(x)h)^\nu}{\nu!}[1 + \gamma_\nu(x,h) \sum\limits_{j=1}^{\nu} \dfrac{(L_1(x)h)^j}{j!}] \le \mu < 1,$

3) $[(c_1(x)+1)c_0(x)+1]c_2(x)[L_2(x)+L_3(x)] \int_0^x c(\xi) e^{\int_0^\xi c(\xi_1)d\xi_1} \times$

$$\times \int_0^\xi \max\{1, L_1(\xi_1)\}[(c_1(\xi_1)+1)c_0(\xi_1)+1]\gamma_\nu(\xi_1,h) \times$$

$$\times \left\| Q_{\nu,h}(\xi_1, u^{(0)}(\xi_1,[\cdot]), \tilde{v}^{(0)}(\xi_1,[\cdot]), \lambda^{(0)}(\xi_1), w^{(0)}(\xi_1,[\cdot])) \right\| d\xi_1 d\xi +$$

$$+ [c_0(x)c_1(x)+1]\gamma_\nu(x,h) \times$$

$$\times \left\| Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x), w^{(0)}(x,[\cdot])) \right\| < \rho(x),$$

4) $[c_0(x)+1]c_2(x)[L_2(x)+L_3(x)] \int_0^x c(\xi) e^{\int_0^\xi c(\xi_1)d\xi_1} \times$

$$\times \int_0^\xi \max\{1, L_1(\xi_1)\}[c_0(\xi_1)c_1(\xi_1)+c_0(\xi_1)+1]\gamma_\nu(\xi_1,h) \times$$

$$\times \left\| Q_{\nu,h}(\xi_1, u^{(0)}(\xi_1,[\cdot]), \tilde{v}^{(0)}(\xi_1,[\cdot]), \lambda^{(0)}(\xi_1), w^{(0)}(\xi_1,[\cdot])) \right\| d\xi_1 d\xi +$$

$$+ c_0(x)\gamma_\nu(x,h) \left\| Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x), w^{(0)}(x,[\cdot])) \right\| <$$

5) $c(x) e^{\int_0^x c(\xi)d\xi} \int_0^x \max\{1, L_1(\xi)\}[c_0(\xi)c_1(\xi)+c_0(\xi)+$

$$+1]\gamma_\nu(\xi,h) \left\| Q_{\nu,h}(\xi, u^{(0)}(\xi,[\cdot]), \tilde{v}^{(0)}(\xi,[\cdot]), \lambda^{(0)}(\xi), w^{(0)}(\xi,[\cdot])) \right\| d\xi +$$

$$+ [c_0(x)c_1(x)+c_0(x)+1]\max\{1, L_1(x)\}\gamma_\nu(x,h) \times$$

$$\times \left\| Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x), w^{(0)}(x,[\cdot])) \right\| < \phi(x),$$

where

$$c(x)=\max\{[c_0(x)c_1(x)+2c_0(x)+2]c_2(x), L_1(x)[c_0(x)c_1(x)+$$

$$+2c_0(x)+2]c_2(x)+1\}[L_2(x)+L_3(x)],\quad c_0(x)=\frac{1}{1-q_\nu(x,h)}\sum_{j=1}^{\nu}\frac{(L_1(x)h)^j}{j!},$$

$$c_1(x)=\gamma_\nu(x,h)\frac{(L_1(x)h)^\nu}{\nu!},\quad c_2(x)=\frac{1}{1-q_\nu(x,h)}\sum_{j=0}^{\nu}\frac{(L_1(x)h)^j}{j!}h.$$

Then, defined by the algorithm the sequence of functions $(\lambda^{(k)}(x), \tilde{v}^{(k)}(x,[t]), \ u^{(k)}(x,[t]), \ w^{(k)}(x,[t])), \ k=1,2,\ldots,$ *is contained in* $S(\lambda^{(0)}(x),\rho(x))\times S(\tilde{v}^{(0)}(x,[t]),\theta(x))\times S(u^{(0)}(x,[t]),\phi(x))\times$ $\times S(w^{(0)}(x,[t]),\phi(x))$, *converges to* $(\lambda^*(x), \ \tilde{v}^*(x,[t]), \ u^*(x,[t]), \ w^*(x,[t]))$ *- solution of the problem (2.27)-(2.32) and the estimates are valid:*

a)

$$\max\Big\{\max_{r=\overline{1,N}}\|\lambda_r^*(x)-\lambda_r^{(0)}(x)\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r^*(x,t)-\tilde{v}_r^{(0)}(x,t)\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\Big\|\frac{\partial\tilde{v}_r^*(x,t)}{\partial t}-\frac{\partial\tilde{v}_r^{(0)}(x,t)}{\partial t}\Big\|\Big\}\leq$$

$$\leq c(x)e^{\int_0^x c(\xi)d\xi}\int_0^x \max\{1,L_1(\xi)\}[c_0(\xi)c_1(\xi)+c_0(\xi)+1]\gamma_\nu(\xi,h)\times$$

$$\times\|Q_{\nu,h}(\xi,u^{(0)}(\xi,[\cdot]),\tilde{v}^{(0)}(\xi,[\cdot]),\lambda^{(0)}(\xi),w^{(0)}(\xi,[\cdot]))\|d\xi+$$

$$+[c_0(x)c_1(x)+c_0(x)+1]\max\{1,L_1(x)\}\gamma_\nu(x,h)\times$$

$$\times\|Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x),w^{(0)}(x,[\cdot]))\|,$$

b)
$$\max\Big\{\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|u_r^*(x,t)-u_r^{(0)}(x,t)\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|w_r^*(x,t)-w_r^{(0)}(x,t)\|\Big\}\leq$$

$$\leq\int_0^x\max\Big\{\max_{r=\overline{1,N}}\|\lambda_r^*(\xi)-\lambda_r^{(0)}(\xi)\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\|\tilde{v}_r^*(\xi,t)-\tilde{v}_r^{(0)}(\xi,t)\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\Big\|\frac{\partial\tilde{v}_r^*(\xi,t)}{\partial t}-\frac{\partial\tilde{v}_r^{(0)}(\xi,t)}{\partial t}\Big\|\Big\}d\xi.$$

Moreover, any solution $(u(x,[t]),\tilde{v}(x,[t]),\lambda(x),w(x,[t]))$ *of*

problem (2.27)-(2.32) in
$S(\lambda^{(0)}(x),\rho(x))\times S(\tilde{v}^{(0)}(x,[t]),\theta(x))\times S(u^{(0)}(x,[t]),\phi(x))\times$
$\times S(w^{(0)}(x,[t]),\phi(x))$ *isolated.*

The functions $v^{(k)}(x,t), u^{(k)}(x,t), w^{(k)}(x,t)$, $k=1,2,\ldots$, defined by equalities:

$$v^{(k)}(x,t)=\begin{cases}\lambda_r^{(k)}(x)+\tilde{v}_r^{(k)}(x,t), & at\ (x,t)\in\Omega_r,\ r=\overline{1,N},\\ \lambda_N^{(k)}(x)+\lim\limits_{t\to T-0}\tilde{v}_N^{(k)}(x,t), & at\ t=Nh,\end{cases}$$

$u^{(k)}(x,t)=\psi(t)+\int_0^x v^{(k)}(\xi,t)d\xi,\ \ w^{(k)}(x,t)=\dot{\psi}(t)+\int_0^x \frac{\partial v^{(k)}(\xi,t)}{\partial t}d\xi$ and through $S_1(u^{(0)}(x,[t]),\rho(x),\theta(x),\phi(x))$ denote the set of functions $u:\overline{\Omega}\to R^n$, piecewise continuously differentiable in x,t and satisfying the inequalities

$$\|u(x,t)-u^{(0)}(x,t)\|<\phi(x),$$
$$\|u(x,T)-u^{(0)}(x,T)\|<\phi(x),$$
$$\|u_x(x,t)-u_x^{(0)}(x,t)\|<\rho(x)+\theta(x),$$
$$\|u_x(x,T)-u_x^{(0)}(x,T)\|<\rho(x)+\theta(x),$$
$$\|u_t(x,t)-u_t^{(0)}(x,t)\|<\phi(x),$$
$$\|u_t(x,T)-u_t^{(0)}(x,T)\|<\phi(x).$$

In view of the equivalence of problems (2.1)-(2.3) and (2.27)-(2.32), Theorem 14 follows from Theorem 13

Due to the equivalence of problems (2.1)-(2.3) and (2.27)-(2.32) from Theorem 13 follows

Theorem 14. *If the conditions of Theorem 13 are satisfied, then the sequence of functions* $\{u^{(k)}(x,t)\}, k=1,2,\ldots,$ *is contained in* $S_1(u^{(0)}(x,[t]),\rho(x),\theta(x),\ \phi(x))$, *converges to* $u^*(x,t)$ *- solution of problem (2.1)-(2.3) in* $S_1(u^{(0)}(x,[t]),\rho(x),$ $\theta(x),\phi(x))$ *and the inequality is valid*

$$\|u^*(x,t)-u^{(k)}(x,t)\|\le\int_0^x c(\xi)\sum_{j=k-1}^{\infty}\frac{1}{j!}\Big(\int_0^{\xi}c(\xi_1)d\xi_1\Big)^j\times$$

$$\times\int_0^{\xi}\max\{1,L_1(\xi_1)\}[c_0(\xi_1)c_1(\xi_1)+c_0(\xi_1)+1]\gamma_v(\xi_1,h)\times$$

$$\times\left\|Q_{\nu,h}(\xi_1,u^{(0)}(\xi_1,[\cdot]),\tilde{v}^{(0)}(\xi_1,[\cdot]),\lambda^{(0)}(\xi_1),w^{(0)}(\xi_1,[\cdot]))\right\|d\xi_1 d\xi,$$

$(x,t)\in\overline{\Omega}$. *Moreover, any solution of problem (2.1)-(2.3) in* $S_1(u^{(0)}(x,[t]),\rho(x),\ \theta(x),\phi(x))$ *isolated.*

2.3 About one algorithm of finding of the "isolated" solution of a semi-periodic boundary value problem for the systems of the nonlinear hyperbolic equations

In subsection 2.3, the method of a parametrization is used to study the system of the form

$$\frac{\partial^2 u}{\partial x\partial t} = f(x,t,u,\frac{\partial u}{\partial x}), \quad (x,t)\in\overline{\Omega}=[0,\omega]\times[0,T], \quad u\in R^n \quad (2.36)$$

with conditions (2.2), (2.3).

To find a solution to the problem (2.36), (2.2), (2.3) efficient algorithms are constructed and sufficient conditions for their convergence are established. These conditions ensure the existence of an isolated solution to the problem under study. Thus, similar to subsection 2.2 entering the function $v(x,t) = \dfrac{\partial u(x,t)}{\partial x}$ and applying the method of a parametrization scheme, the problem (1.36), (2.2), (2.3) we reduce the equivalent boundary value problem with unknown functions $\lambda_r(x)$:

$$\frac{\partial \tilde{v}_r}{\partial t} = f(x,t,u_r,\tilde{v}_r+\lambda_r(x)), \quad (x,t)\in\Omega_r, \qquad (2.37)$$

$$\tilde{v}_r(x,(r-1)h) = 0, \quad x\in[0,\omega], \quad r=\overline{1,N}, \qquad (2.38)$$

$$\lambda_1(x)-\lambda_N(x)-\lim_{t\to T-0}\tilde{v}_N(x,t) = 0, \quad x\in[0,\omega], \qquad (2.39)$$

$$\lambda_s(x)+\lim_{t\to sh-0}\tilde{v}_s(x,t)-\lambda_{s+1}(x) = 0, \quad x\in[0,\omega], s=\overline{1,N-1}, \qquad (2.40)$$

$$u_r(x,t) = \psi(t)+\int_0^x \tilde{v}_r(\xi,t)d\xi+\int_0^x \lambda_r(\xi)d\xi, (x,t)\in\Omega_r, r=\overline{1,N}. \qquad (2.41)$$

Problem (2.37), (2.38) with fixed $\lambda_r(x), u_r(x,t)$, is a one-parameter family of Cauchy problems for systems ordinary differential equations where $x\in[0,\omega]$, and is equivalent to a nonlinear integral equation

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} f(x,\tau,u_r(x,\tau),\tilde{v}_r(x,\tau) + \lambda_r(x))d\tau. \qquad (2.42)$$

Instead $\tilde{v}_r(x,\tau)$ we substitute the corresponding right-hand side (2.42) and repeating this process ν $(\nu = 1,2,...)$ times we obtain

$$\tilde{v}_r(x,t) = \int_{(r-1)h}^{t} f(x,\tau_1,u_r(x,\tau_1), \int_{(r-1)h}^{\tau_1} f(x,\tau_2,u_r(x,\tau_2),...$$

$$...\int_{(r-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_r(x,\tau_\nu),\tilde{v}_r(x,\tau_\nu) + \lambda_r(x))d\tau_\nu +$$

$$... + \lambda_r(x))d\tau_2 + \lambda_r(x))d\tau_1. \qquad (2.43)$$

Hence defining $\lim\limits_{t\to rh-0} \tilde{v}_r(x,t),$ substituting them in (2.39), (2.40) we obtain a system of nonlinear equations regarding $\lambda_r(x)$:

$$\lambda_1(x) - \lambda_N(x) - \int_{(N-1)h}^{Nh} f(x,\tau_1,u_N(x,\tau_1), \int_{(N-1)h}^{\tau_1} f(x,\tau_2,u_N(x,\tau_2),...$$

$$...\int_{(N-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_N(x,\tau_\nu),\tilde{v}_N(x,\tau_\nu) + \lambda_N(x))d\tau_\nu + ...$$

$$... + \lambda_N(x))d\tau_2 + \lambda_N(x))d\tau_1 = 0, \quad x\in[0,\omega],$$

$$\lambda_s(x) + \int_{(s-1)h}^{sh} f(x,\tau_1,u_s(x,\tau_1), \int_{(s-1)h}^{\tau_1} f(x,\tau_2,u_s(x,\tau_2),...$$

$$...\int_{(s-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_s(x,\tau_\nu),\tilde{v}_s(x,\tau_\nu) + \lambda_s(x))d\tau_\nu + ...$$

$$... + \lambda_s(x))d\tau_2 + \lambda_s(x))d\tau_1 - \lambda_{s+1}(x) = 0, \quad x\in[0,\omega], \quad s = \overline{1,N-1},$$

which we write in the form

$$Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x)) = 0. \qquad (2.44)$$

For finding of a system from three functions $\{\lambda_r(x),\tilde{v}_r(x,t),u_r(x,t)\}, r = \overline{1,N},$ we have a vice-system consisting of of equations (2.44), (2.43) and (2.41), defined by function f, to splitting step $h > 0$ and the number of substitutions ν.

Choose a step $h > 0$: $Nh = T$ $(N = 1,2,...)$, vector function $\lambda^{(0)}(x) = (\lambda_1^{(0)}(x),...,\lambda_N^{(0)}(x))' \in C([0,\omega],R^{Nn})$ and suppose that the problem (2.37)-(2.41) when $\lambda_r(x) = \lambda_r^{(0)}(x),$ $r = \overline{1,N},$ has a solution

$$u_r^{(0)}(x,t)\in\tilde{C}(\Omega_r,R^n), \tilde{v}_r^{(0)}(x,t)\in\tilde{C}(\Omega_r,R^n), r = \overline{1,N}.$$

The set of such $\lambda^{(0)}(x)\in C([0,\omega],R^{nN})$ is denoted $G_0(f,x,h),$ and the corresponding $\lambda^{(0)}(x)$ solution system of problems (2.37)-(2.41) by

$$\tilde{v}^{(0)}(x,[t]) = (\tilde{v}_1^{(0)}(x,t),\ldots,\tilde{v}_N^{(0)}(x,t))',$$
$$u^{(0)}(x,[t]) = (u_1^{(0)}(x,t),\ldots,u_N^{(0)}(x,t))'.$$

Taking the function we construct the sets. Taking $\lambda^{(0)}(x) \in G_0(f,x,h)$, $u^{(0)}(x,[t])$, $\tilde{v}^{(0)}(x,[t])$, the function $\rho(x) > 0$, we construct the sets:

$$S(\lambda^{(0)}(x),\rho(x)) = \{(\lambda_1(x),\ldots,\lambda_N(x))' \in C([0,\omega],R^{nN}):$$
$$\|\lambda_r(x) - \lambda_r^{(0)}(x)\| < \rho(x), r = \overline{1,N}\},$$
$$S(\lambda^{(0)}(x) + \tilde{v}^{(0)}(x,[t]),\rho(x)) = \{\lambda(x) + \tilde{v}(x,[t]),$$
$$\lambda(x) = (\lambda_1(x),\ldots,\lambda_N(x))' \in C([0,\omega],R^{nN}), \quad (\tilde{v}_1(x,t),\ldots,\tilde{v}_N(x,t))',$$
$$\tilde{v}_r(x,t) \in \tilde{C}(\Omega_r,R^n) : \|\lambda_r(x) - \lambda_r^{(0)}(x)\| +$$
$$+ \|\tilde{v}_r(x,t) - \tilde{v}_r^{(0)}(x,t)\| < \rho(x), r = \overline{1,N}\},$$
$$S(u^{(0)}(x,[t]),\omega\rho(x)) = \{(u_1(x,t),\ldots,u_N(x,t))', \quad u_r(x,t) \in \tilde{C}(\Omega_r,R^n):$$
$$\|u_r(x,t) - u_r^{(0)}(x,t)\| < \omega\rho(x), \quad (x,t) \in \Omega_r, \quad r = \overline{1,N}\},$$
$$G_0(\omega,\rho(x)) = \{(x,t,u,v) : (x,t) \in \overline{\Omega},$$
$$\|u - u_r^{(0)}(x,t)\| < \omega\rho(x), \quad (x,t) \in \Omega_r,$$
$$r = \overline{1,N}, \quad \|u - \lim_{t\to Nh-0} u_N^{(0)}(x,t)\| < \omega\rho(x), \quad t = T, \quad x \in [0,\omega],$$
$$\|v - \lambda_r^{(0)}(x) - \tilde{v}_r^{(0)}(x,t)\| < \rho(x), \quad (x,t) \in \Omega_r, \quad r = \overline{1,N},$$
$$\|v - \lambda_N^{(0)}(x) - \lim_{t\to Nh-0} \tilde{v}_N^{(0)}(x,t)\| < \rho(x), \quad t = T, \quad x \in [0,\omega]\}.$$

Through $D_0(f, L_1(x), L_2(x), x, h)$ denote the set $(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]),$ $u^{(0)}(x,[t]), \omega, \rho(x))$ at which the function $f(x,t,u,v)$ in $G_0(\omega,\rho(x))$ has continuous private derivatives $f_v'(x,t,u,v)$, $f_u'(x,t,u,v)$ and $\| f_v'(x,t,u,v)\| \le L_1(x)$, $\| f_u'(x,t,u,v)\| \le L_2(x)$.

On system $\{\lambda_r(x), \tilde{v}_r(x,t), u_r(x,t)\}$, $r = \overline{1,N}$, we make up a triple $\{\lambda(x), \tilde{v}(x,[t]),$ $u(x,[t])\}$, where
$$\lambda(x) = (\lambda_1(x),\ldots,\lambda_N(x))', \quad \tilde{v}(x,[t]) = (\tilde{v}_1(x,t),\ldots,\tilde{v}_N(x,t))',$$
$$u(x,[t]) = (u_1(x,t),\ldots,u_N(x,t))'.$$

Assuming the existence $\lambda^{(0)}(x) \in G(f, x, h)$, for the initial approximation of the problem (2.37)-(2.41) we take the system of triples $(\lambda_r^{(0)}(x), u_r^{(0)}(x,t), \tilde{v}_r^{(0)}(x,t))$, $r = \overline{1, N}$, and we construct successive approximations according to the following algorithm:

Step 1. Substituting $u_r^{(0)}(x,t), \tilde{v}_r^{(0)}(x,t), r = \overline{1, N}$, into (2.44) from a system of functional equations

$$Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda(x)) = 0$$

define $\lambda_r^{(1)}(x)$, $r = \overline{1, N}$. Substituting in (2.43) instead of $u_r(x,t)$, $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $u_r^{(0)}(x,t)$, $\tilde{v}_r^{(0)}(x,t)$, $\lambda_r^{(1)}(x)$, $r = \overline{1, N}$, we obtain $\{\tilde{v}_r^{(1)}(x,t)\}$, $r = \overline{1, N}$. Functions $u_r^{(1)}(x,t)$ determined from ratios (2.41), where

$$\lambda_r(x) = \lambda_r^{(1)}(x), \quad \tilde{v}_r(x,t) = \tilde{v}_r^{(1)}(x,t), \quad r = \overline{1, N}.$$

Step 2. Function $\lambda^{(2)}(x) = (\lambda_1^{(2)}(x), \lambda_2^{(2)}(x), \ldots, \lambda_N^{(2)}(x))'$ we define as the solution of the system of equations (2.44), where

$u_r(x,t) = u_r^{(1)}(x,t), \quad \tilde{v}_r(x,t) = \tilde{v}_r^{(1)}(x,t), \quad r = \overline{1, N}.$

Again substituting in (2.43) instead of $u_r(x,t)$, $\tilde{v}_r(x,t)$, $\lambda_r(x)$, respectively, $u_r^{(1)}(x,t)$, $\tilde{v}_r^{(1)}(x,t)$, $\lambda_r^{(2)}(x), r = \overline{1, N}$, we obtain $\{\tilde{v}_r^{(2)}(x,t)\}, r = \overline{1, N}$. Functions $u_r^{(2)}(x,t)$ are defined from relations (2.41), where

$$\lambda_r(x) = \lambda_r^{(2)}(x), \quad \tilde{v}_r(x,t) = \tilde{v}_r^{(2)}(x,t), \quad r = \overline{1, N}.$$

Continuing the process on k step we obtain system of triples $\{\lambda_r^{(k)}(x), \tilde{v}_r^{(k)}(x,t), u_r^{(k)}(x,t)\}, r = \overline{1, N}$.

Sufficient conditions for the feasibility, convergence of the algorithm and the existence of a solution to a multicharacteristic boundary value problem with functional parameters (2.37)-(2.41) establish

Theorem 15. *Let exist* $h > 0 : Nh = T, (N = 1,2,\ldots)$, $\nu \in N$, $(\lambda^{(0)}(x), \tilde{v}^{(0)}(x,[t]), u^{(0)}(x,[t]), \omega, \rho(x)) \in D_0(f, L_1(x), L_2(x), x, h)$, *at which the Jacobi matrix* $\dfrac{\partial Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x))}{\partial\lambda}$ *is invertible for all* $(x,u(x,[\cdot]),\ \tilde{v}(x,[\cdot]),\lambda(x))$, *where* $x\in[0,\omega],(\lambda(x)+\tilde{v}(x,[t]),u(x,[t]))\in S(\lambda^{(0)}(x)+\tilde{v}^{(0)}(x,[t]),\rho(x))\times S(u^{(0)}(x,[t]),\omega\rho(x))$ *and the following inequalities are executed:*

1) $\left\|\left[\dfrac{\partial Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x))}{\partial\lambda}\right]^{-1}\right\|\le\gamma_\nu(x,h),$

2) $q_\nu(x,h)=\dfrac{(L_1(x)h)^\nu}{\nu!}[1+\gamma_\nu(x,h)\sum\limits_{j=0}^{\nu}\dfrac{(L_1(x)h)^j}{j!}]\le\mu<\dfrac{1}{2},$

3) $$\left[\frac{(2\mu)^2}{1-2\mu}\frac{(\frac{\tilde{\chi}\omega}{\mu})^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}+1\right]\sum_{j=0}^{\nu}\frac{(\tilde{L}_1h)^j}{j!}\max_{x\in[0,\omega]}\gamma_\nu(x,h)\times$$

$$\times\left\|Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\|<\rho(x),$$

where $\tilde{L}_1=\max\limits_{x\in[0,\omega]}L_1(x),\ \ \tilde{\chi}=\max\limits_{x\in[0,\omega]}\chi(x),$

$$\chi(x)=\sum_{j=0}^{\nu-1}\frac{(L_1(x)h)^j}{j!}L_2(x)h[1+\gamma_\nu(x,h)\sum_{j=0}^{\nu}\frac{(L_1(x)h)^j}{j!}].$$

Then the sequence of couple $(\lambda^{(k)}(x)+\tilde{v}^{(k)}(x,[t]),u^{(k)}(x,[t]))$, $k=0,1,\ldots,$ *determined by the algorithm is contained in* $S(\lambda^{(0)}(x)+\tilde{v}^{(0)}(x,[t]),\rho(x))\times S(u^{(0)}(x,[t]),\omega\rho(x))$, *converges to* $(\lambda^{*}(x)+\tilde{v}^{*}(x,[t]),u^{*}(x,[t]))$ *- solution of the problem (2.37)-(2.41) and the estimates are valid:*

a)

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{*}(x)-\lambda_r^{(k)}(x)\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{*}(x,t)-\tilde{v}_r^{(k)}(x,t)\right\|\le$$

$$\le\left[\frac{(2\mu)^{k+1}}{1-2\mu}\frac{(\frac{\tilde{\chi}\omega}{\mu})^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}+1\right]\sum_{j=0}^{\nu}\frac{(\tilde{L}_1h)^j}{j!}\max_{x\in[0,\omega]}\gamma_\nu(x,h)\times$$

$$\times\max_{x\in[0,\omega]}\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\|,$$

where $\left[\dfrac{\chi\omega}{\mu}\right]$ *- the largest integer not exceeding* $\dfrac{\tilde{\chi}\omega}{\mu}$.

b) $\max\limits_{r=\overline{1,N}} \sup\limits_{t\in[(r-1)h,rh)} \left\| u_r^{\star}(x,t) - u_r^{(k)}(x,t) \right\| \le$

$$\le \int_0^x \left(\max_{r=\overline{1,N}} \left\| \lambda_r^{\star}(\xi) - \lambda_r^{(k)}(\xi) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{\star}(\xi,t) - \tilde{v}_r^{(k)}(\xi,t) \right\| \right) d\xi.$$

And any solution $u(x,t),\, \tilde{v}(x,t),\, \lambda(x)$ *problems (2.37)-(2.41), where* $\tilde{v}(x,t) + \lambda(x) \in S(\lambda^{(0)}(x) + \tilde{v}^{(0)}(x,[t]), \rho(x)),$ $u(x,t) \in S(u^{(0)}(x,[t]), \omega\rho(x))$ *isolated.*

Proof. Believing that $u_r(x,t) = u_r^{(0)}(x,t), \tilde{v}_r(x,t) = \tilde{v}_r^{(0)}(x,t)$ and solving equations (2.44), we find

$$\lambda^{(1)}(x) = (\lambda_1^{(1)}(x),\, \lambda_2^{(1)}(x), \ldots, \lambda_N^{(1)}(x))'.$$

By owing of the inequality 3) of the Theorem, with fixed $\overline{x} \in [0,\omega],$ exist a number $\varepsilon_0 > 0,$ satisfying inequalities

$$\varepsilon_0 \gamma_\nu(\overline{x},h) < 1/2,$$

$$\frac{\gamma_\nu(\overline{x},h)}{1 - \varepsilon_0 \gamma_\nu(\overline{x},h)} \left\| Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x},[\cdot]), \tilde{v}^{(0)}(\overline{x},[\cdot]), \lambda^{(0)}(\overline{x})) \right\| < \rho(\overline{x}),$$

and matrix of Jacobi $\dfrac{\partial Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x},[\cdot]), \tilde{v}^{(0)}(\overline{x},[\cdot]), \lambda^{(0)}(\overline{x}))}{\partial \lambda}$

uniformly continuous in $S(\lambda^{(0)}(\overline{x}), \rho(\overline{x}))$ and for $\varepsilon_0 > 0$ find

$\delta_0 \in (0, \dfrac{\rho(\overline{x})}{2})$ such that

$$\left\| \frac{\partial Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x},[\cdot]), \tilde{v}^{(0)}(\overline{x},[\cdot]), \lambda(\overline{x}))}{\partial \lambda} - \frac{\partial Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x},[\cdot]), \tilde{v}^{(0)}(\overline{x},[\cdot]),}{\partial \lambda} \right.$$

when for $\lambda(\overline{x}), \tilde{\lambda}(\overline{x}) \in S(\lambda^{(0)}(\overline{x}), \rho(\overline{x})),$ the inequality is valid

$$\left\| \lambda(\overline{x}) - \tilde{\lambda}(\overline{x}) \right\| < \delta_0, \quad \overline{x} \in [0,\omega].$$

Choosing a number

$$\alpha \ge \alpha_0 = \max\left\{ 1, \frac{\gamma_\nu(\overline{x},h) \left\| Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x},[\cdot]), \tilde{v}^{(0)}(\overline{x},[\cdot]), \lambda^{(0)}(\overline{x})) \right\|}{\delta_0} \right\}$$

let's construct a repetitive process:

$$\lambda^{(1,0)}(\overline{x}) = \lambda^{(0)}(\overline{x}),$$

$$\lambda^{(1,m+1)}(\bar{x}) = \lambda^{(1,m)}(\bar{x}) -$$

$$- \frac{1}{\alpha}\left[\frac{\partial Q_{\nu,h}(\bar{x}, u^{(0)}(\bar{x},[\cdot]), \tilde{v}^{(0)}(\bar{x},[\cdot]), \lambda^{(1,m)}(\bar{x}))}{\partial \lambda}\right]^{-1} \times$$

$$\times Q_{\nu,h}(\bar{x}, u^{(0)}(\bar{x},[\cdot]), \tilde{v}^{(0)}(\bar{x},[\cdot]), \lambda^{(1,m)}(\bar{x})), \bar{x} \in [0,\omega], m = 0,1,2,....$$

(2.45)

By Theorem 1 of [87] the iterative process (2.45) converges to $\lambda^{(1)}(\bar{x})$ - isolated solution of the equation

$$Q_{\nu,h}(\bar{x}, u^{(0)}(\bar{x},[\cdot]), \tilde{v}^{(0)}(\bar{x},[\cdot]), \lambda(\bar{x})) = 0$$

in $S(\lambda^{(0)}(\bar{x}), \rho(\bar{x}))$ and

$$\left\|\lambda^{(1)}(\bar{x}) - \lambda^{(0)}(\bar{x})\right\| \le$$

$$\le \gamma_\nu(\bar{x}, h)\left\|Q_{\nu,h}(\bar{x}, u^{(0)}(\bar{x},[\cdot]), \tilde{v}^{(0)}(\bar{x},[\cdot]), \lambda^{(0)}(\bar{x}))\right\| < \rho(\bar{x}). \tag{2.46}$$

Due to the arbitrariness of $\bar{x}$ estimate (2.46) is valid for all $x \in [0,\omega]$.

Functions $\tilde{v}_r^{(1)}(x,t), r = \overline{1,N}$, determined from ratios

$$\tilde{v}_r^{(1)}(x,t) = \int\limits_{(r-1)h}^{t} f(x,\tau_1,u_r^{(0)}(x,\tau_1),...$$

$$...\int\limits_{(r-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,u_r^{(0)}(x,\tau_\nu),\tilde{v}_r^{(0)}(x,\tau_\nu) +$$

$$+\lambda_r^{(1)}(x))d\tau_\nu + ... + \lambda_r^{(1)}(x))d\tau_1, r = \overline{1,N}.$$

Then

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\|\tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t)\right\| \le$$

$$\le \sum_{j=1}^{\nu} \frac{(L_1(x)h)^j}{j!} \max_{r=\overline{1,N}} \left\|\lambda_r^{(1)}(x) - \lambda_r^{(0)}(x)\right\|. \tag{2.47}$$

It follows that

$$\Delta^{(1)}(x) \le \sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!} \max_{r=\overline{1,N}} \left\|\lambda_r^{(1)}(x) - \lambda_r^{(0)}(x)\right\| \le$$

$$\le \sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!} \gamma_\nu(x,h)\left\|Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x))\right\| < \rho(x).(2$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\|u_r^{(1)}(x,t) - u_r^{(0)}(x,t)\right\| \le \int\limits_0^x \max_{r=\overline{1,N}} \left\|\lambda_r^{(1)}(\xi) - \lambda_r^{(0)}(\xi)\right\| d\xi +$$

$$+\int_0^x \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\|\tilde{v}_r^{(1)}(\xi,t)-\tilde{v}_r^{(0)}(\xi,t)\right\| d\xi < \omega\rho(x).$$

From estimates it is visible that a system

$$(\lambda^{(1)}(x)+\tilde{v}^{(1)}(x,[t]))\in S(\lambda^{(0)}(x)+\tilde{v}^{(0)}(x,[t]),\ \rho(x)),$$

$$u^{(1)}(x,[t])\in S(u^{(0)}(x,[t]),\omega\rho(x)).$$

From the structure of the operator $Q_{\nu,h}(x,u^{(1)}(x,[\cdot]),\tilde{v}^{(1)}(x,[\cdot]),\lambda(x))$ and equality

$$Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(1)}(x))=0$$

follows that

$$\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,u^{(1)}(x,[\cdot]),\tilde{v}^{(1)}(x,[\cdot]),\lambda^{(1)}(x))\right\| \le$$

$$\le \gamma_\nu(x,h)\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \Bigg[\int_{(r-1)h}^{rh} L_2(x)\left\|u_r^{(1)}(x,\tau_1)-u_r^{(0)}(x,\tau_1)\right\|d\tau_1 +$$

$$+\int_{(r-1)h}^{rh} L_1(x)\int_{(r-1)h}^{\tau_1} L_2(x)\left\|u_r^{(1)}(x,\tau_2)-u_r^{(0)}(x,\tau_2)\right\|d\tau_2\tau_1 + \ldots +$$

$$+\int_{(r-1)h}^{rh} L_1(x)\int_{(r-1)h}^{\tau_1} L_1(x)\ldots\int_{(r-1)h}^{\tau_{\nu-1}} L_2(x)\left\|u_r^{(1)}(x,\tau_\nu)-u_r^{(0)}(x,\tau_\nu)\right\|d\tau_\nu\ldots d\tau_2 d\tau_1 +$$

$$+\int_{(r-1)h}^{rh} L_1(x)\int_{(r-1)h}^{\tau_1} L_1(x)\ldots\int_{(r-1)h}^{\tau_{\nu-1}} L_1(x)\left\|\tilde{v}_r^{(1)}(x,\tau_\nu)-\tilde{v}_r^{(0)}(x,\tau_\nu)\right\|d\tau_\nu\ldots d\tau_2 d\tau_1\Bigg] \le$$

$$\le \gamma_\nu(x,h)\sum_{j=0}^{\nu-1}\frac{(L_1(x)h)^j}{j!}L_2(x)h\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\|u_r^{(1)}(x,t)-u_r^{(0)}(x,t)\right\| +$$

$$+\gamma_\nu(x,h)\frac{(L_1(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\|\tilde{v}_r^{(1)}(x,t)-\tilde{v}_r^{(0)}(x,t)\right\| \le$$

$$\le \chi(x)\int_0^x \Delta^{(1)}(\xi)d\xi + q_\nu(x,h)\Delta^{(1)}(x).$$

Thus

$$\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,u^{(1)}(x,[\cdot]),\tilde{v}^{(1)}(x,[\cdot]),\lambda^{(1)}(x))\right\| \le$$

$$\le \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}(2\mu)^2\max_{x\in[0,\omega]}\Delta^{(1)}(x). \qquad (2.49)$$

Choose number $\tilde{\varepsilon}>0$, satisfying for all $\bar{x}\in[0,\omega]$, inequality

$$\left[\frac{(2\mu)^2}{1-2\mu}\cdot\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}+1\right]\sum_{j=0}^{\nu}\frac{(L_1(\bar{x})h)^j}{j!}\times$$

$$\times\gamma_\nu(\bar{x},h)\left\|Q_{\nu,h}(\bar{x},u^{(0)}(\bar{x},[\cdot]),\tilde{v}^{(0)}(\bar{x},[\cdot]),\lambda^{(0)}(\bar{x}))\right\| + \tilde{\varepsilon} < \rho(\bar{x}).$$

If $\lambda(\bar{x}) \in S(\lambda^{(1)}(\bar{x}), \rho^{(1)}(\bar{x}) + \tilde{\varepsilon})$, where

$$\rho^{(1)}(\bar{x}) = \gamma_{\nu}(\bar{x}, h) \left\| Q_{\nu,h}(\bar{x}, u^{(1)}(\bar{x},[\cdot]), \tilde{v}^{(1)}(\bar{x},[\cdot]), \lambda^{(1)}(\bar{x})) \right\|,$$

then, by owing of inequalities 2), 3) of the theorem and (2.47), (2.49), the estimate

$$\max_{r=\overline{1,N}} \left\| \lambda_r(\bar{x}) - \lambda_r^{(0)}(\bar{x}) \right\| \leq$$

$$\leq \max_{r=\overline{1,N}} \left\| \lambda_r(\bar{x}) - \lambda_r^{(0)}(\bar{x}) \right\| + \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \tilde{v}_r(\bar{x}, t) - \tilde{v}_r^{(0)}(\bar{x}, t) \right\| \leq$$

$$\leq \max_{r=\overline{1,N}} \left\| \lambda_r(\bar{x}) - \lambda_r^{(1)}(\bar{x}) \right\| + \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \tilde{v}_r(\bar{x}, t) - \tilde{v}_r^{(1)}(\bar{x}, t) \right\| +$$

$$+ \max_{r=\overline{1,N}} \left\| \lambda_r^{(1)}(\bar{x}) - \lambda_r^{(0)}(\bar{x}) \right\| + \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \tilde{v}_r^{(1)}(\bar{x}, t) - \tilde{v}_r^{(0)}(\bar{x}, t) \right\| \leq$$

$$\leq (2\beta)^2 \frac{\left(\frac{\tilde{\chi}\omega}{\mu} \right)^{\left[\frac{\tilde{\chi}\omega}{\mu} \right]}}{\left[\frac{\tilde{\chi}\omega}{\mu} \right]!} \Delta^{(1)}(\bar{x}) + \Delta^{(1)}(\bar{x}) + \tilde{\varepsilon} \leq$$

$$\leq [(2\mu)^2 \frac{\left(\frac{\tilde{\chi}\omega}{\mu} \right)^{\left[\frac{\tilde{\chi}\omega}{\mu} \right]}}{\left[\frac{\tilde{\chi}\omega}{\mu} \right]!} + 1] \Delta^{(1)}(\bar{x}) + \tilde{\varepsilon} \leq$$

$$\leq \left[\frac{(2\mu)^2}{1 - 2\mu} \cdot \frac{\left(\frac{\tilde{\chi}\omega}{\mu} \right)^{\left[\frac{\tilde{\chi}\omega}{\mu} \right]}}{\left[\frac{\tilde{\chi}\omega}{\mu} \right]!} + 1 \right] \sum_{j=0}^{\nu} \frac{(L_1(\bar{x})h)^j}{j!} \times$$

$$\times \gamma_{\nu}(\bar{x}, h) \left\| Q_{\nu,h}(\bar{x}, u^{(0)}(\bar{x},[\cdot]), \tilde{v}^{(0)}(\bar{x},[\cdot]), \lambda^{(0)}(\bar{x})) \right\| + \tilde{\varepsilon} < \rho(\bar{x}),$$

i.e. $S(\lambda^{(1)}(\bar{x}), \rho^{(1)}(\bar{x}) + \tilde{\varepsilon}) \subset S(\lambda^{(0)}(\bar{x}), \rho(\bar{x}))$. From the conditions of the theorem it follows that the operator $Q_{\nu,h}(\bar{x}, u^{(1)}(\bar{x},[\cdot]), \tilde{v}^{(1)}(\bar{x},[\cdot]), \lambda(\bar{x}))$ to $S(\lambda^{(1)}(\bar{x}), \rho^{(1)}(\bar{x}) + \tilde{\varepsilon})$ satisfies all the conditions of the Theorem 1 [87]. Therefore, the iterative process:

$$\lambda^{(2,0)}(\bar{x}) = \lambda^{(1)}(\bar{x}),$$

$$\lambda^{(2,m+1)}(\bar{x}) =$$

$$=\lambda^{(2,m)}(\bar{x}) - \frac{1}{\alpha}\left[\frac{\partial Q_{\nu,h}(\bar{x},u^{(1)}(\bar{x},[\cdot]),\tilde{v}^{(1)}(\bar{x},[\cdot]),\lambda^{(2,m)}(\bar{x}))}{\partial\lambda}\right]^{-1}\times$$

$$\times Q_{\nu,h}(\bar{x},u^{(1)}(\bar{x},[\cdot]),\tilde{v}^{(1)}(\bar{x},[\cdot]),\lambda^{(2,m)}(\bar{x})),\quad \bar{x}\in[0,\omega],\quad m=0,1,2,...,$$

converges to $\lambda^{(2)}(\bar{x})$ - isolated solution of the equation

$$Q_{\nu,h}(\bar{x},u^{(1)}(\bar{x},[\cdot]),\tilde{v}^{(1)}(\bar{x},[\cdot]),\lambda(\bar{x}))=0$$

in the set $S(\lambda^{(1)}(\bar{x}),\rho_1(\bar{x})+\tilde{\varepsilon}),\ \bar{x}\in[0,\omega]$ and

$$\left\|\lambda^{(2)}(\bar{x}) - \lambda^{(1)}(\bar{x})\right\|\le\gamma_\nu(\bar{x},h)\left\|Q_{\nu,h}(\bar{x},u^{(1)}(\bar{x},[\cdot]),\tilde{v}^{(1)}(\bar{x},[\cdot]),\lambda^{(1)}(\bar{x}))\right\|,$$

from here, from (2.48), by owing of conditions 2), 3) of Theorem 1, it follows that

$$\left\|\lambda^{(2)}(\bar{x}) - \lambda^{(0)}(\bar{x})\right\|\le\left[\frac{(2\mu)^2}{1-2\mu}\cdot\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}+1\right]\sum_{j=0}^{\nu}\frac{(L_1(\bar{x})h)^j}{j!}\times$$

$$\times\gamma_\nu(\bar{x},h)\left\|Q_{\nu,h}(\bar{x},u^{(0)}(\bar{x},[\cdot]),\tilde{v}^{(0)}(\bar{x},[\cdot]),\lambda^{(0)}(\bar{x}))\right\|<\rho(\bar{x}).\qquad(2.50)$$

By owing of arbitrariness $\bar{x}$ assessment (2.50) is valid for all $x\in[0,\omega]$. It is not difficult to establish that $\lambda^{(2)}(x)$ is continuous on $[0,\omega]$.

Functions $\tilde{v}_r^{(2)}(x,t), r=\overline{1,N}$, determined from ratios

$$\tilde{v}_r^{(2)}(x,t)=\int_{(r-1)h}^{t}f(x,\tau_1,u_r^{(1)}(x,\tau_1),...$$

$$...\int_{(r-1)h}^{\tau_{\nu-1}}f(x,\tau_\nu,u_r^{(1)}(x,\tau_\nu),\tilde{v}_r^{(1)}(x,\tau_\nu)+\lambda_r^{(2)}(x))d\tau_\nu+...+\lambda_r^{(2)}(x))d\tau_1,$$

$r=\overline{1,N}$. With assumptions of the theorem there are estimates

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(2)}(x,t) - \tilde{v}_r^{(1)}(x,t)\right\|\le$$

$$\le\sum_{j=0}^{\nu-1}\frac{(L_1(x)h)^j}{j!}L_2(x)h\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(1)}(x,t) - u_r^{(0)}(x,t)\right\|+$$

$$+\frac{(L_1(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t)\right\|+$$

$$+\sum_{j=1}^{\nu}\frac{(L_1(x)h)^j}{j!}\max_{r=\overline{1,N}}\left\|\lambda_r^{(2)}(x) - \lambda_r^{(1)}(x)\right\|\le$$

$$\leq \sum_{j=0}^{\nu-1} \frac{(L_1(x)h)^j}{j!} L_2(x)h[1+\gamma_\nu(x,h)\sum_{j=1}^{\nu} \frac{(L_1(x)h)^j}{j!}] \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(1)}(x,t) - u_r^{(0)}(x,t) \right\| +$$

$$+ \frac{(L_1(x)h)^\nu}{\nu!} \left[1 + \gamma_\nu(x,h) \sum_{j=1}^{\nu} \frac{(L_1(x)h)^j}{j!} \right] \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t). \right\|$$ Thus,

$$\Delta^{(2)}(x) = \max_{r=\overline{1,N}} \left\| \lambda_r^{(2)}(x) - \lambda_r^{(1)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(2)}(x,t) - \tilde{v}_r^{(1)}(x,t) \right\| \leq$$

$$\leq \sum_{j=0}^{\nu-1} \frac{(L_1(x)h)^j}{j!} L_2(x)h[1+\gamma_\nu(x,h)\sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!}] \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(1)}(x,t) - u_r^{(0)}(x,t) \right\| +$$

$$+ \frac{(L_1(x)h)^\nu}{\nu!} \left[1 + \gamma_\nu(x,h) \sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!} \right] \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\| \leq$$

$$\leq \chi(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(1)}(x,t) - u_r^{(0)}(x,t) \right\| +$$

$$+ q_\nu(x,h) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(1)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\| \leq$$

$$\leq \frac{\left(\frac{\tilde{\chi}\omega}{\mu} \right)^{\left[\frac{\tilde{\chi}\omega}{\mu} \right]}}{\left[\frac{\tilde{\chi}\omega}{\mu} \right]!} (2\mu)^2 \max_{x\in[0,\omega]} \Delta^{(1)}(x).$$

Then there are inequalities

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(2)}(x,t) - u_r^{(1)}(x,t) \right\| \leq$$

$$\leq \int_0^x \left(\max_{r=\overline{1,N}} \left\| \lambda_r^{(2)}(\xi) - \lambda_r^{(1)}(\xi) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(2)}(\xi,t) - \tilde{v}_r^{(1)}(\xi,t) \right\| \right) d\xi,$$

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(2)}(x) - \lambda_r^{(0)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(2)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\| \leq$$

$$\le \left[(2\mu)^2 \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} + 1 \right] \max_{x\in[0,\omega]} \Delta^{(1)}(x) \le$$

$$\le \left[\frac{(2\mu)^2}{1-2\mu} \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} + 1 \right] \sum_{j=0}^{\nu} \frac{(\tilde{L}_1 h)^j}{j!} \times$$

$$\times \max_{x\in[0,\omega]} \gamma_\nu(x,h) \left\| Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), v^{(0)}(x,[\cdot]), \lambda^{(0)}(x)) \right\| < \rho(x).$$

Assuming that are defined $\lambda_r^{(k-1)}(x), \tilde{v}_r^{(k-1)}(x,[t]), u_r^{(k-1)}(x,[t]),$ where $(\lambda_r^{(k-1)}(x) + \tilde{v}_r^{(k-1)}(x,[t]), u_r^{(k-1)}(x,[t])) \in S(\lambda_r^{(0)}(x) + \tilde{v}_r^{(0)}(x,[t]), \rho(x)) \times$ $\times S(u_r^{(0)}(x,[t]), \omega\rho(x)),$ and estimates are established

$$\Delta^{(k-1)}(x) = \max_{r=\overline{1,N}} \left\| \lambda_r^{(k-1)}(x) - \lambda_r^{(k-2)}(x) \right\| + \qquad (2.51)$$

$$+ \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k-1)}(x,t) - \tilde{v}_r^{(k-2)}(x,t) \right\| \le$$

$$\le (2\mu)^{k-1} \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} \max_{x\in[0,\omega]} \Delta^{(1)}(x),$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k-1)}(x,t) - u_r^{(k-2)}(x,t) \right\| \le \int_0^x \Delta^{(k-1)}(\xi)d\xi. \qquad (2.52)$$

k an approximation in the functional parameter $\lambda^{(k)}(x),$ find from the equation

$$Q_{\nu,h}(x, u^{(k-1)}(x,[\cdot]), \tilde{v}^{(k-1)}(x,[\cdot]), \lambda(x)) = 0.$$

Using (2.51), (2.52) and equality

$$Q_{\nu,h}(x, u^{(k-2)}(x,[\cdot]), \tilde{v}^{(k-2)}(x,[\cdot]), \lambda^{(k-1)}(x)) = 0,$$

install the inequality is valid

$$\gamma_\nu(x,h) \left\| Q_{\nu,h}(x, u^{(k-1)}(x,[\cdot]), \tilde{v}^{(k-1)}(x,[\cdot]), \lambda^{(k-1)}(x)) \right\| \le$$

$$\le \gamma_\nu(x,h) \sum_{j=0}^{\nu-1} \frac{(L_1(x)h)^j}{j!} L_2(x)h \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k-1)}(x,t) - u_r^{(k-2)}(x,t) \right\| +$$

$$+\gamma_\nu(x,h)\frac{(L_1(x)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(k-1)}(x,t)-\tilde{v}_r^{(k-2)}(x,t)\right\|\le$$

$$\le\chi(x)\int_0^x\Delta^{(k-1)}(\xi)d\xi+q_\nu(x,h)\Delta^{(k-1)}(x)\le$$

$$\le(2\mu)^k\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\max_{x\in[0,\omega]}\Delta^{(1)}(x). \qquad (2.53)$$

Take

$$\rho^{(k-1)}(\bar{x})=\gamma_\nu(\bar{x},h)\left\|Q_{\nu,h}(\bar{x},u^{(k-1)}(\bar{x},[\cdot]),\tilde{v}^{(k-1)}(\bar{x},[\cdot]),\lambda^{(k-1)}(\bar{x}))\right\|,$$

at $\bar{x}\in[0,\omega]$ and show that

$$S(\lambda^{(k-1)}(\bar{x}),\rho^{(k-1)}(\bar{x})+\tilde{\varepsilon})\subset S(\lambda^{(0)}(\bar{x}),\rho(\bar{x})).$$

Really, in view of inequalities (2.51), (2.53), 3)

$$\max_{r=\overline{1,N}}\left\|\lambda_r(\bar{x})-\lambda_r^{(0)}(\bar{x})\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r(\bar{x},t)-\tilde{v}_r^{(0)}(\bar{x},t)\right\|\le$$

$$\le\max_{r=\overline{1,N}}\left\|\lambda_r(\bar{x})-\lambda_r^{(k-1)}(\bar{x})\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r(\bar{x},t)-\tilde{v}_r^{(k-1)}(\bar{x},t)\right\|+$$

$$+\max_{r=\overline{1,N}}\left\|\lambda_r^{(k-1)}(\bar{x})-\lambda_r^{(k-2)}(\bar{x})\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(k-1)}(\bar{x},t)-\tilde{v}_r^{(k-2)}(\bar{x},t)\right\|$$

$$+\ldots+\max_{r=\overline{1,N}}\left\|\lambda_r^{(1)}(\bar{x})-\lambda_r^{(0)}(\bar{x})\right\|+\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(1)}(\bar{x},t)-\tilde{v}_r^{(0)}(\bar{x},t)\right\|\le$$

$$\le\rho^{(k-1)}(\bar{x})+\tilde{\varepsilon}+(2\mu)^{k-1}\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\Delta^{(1)}(\bar{x})+$$

$$+(2\mu)^{k-2}\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\Delta^{(1)}(\bar{x})+\ldots+\Delta^{(1)}(\bar{x})\le$$

$$\leq [\frac{(2\mu)^2}{1-2\mu} \frac{\left(\frac{\widetilde{\chi}\omega}{\mu}\right)^{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]!} + 1]\Delta^{(1)}(\overline{x}) + \widetilde{\varepsilon} \leq \leq [\frac{(2\mu)^2}{1-2\mu} \frac{\left(\frac{\widetilde{\chi}\omega}{\mu}\right)^{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]!} + 1]\sum_{j=0}^{\nu} \frac{(L(\overline{x})_1 h)^j}{j!} \times$$

$$\times \gamma_\nu(\overline{x}, h) \left\| Q_{\nu,h}(\overline{x}, u^{(0)}(\overline{x}, [\cdot]), \widetilde{v}^{(0)}(\overline{x}, [\cdot]), \lambda^{(0)}(\overline{x})) \right\| + \widetilde{\varepsilon} < \rho(\overline{x}).$$

Because $Q_{\nu,h}(x, u^{(k-1)}(\overline{x}, [\cdot]), \widetilde{v}^{(k-1)}(\overline{x}, [\cdot]), \lambda(\overline{x}))$ in $S(\lambda^{(k-1)}(\overline{x}), \rho^{(k-1)}(\overline{x}) + \widetilde{\varepsilon})$ satisfies all the conditions of Theorem 1 of [87], then there exists $\lambda^{(k)}(\overline{x})$ - solution of the equation

$$Q_{\nu,h}(x, u^{(k-1)}(\overline{x}, [\cdot]), \widetilde{v}^{(k-1)}(\overline{x}, [\cdot]), \lambda(\overline{x})) = 0$$

in $S(\lambda^{(k-1)}(\overline{x}), \rho_{k-1}(\overline{x}) + \widetilde{\varepsilon})$ and estimate is valid

$$\left\| \lambda^{(k)}(\overline{x}) - \lambda^{(k-1)}(\overline{x}) \right\| \leq$$

$$\leq \gamma_\nu(\overline{x}, h) \left\| Q_{\nu,h}(\overline{x}, u^{(k-1)}(\overline{x}, [\cdot]), \widetilde{v}^{(k-1)}(\overline{x}, [\cdot]), \lambda^{(k-1)}(\overline{x})) \right\|. \tag{2.54}$$

By owing of arbitrariness $\overline{x}$ estimate (2.54) is valid for all $x \in [0, \omega]$. It is not difficult to establish that $\lambda^{(k)}(x)$ is continuous on $[0, \omega]$. At assumptions of the theorem there are estimates

$$\Delta^{(k)}(x) = \max_{r=\overline{1,N}} \left\| \lambda_r^{(k)}(x) - \lambda_r^{(k-1)}(x) \right\| +$$

$$+ \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \widetilde{v}_r^{(k)}(x,t) - \widetilde{v}_r^{(k-1)}(x,t) \right\| \leq$$

$$\leq (2\mu)^k \frac{\left(\frac{\widetilde{\chi}\omega}{\mu}\right)^{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\widetilde{\chi}\omega}{\mu}\right]!} \max_{x \in [0,\omega]} \Delta^{(1)}(x),$$

$$\max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| \leq \int_0^x \Delta^{(k)}(\xi) d\xi.$$

For systems of differences

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(x) - \lambda_r^{(k)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| \widetilde{v}_r^{(k+1)}(x,t) - \widetilde{v}_r^{(k)}(x,t) \right\|,$$

$$\max_{r=\overline{1,N}} \sup_{t \in [(r-1)h, rh)} \left\| u_r^{(k+1)}(x,t) - u_r^{(k)}(x,t) \right\|,$$

$r = \overline{1, N}$, $k = 1, 2, \ldots$ the inequality is valid

$$\Delta^{(k+1)}(x) \le \sum_{j=0}^{\nu-1} \frac{(L_1(x)h)^j}{j!} L_2(x)h[1+\gamma_\nu(x,h)\sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!}] \times$$

$$\times \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| +$$

$$+\frac{(L_1(x)h)^\nu}{\nu!}[1+\gamma_\nu(x,h)\sum_{j=0}^{\nu} \frac{(L_1(x)h)^j}{j!}] \ldots$$

$$\ldots \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k)}(x,t) - \tilde{v}_r^{(k-1)}(x,t) \right\| \le$$

$$\le \chi(x) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k)}(x,t) - u_r^{(k-1)}(x,t) \right\| +$$

$$+ q_\nu(x,h) \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k)}(x,t) - \tilde{v}_r^{(k-1)}(x,t) \right\| \le$$

$$\le (2\mu)^{k+1} \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} \max_{x\in[0,\omega]} \Delta^{(1)}(x), \qquad (2.55)$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k+1)}(x,t) - u_r^{(k)}(x,t) \right\| \le \int_0^x \Delta^{(k+1)}(\xi)d\xi, \qquad (2.56)$$

$$\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(x) - \lambda_r^{(0)}(x) \right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1)}(x,t) - \tilde{v}_r^{(0)}(x,t) \right\| \le$$

$$\le \Delta^{(k+1)}(x) + \Delta^{(k)}(x) + \ldots + \Delta^{(1)}(x) \le$$

$$\le [\frac{(2\mu)^2}{1-2\mu} \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} + 1] \sum_{j=0}^{\nu} \frac{(\tilde{L}_1 h)^j}{j!} \times$$

$$\times \max_{x\in[0,\omega]} \gamma_\nu(x,h) \left\| Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x)) \right\| < \rho(x),$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| u_r^{(k+1)}(x,t) - u_r^{(0)}(x,t) \right\| \le$$

$$\le \int_0^x \left(\max_{r=\overline{1,N}} \left\| \lambda_r^{(k+1)}(\xi) - \lambda_r^{(0)}(\xi) \right\| + \right.$$

$$\left. + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)} \left\| \tilde{v}_r^{(k+1)}(\xi,t) - \tilde{v}_r^{(0)}(\xi,t) \right\| \right) d\xi \le$$

$$\le \omega [\frac{(2\mu)^2}{1-2\mu} \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} + 1] \sum_{j=0}^{\nu} \frac{(\tilde{L}_1 h)^j}{j!} \times$$

$$\times \max_{x\in[0,\omega]} \gamma_\nu(x,h)\left\| Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\| < \omega\rho(x).$$

From inequalities (2.55), (2.56) and $q_\nu(x,h) < 1/2$ it follows that the sequence $(\lambda^{(k)}(x)+\tilde{v}^{(k)}(x,[t]),u^{(k)}(x,[t]))$ at $k\to\infty$ converges to $(\lambda^*(x)+\tilde{v}^*(x,[t]),\ u^*(x,[t]))$ - solution of the problem (2.37)-(2.41) in $S(\lambda^{(0)}(x)+\tilde{v}^{(0)}(x,[t]),\rho(x))$ $\times S(u^{(0)}(x,[t]),\omega\rho(x))$. Install of inequality

$$\max_{r=\overline{1,N}}\left\|\lambda_r^{(k+p)}(x)-\lambda_r^{(k)}(x)\right\| + \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(k+p)}(x,t)-\tilde{v}_r^{(k)}(x,t)\right\| \le$$

$$\le [(2\mu)^{p-1}+(2\mu)^{p-2}+\ldots+1](2\mu)^{k+1}\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\max_{x\in[0,\omega]}\Delta^{(1)}(x) \le$$

$$\le \frac{(2\mu)^{k+1}}{1-2\mu}\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\max_{x\in[0,\omega]}\Delta^{(1)}(x) \le$$

$$\le \frac{(2\mu)^{k+1}}{1-2\mu}\frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!}\sum_{j=0}^{\nu}\frac{(\tilde{L}_1 h)^j}{j!}\times$$

$$\times \max_{x\in[0,\omega]} \gamma_\nu(x,h)\left\| Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x))\right\|, \tag{2.57}$$

$$\max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)}\left\|u_r^{(k+p)}(x,t)-u_r^{(k)}(x,t)\right\| \le$$

$$\le \int_0^x \max_{r=\overline{1,N}}\left\|\lambda_r^{(k+p)}(\xi)-\lambda_r^{(k)}(\xi)\right\| d\xi +$$

$$+ \int_0^x \max_{r=\overline{1,N}} \sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(k+p)}(\xi,t)-\tilde{v}_r^{(k)}(\xi,t)\right\| d\xi. \tag{2.58}$$

In inequalities (2.57), (2.58) passing to the limit at $p\to\infty$, obtain estimates a), b).

The isolation is established similarly to the proof of Theorem 13. Theorem 15 is proved.

Functions $v^{(k)}(x,t), u^{(k)}(x,t)$, $k=0,1,2,\ldots$, are defined by equations:

$$v^{(k)}(x,t) = \begin{cases} \lambda_r^{(k)}(x) + \tilde{v}_r^{(k)}(x,t), & at \ (x,t) \in \Omega_r, \ r = \overline{1,N}, \\ \lambda_N^{(k)}(x) + \lim\limits_{t \to T-0} \tilde{v}_N^{(k)}(x,t), & at \ t = Nh, \end{cases}$$

$$u^{(k)}(x,t) = \psi(t) + \int_0^x v^{(k)}(\xi,t)d\xi$$

and through $S(u^{(0)}(x,t),[\omega+1]\rho(x))$ denote the set piecewise-continuously differentiable at x,t functions $u:\overline{\Omega} \to R^n$, satisfying inequalities

$$\|u(x,t) - u^{(0)}(x,t)\| < [\omega+1]\rho(x),$$

$$\|v(x,t) - \lambda^{(0)}(x) - \tilde{v}^{(0)}(x,t)\| < [\omega+1]\rho(x),$$

$$\|u(x,T) - u^{(0)}(x,T)\| < [\omega+1]\rho(x),$$

$$\|v(x,T) - \lambda^{(0)}(x) - \tilde{v}^{(0)}(x,T)\| < [\omega+1]\rho(x),$$

where $v(x,t) = \dfrac{\partial u(x,t)}{\partial x}$. In view of the equivalence of problems (2.36), (2.2), (2.3) and (2.37)-(2.41) from theorem 15 follows

Theorem 16. *If the conditions of Theorem 15 are satisfied, then the sequence of functions* $(u^{(k)}(x,t))$, $k = 0,1,\ldots,$ *is contained in* $S(u^{(0)}(x,t),[\omega+1]\rho(x))$, *converges to* $u^*(x,t)$ *- problem solving (2.36) ,(2.2), (2.3) to* $S(u^{(0)}(x,t),[\omega+1]\rho(x))$ *and the inequality is valid*

$$\|u^*(x,t) - u^{(k)}(x,t)\| \le \omega \frac{(2\mu)^{k+1}}{1-2\mu} \cdot \frac{\left(\frac{\tilde{\chi}\omega}{\mu}\right)^{\left[\frac{\tilde{\chi}\omega}{\mu}\right]}}{\left[\frac{\tilde{\chi}\omega}{\mu}\right]!} \sum_{j=0}^{\nu} \frac{(\tilde{L}_1 h)^j}{j!} \times$$

$$\times \max_{x \in [0,\omega]} \gamma_\nu(x,h) \|Q_{\nu,h}(x, u^{(0)}(x,[\cdot]), \tilde{v}^{(0)}(x,[\cdot]), \lambda^{(0)}(x))\|, (x,t) \in \overline{\Omega}. (2.5$$

Moreover, any solution to the problem (2.36), (2.2), (2.3) to $S(u^{(0)}(x,t),[\omega+1]\rho(x))$ *isolated.*

Proof. Suppose that the conditions of Theorem 15 are satisfied. Because for all $k = 0,1,2,\ldots,$ $(\lambda^{(k)}(x) + \tilde{v}^{(k)}(x,[t]), u^{(k)}(x,[t]))$ is contained to $S(\lambda^{(0)}(x) + \tilde{v}^{(0)}(x,[t]), \rho(x))$ $S(u^{(0)}(x,[t]), \omega\rho(x))$, converges to $(\lambda^*(x) + \tilde{v}^*(x,[t]),$

$u^*(x,[t]))\in S(\lambda^{(0)}(x)+\tilde{v}^{(0)}(x,[t]),\rho(x))\times S(u^{(0)}(x,[t]),\omega\rho(x))$ - solve problems (2.37)-(2.41), then we determined the sequence of functions $(u^{(k)}(x,t))$, $k=1,2,\dots,$ is contained in $S(u^{(0)}(x,t),\ [\omega+1]\ \rho(x))$, converges to $u^*(x,t)\in$ $\in S(u^{(0)}(x,t),[\omega+1]\rho(x))$ - solve problems (2.36), (2.2), (2.3). By owing of estimates a), b) Theorem 15 at $(x,t)\in\overline{\Omega}$, takes place inequality (2.59).

Theorem 16 is proved.

2.4 The problem of choosing the initial approximation

In this subsection, an estimate of the difference between $\lambda^{(0)}(x)$ $\lambda^*(x)$, $x\in[0,\omega]$, the components of which are composed of the values of the solutions of the boundary value problem (2.36), (2.5)-(2.7) at the points of partition of the segment $[0,T]$, and $\lambda^{(0)}(x)$ - system solution

$$Q_{\nu,h}(x,\psi([\cdot])+\int_0^x\lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+$$
$$+\int_0^{\xi}\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)=0,\quad \lambda(x)\in C([0,\omega],R^{nN}).\quad (2.60)$$

This system of equations, in contrast to ordinary differential equations, consists from nN non-linear integral equations and fully determined according to the initial data of the problem and differ in the choice of values $h>0$ and $\nu\in\mathbb{N}$. Since the values of $h>0$ and ν affect the dimension and complexity of the system of equations, we propose algorithms for finding solutions to this system, based on the transition from simple and smaller dimensions of the system to a more complex and larger system.

Let $\lambda^{(0)}(x)\in G_0(f,x,h)$ - decision (2.60) and

$$(\lambda^{(0)}(x),\tilde{v}^{(0)}(x,[t]),u^{(0)}(x,[t]),w^{(0)}(x,[t]),\rho(x),\theta(x),\phi(x))\in$$
$$\in U_0(f,L_1(x),L_2(x),L_3(x),x,h).$$

Theorem 17. *If matrix* $\dfrac{\partial Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x),w(x,[\cdot]))}{\partial\lambda}$ *Jacobi invertible* *for all* $(x,\ u(x,[\cdot]),\ \tilde{v}(x,[\cdot]),\ \lambda(x),\ w(x,[\cdot]))$, *where* $x\in[0,\omega]$, $(\lambda(x),\ \tilde{v}(x,[t]), u(x,[t]), w(x,[t]))\in S(\lambda^{(0)}(x), \rho(x))\times S(\tilde{v}^{(0)}(x,[t]), \theta(x))\times S(u^{(0)}(x,[t]), \phi(x))\times S(w^{(0)}(x,[t]),\phi(x))$, *and performed inequalities 1)-5) of Theorem 13, that estimates takes place*

$$\left\|\lambda^{*}(x)-\lambda^{(0)}(x)\right\|\le$$

$$\le[c_1(x)c_0(x)+c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int_0^x c(\xi)e^{\int_0^{\xi}c(\xi_1)d\xi_1}\times$$

$$\times\int_0^{\xi}\max\{1,L_1(\xi_1)\}[c_1(\xi_1)c_0(\xi_1)+c_0(\xi_1)+1]\times$$

$$\times\gamma_{\nu}(\xi_1,h)[\sum_{j=0}^{\nu-1}\frac{(L_1(\xi_1)h)^j}{j!}[L_2(\xi_1)+L_3(\xi_1)]h\times\times\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\int_0^{\xi_1}\max\{1,L_1(\xi_2)\}\int_{(r-1)h}^{t}f(\xi_2,\tau_1,\psi(\tau_1)+\right.$$

$$+\int_0^{\xi_2}\lambda_r^{(0)}(\xi_3)d\xi_3,\int_{(r-1)h}^{\tau_1}f(\xi_2,\tau_2,\psi(\tau_2)+\int_0^{\xi_2}\lambda_r^{(0)}(\xi_3)d\xi_3,\dots\dots\int_{(r-1)h}^{\tau_{\nu-1}}f(\xi_2,\tau_{\nu},\psi(\tau_{\nu})+\int_0^{\xi_2}\lambda_r^{(0)}(\xi_3)d\xi_3,\lambda_r^{(0)}(\xi_2),\dot{\psi}(\tau_{\nu})+$$

$$+\int_0^{\xi_2}f(\xi_3,\tau_{\nu},\psi(\tau_{\nu})+\int_0^{\xi_3}\lambda^{(0)}(\xi_4)d\xi_4,\lambda^{(0)}(\xi_3),\dot{\psi}(\tau_{\nu}))d\xi_3)d\tau_{\nu}+\dots$$

$$\dots+\lambda_r^{(0)}(\xi_2),\dot{\psi}(\tau_2)+\int_0^{\xi_2}f(\xi_3,\tau_2,\psi(\tau_2)+$$

$$+\int_0^{\xi_3}\lambda^{(0)}(\xi_4)d\xi_4,\lambda^{(0)}(\xi_3),\dot{\psi}(\tau_2))d\xi_3)d\tau_2+\lambda_r^{(0)}(\xi_2),\dot{\psi}(\tau_1)+$$

$$\left.\int_0^{\xi_2}f(\xi_3,\tau_1,\psi(\tau_1)+\int_0^{\xi_3}\lambda^{(0)}(\xi_4)d\xi_4,\lambda^{(0)}(\xi_3),\dot{\psi}(\tau_1))d\xi_3)d\tau_1 d\xi_2\right\|d\xi_1 d\xi+$$

$$+[c_0(x)c_1(x)+1]\gamma_{\nu}(x,h)[\sum_{j=0}^{\nu-1}\frac{(L_1(x)h)^j}{j!}[L_2(x)+L_3(x)]h\times$$

$$\times\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left[\int_0^{x}\max\{1,L_1(\xi)\}\int_{(r-1)h}^{t}f(\xi,\tau_1,\psi(\tau_1)+\right.$$

$$+\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\int_{(r-1)h}^{\tau_1}f(\xi,\tau_2,\psi(\tau_2)+\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\dots\int_{(r-1)h}^{\tau_{\nu-1}}f(\xi,\tau_{\nu},\psi(\tau_{\nu})++\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\lambda_r^{(0)}(x),\dot{\psi}(\tau_{\nu})+\int_0^{\xi}f(\xi_1,\tau_{\nu},\psi(\tau_{\nu})+$$

$$+\int_0^{\xi_1}\lambda^{(0)}(\xi_2)d\xi_2,\lambda^{(0)}(\xi_1),\dot{\psi}(\tau_{\nu}))d\xi_1)d\tau_{\nu}+\dots$$

$$\dots+\lambda_r^{(0)}(\xi),\dot{\psi}(\tau_2)+\int_0^{\xi}f(\xi_1,\tau_2,\psi(\tau_2)+$$

$$+\int_0^{\xi_1}\lambda^{(0)}(\xi_2)d\xi_2,\lambda^{(0)}(\xi_1),\dot{\psi}(\tau_2))d\xi_1)d\tau_2+$$

$$+\lambda_r^{(0)}(\xi),\dot{\psi}(\tau_1)+\int_0^{\xi}f(\xi_1,\tau_1,\psi(\tau_1)+$$

$$\left.+\int_0^{\xi_1}\lambda^{(0)}(\xi_2)d\xi_2,\lambda^{(0)}(\xi_1),\dot{\psi}(\tau_1))d\xi_1)d\tau_1 d\xi\right]. \qquad (2.61)$$

Proof. Considering structure operator $Q_{\nu,h}(x,u(x,[\cdot]),\tilde{v}(x,[\cdot]),\lambda(x),w(x,[\cdot]))$ and equality

$$Q_{\nu,h}(x,\psi([\cdot])+\int_0^x\lambda^{(0)}(\xi)d\xi,0,\lambda^{(0)}(x),\dot{\psi}([\cdot])+$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}([\cdot]))d\xi)=0,$$

from the inequality a) of Theorem 13 we obtain

$$\max_{r=\overline{1,N}}\left\|\lambda_r^*(x)-\lambda_r^{(0)}(x)\right\|\le$$

$$\le[c_1(x)c_0(x)+c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int_0^x c(\xi)e^{\int_0^{\xi}c(\xi_1)d\xi_1}\times$$

$$\times\int_0^{\xi}\max\{1,L_1(\xi_1)\}[c_1(x)c_0(x)+c_0(x)+1]\gamma_\nu(\xi_1,h)\times$$

$$\times\left\|Q_{\nu,h}(\xi_1,u^{(0)}(\xi_1,[\cdot]),\tilde{v}^{(0)}(\xi_1,[\cdot]),\lambda^{(0)}(\xi_1),w^{(0)}(\xi_1,[\cdot]))-\right.$$

$$-Q_{\nu,h}(\xi_1,\psi([\cdot])+\int_0^{\xi_1}\lambda^{(0)}(\xi_2)d\xi_2,0,\lambda^{(0)}(\xi_1),\dot{\psi}([\cdot])+$$

$$\left.+\int_0^{\xi_1}f(\xi_2,[\cdot],\psi([\cdot])+\int_0^{\xi_2}\lambda^{(0)}(\xi_3)d\xi_3,\lambda^{(0)}(\xi_2),\dot{\psi}([\cdot]))d\xi_2)\right\|d\xi_1 d\xi+$$

$$+[c_0(x)c_1(x)+1]\gamma_\nu(x,h)\left\|Q_{\nu,h}(x,u^{(0)}(x,[\cdot]),\tilde{v}^{(0)}(x,[\cdot]),\lambda^{(0)}(x),w^{(0)}(x,\right.$$

$$-Q_{\nu,h}(x,\psi([\cdot])+\int_0^x\lambda^{(0)}(\xi)d\xi,0,\lambda^{(0)}(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+$$

$$\left.+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}([\cdot]))d\xi)\right\|\le$$

$$\le[c_1(x)c_0(x)+c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int_0^x c(\xi)e^{\int_0^{\xi}c(\xi_1)d\xi_1}\times$$

$$\times\int_0^{\xi}\max\{1,L_1(\xi_1)\}[c_1(\xi_1)c_0(\xi_1)+c_0(\xi_1)+1]\gamma_\nu(\xi_1,h)\times$$

$$\times\left(\sum_{j=0}^{\nu-1}\frac{(L_1(\xi_1)h)^j}{j!}[L_2(\xi_1)+L_3(\xi_1)]h\times\right.$$

$$\times\max\left\{\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(\xi_1,t)-\psi(t)-\int_0^{\xi_1}\lambda_r^{(0)}(\xi_2)d\xi_2\right\|,\right.$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(0)}(\xi_1,t)-\dot{\psi}(t)-\right.$$

$$\left.\left.-\int_0^{\xi_1}f(\xi_2,t,\psi(t)+\int_0^{\xi_2}\lambda^{(0)}(\xi_3)d\xi_3,\lambda^{(0)}(\xi_2),\dot{\psi}(t))d\xi_2\right\|\right\}+$$

$$\left.+\gamma_\nu(\xi_2,h)\frac{(L_1(\xi_2)h)^\nu}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\tilde{v}_r^{(0)}(\xi_2,t)\right\|\right)d\xi_2 d\xi_1+$$

$$+[c_0(x)c_1(x)+1]\gamma_\nu(x,h)\left(\sum_{j=0}^{\nu-1}\frac{(L_1(x)h)^j}{j!}[L_2(x)+L_3(x)]h\times\right.$$

$$\times\max\left\{\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)-\int_0^x\lambda_r^{(0)}(\xi)d\xi\right\|,\right.$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|w_r^{(0)}(x,t)-\dot{\psi}(t)-\right.$$

$$\left.\left.-\int_0^x f(\xi,t,\psi(t)+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}(t))d\xi\right\|\right\}+$$

$$+\gamma_{\nu}(x,h)\frac{(L_1(x)h)^{\nu}}{\nu!}\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\|\Bigg). \qquad (2.62)$$

As $\widetilde{v}_r^{(0)}(x,t), u_r^{(0)}(x,t), w_r^{(0)}(x,t)$ are solutions to Goursat problems (2.27)-(2.32), at $\lambda_r(x)=\lambda_r^{(0)}(x), x\in[0,\omega]$, then on $\Omega_r, r=\overline{1,N}$, estimates take place

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\widetilde{v}_r^{(0)}(x,t)\right\| \le \max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\Bigg\|\int_{(r-1)h}^{t} f(x,\tau_1,\psi(\tau_1)+$$

$$+\int_0^x \lambda_r^{(0)}(\xi)d\xi, \int_{(r-1)h}^{\tau_1} f(x,\tau_2,\psi(\tau_2)+\int_0^x \lambda_r^{(0)}(\xi)d\xi,\ldots$$

$$\ldots\int_{(r-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,\psi(\tau_\nu)+\int_0^x \lambda_r^{(0)}(\xi)d\xi,\lambda_r^{(0)}(x),\dot{\psi}(\tau_\nu)+\int_0^x f(\xi,\tau_\nu,\psi(\tau_\nu)+$$

$$+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}(\tau_\nu))d\xi)d\tau_\nu+\ldots$$

$$\ldots+\lambda_r^{(0)}(x),\dot{\psi}(\tau_2)+\int_0^x f(\xi,\tau_2,\psi(\tau_2)+$$

$$+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}(\tau_2))d\xi)d\tau_2+\lambda_r^{(0)}(x),\dot{\psi}(\tau_1)+\int_0^x f(\xi,\tau_1,\psi(\tau_1)+\ +\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}(\tau_1))d\xi)d\tau_1\Bigg\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|u_r^{(0)}(x,t)-\psi(t)-\int_0^x\lambda_r^{(0)}(\xi)d\xi\right\| \le$$

$$\le\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\psi(t)+\int_0^x\lambda_r^{(0)}(\xi)d\xi+\int_0^x \widetilde{v}_r^{(0)}(\xi,t)d\xi-\psi(t)-\int_0^x\lambda_r^{(0)}(\xi)d\xi\right\| \le \le \max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\left\|\int_0^x \widetilde{v}_r^{(0)}(\xi,t)d\xi\right\|,$$

$$\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\Bigg\|w_r^{(0)}(x,t)-\dot{\psi}(t)-$$

$$-\int_0^x f(\xi,t,\psi(t)+\int_0^{\xi}\lambda^{(0)}(\xi_1)d\xi_1,\lambda^{(0)}(\xi),\dot{\psi}(t))d\xi\Bigg\| \le$$

$$\le\max_{r=\overline{1,N}}\sup_{t\in[(r-1)h,rh)}\Bigg\|\int_0^x L_1(\xi)\int_{(r-1)h}^{t} f(\xi,\tau_1,\psi(\tau_1)+$$

$$+\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\int_{(r-1)h}^{\tau_1} f(\xi,\tau_2,\psi(\tau_2)+\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\ldots\int_{(r-1)h}^{\tau_{\nu-1}} f(\xi,\tau_\nu,\psi(\tau_\nu)++\int_0^{\xi}\lambda_r^{(0)}(\xi_1)d\xi_1,\lambda_r^{(0)}(x),\dot{\psi}(\tau_\nu)+\int_0^{\xi} f(\xi_1,\tau_\nu,\psi(\tau_\nu)+$$

$$+\int_0^{\xi_1}\lambda(\xi_2)d\xi_2,\lambda(\xi_1),\dot{\psi}(\tau_\nu))d\xi_1)d\tau_\nu+\ldots$$

$$\ldots+\lambda_r^{(0)}(\xi),\dot{\psi}(\tau_2)+\int_0^{\xi} f(\xi_1,\tau_2,\psi(\tau_2)+$$

$$+\int_0^{\xi_1}\lambda(\xi_2)d\xi_2,\lambda(\xi_1),\dot{\psi}(\tau_2))d\xi_1)d\tau_2+\lambda_r^{(0)}(\xi),\dot{\psi}(\tau_1)+\int_0^{\xi} f(\xi_1,\tau_1,\psi(\tau_1)+\ +\int_0^{\xi_1}\lambda(\xi_2)d\xi_2,\lambda(\xi_1),\dot{\psi}(\tau_1))d\xi_1)d\tau_1 d\xi\Bigg\|,$$

taking into account which, from (2.62) we will receive validity of inequality (2.61). Theorem 17 is proved.

From the estimate (2.61) shows that the initial approximation of the parameter will be closer to the exact solution $\lambda^*(x)$, the smaller the partition step $h>0$ or the greater the number of substitutions $\nu=1,2,\ldots.$ The initial approximation by the functional parameter - $\lambda^{(0)}(x)\in C([0,\omega],R^{nN})$ will be sought as the solution of the equation (2.60), i.e. the system

$$\lambda_1(x)-\lambda_N(x)-\int\limits_{(N-1)h}^{Nh} f(x,\tau_1,\psi(\tau_1)+\int\limits_0^x \lambda_N(\xi)d\xi,\int\limits_{(N-1)h}^{\tau_1} f(x,\tau_2,\psi(\tau_2)+$$

$$+\int\limits_0^x \lambda_N(\xi)d\xi,\dots\int\limits_{(N-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,\psi(\tau_\nu)+\int\limits_0^x \lambda_N(\xi)d\xi,\lambda_N(x),\dot{\psi}(\tau_\nu)+$$

$$+\int\limits_0^x f(\xi,\tau_\nu,\psi(\tau_\nu)+\int\limits_0^\xi \lambda_N(\xi_2)d\xi_1,\lambda_N(\xi),\dot{\psi}(\tau_\nu))d\xi)d\tau_\nu+\dots+\lambda_N(x),\dot{\psi}(\tau_2)+$$

$$+\int\limits_0^x f(\xi,\tau_2,\psi(\tau_2)+\int\limits_0^\xi \lambda_N(\xi_1)d\xi_1,\lambda_N(\xi),\dot{\psi}(\tau_2))d\xi)d\tau_2+\lambda_N(x),\dot{\psi}(\tau_1)+$$

$$+\int\limits_0^x f(\xi,\tau_1,\psi(\tau_1)+\int\limits_0^\xi \lambda_N(\xi_1)d\xi_1,\lambda_N(\xi),\dot{\psi}(\tau_1))d\xi)d\tau_1=0, \qquad (2.63)$$

$$\lambda_s(x)+\int\limits_{(s-1)h}^{sh} f(x,\tau_1,\psi(\tau_1)+\int\limits_0^x \lambda_s(\xi)d\xi,\int\limits_{(s-1)h}^{\tau_1} f(x,\tau_2,\psi(\tau_2)+$$

$$+\int\limits_0^x \lambda_s(\xi)d\xi,\dots\int\limits_{(s-1)h}^{\tau_{\nu-1}} f(x,\tau_\nu,\psi(\tau_\nu)+$$

$$+\int\limits_0^x \lambda_s(\xi)d\xi,\lambda_s(x),\dot{\psi}(\tau_\nu)+\int\limits_0^x f(\xi,\tau_\nu,\psi(\tau_\nu)+$$

$$+\int\limits_0^\xi \lambda_s(\xi_1)d\xi_1,\lambda_s(\xi),\dot{\psi}(\tau_\nu))d\xi)d\tau_\nu+\dots+\lambda_s(x))d\tau_2+\lambda_s(x),\dot{\psi}(\tau_2)+$$

$$+\int\limits_0^x f(\xi,\tau_2,\psi(\tau_2)+\int\limits_0^\xi \lambda_s(\xi_1)d\xi_1,\lambda_s(\xi),\dot{\psi}(\tau_2))d\xi)d\tau_2+$$

$$+\lambda_s(x),\dot{\psi}(\tau_1)+\int\limits_0^x f(\xi,\tau_1,\psi(\tau_1)+$$

$$+\int\limits_0^\xi \lambda_s(\xi_1)d\xi_1,\lambda_s(\xi),\dot{\psi}(\tau_1))d\xi)d\tau_1-\lambda_{s+1}(x)=0, \qquad (2.64)$$

where $x\in[0,\omega], s=\overline{1,N-1}$. The system of equations (2.63), (2.64) is a complex nonlinear dimension system nN. However, this system is completely determined by the source data of the problem and to find it solutions $\lambda^{(0)}(x)\in C([0,\omega],R^{nN})$ can apply known methods of the theory of nonlinear systems of equations [88, 90]. One of the main methods for finding solutions are iterative methods. Fast converging iterative type methods Newton's method generally implies the existence of a "good" initial approximation. To find such an initial approximation suggests the following approach. Consider first system (2.64) at $h=T(N=1),\nu=1$:

$$Q_{1,T}(x,\psi[\cdot]+\int\limits_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int\limits_0^x f(\xi,[\cdot],\psi([\cdot])+$$

$$+\int\limits_0^\xi \lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)=0, \quad \lambda(x)\in C([0,\omega],R^{nN}) \qquad (2.65)$$

which has the form

$$\int\limits_0^T f(x,\tau,\psi(\tau)+\int\limits_0^x \lambda_{1,T}(\xi)d\xi,\lambda_{1,T}(x),\dot{\psi}(\tau)+\int\limits_0^x f(\xi,\tau,\psi(\tau)+$$

$$+\int\limits_0^\xi \lambda_{1,T}(\xi_1)d\xi_1,\lambda_{1,T}(\xi),\dot{\psi}(\tau))d\xi)d\tau=0, \quad x\in[0,\omega].$$

Let $\lambda_{1,T}(x)-$ decision (2.65). Next, consider the system (2.60) at $h = \frac{T}{2}$ $(N = 2), \nu = 1$, which is written as

$$\lambda_{1,\frac{T}{2},1}(x) - \lambda_{1,\frac{T}{2},2}(x) - \int_{\frac{T}{2}}^{T} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,\frac{T}{2},2}(\xi)d\xi, \lambda_{1,\frac{T}{2},2}(x), \dot{\psi}(\tau) +$$

$$+ \int_0^x f(\xi,\tau,\psi(\tau) + \int_0^\xi \lambda_{1,\frac{T}{2},2}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2},2}(\xi), \dot{\psi}(\tau))d\xi)d\tau = 0, \quad (2.66)$$

$$\lambda_{1,\frac{T}{2},1}(x) + \int_0^{\frac{T}{2}} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,\frac{T}{2},1}(\xi)d\xi, \lambda_{1,\frac{T}{2},1}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+ \int_0^\xi \lambda_{1,\frac{T}{2},1}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2},1}(\xi), \dot{\psi}(\tau))d\xi)d\tau - \lambda_{1,\frac{T}{2},2}(x) = 0, \quad (2.67)$$

where $x \in [0,\omega]$, i.e. initial system approximation (2.66), (2.67) - vector-function $\lambda_{1,\frac{T}{2}}(x) = (\lambda_{1,\frac{T}{2},1}(x), \lambda_{1,\frac{T}{2},2}(x))' \in C([0,\omega], R^{n2})$ we define as follows:

$$\tilde{\lambda}_{1,\frac{T}{2},1}(x) = \lambda_{1,T}(x), \tilde{\lambda}_{1,\frac{T}{2},2}(x) =$$

$$= \lambda_{1,T}(x) + \int_0^{\frac{T}{2}} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,T}(\xi)d\xi, \lambda_{1,T}(x), \dot{\psi}(\tau) +$$

$$+ \int_0^x f(\xi,\tau,\psi(\tau) + \int_0^\xi \lambda_{1,T}(\xi_1)d\xi_1, \lambda_{1,T}(\xi), \dot{\psi}(\tau))d\xi)d\tau.$$

Taking $\tilde{\lambda}_{1,\frac{T}{2}}(x)$, the function $\rho_1(x) > 0$, we construct the sets

$$S(\tilde{\lambda}_{1,\frac{T}{2}}(x), \rho_1(x)) = \{(\tilde{\lambda}_{1,\frac{T}{2},1}(x), \tilde{\lambda}_{1,\frac{T}{2},2}(x))' \in C([0,\omega], R^{n2}):$$

$$\left\| \lambda_{1,\frac{T}{2},r}(x) - \tilde{\lambda}_{1,\frac{T}{2},r}(x) \right\| < \rho_1(x), r = \overline{1,2}\}.$$

$$G_1(\tilde{\lambda}_{1,\frac{T}{2}}(x), \rho_1(x)) = \{(x,t,u,v,w) : (x,t) \in \overline{\Omega}, \left\| u - \tilde{\lambda}_{1,\frac{T}{2},s} \right\| < \rho_1(x),$$

$$\left\| v - \tilde{\lambda}_{1,\frac{T}{2},s} \right\| < \rho_1(x), \quad \left\| w - \tilde{\lambda}_{1,\frac{T}{2},s} \right\| < \rho_1(x),$$

$$x \in [0,\omega], \quad t \in \left[(s-1)\frac{T}{2}, \quad s\frac{T}{2}\right), s = 1,2\}.$$

Statement 1: Let function $f(x,t,u,v,w)$ in $G_1\left(\tilde{\lambda}_{1,\frac{T}{2}}(x), \rho_1(x)\right)$ have uniformly continuous partial derivatives

$f_u'(x,t,u,v,w)$, $f_v'(x,t,u,v,w)$, $f_w'(x,t,u,v,w)$ and matrix Jacobi $\frac{\partial}{\partial\lambda}Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)$ is invertible for all $\lambda(x)\in S(\tilde{\lambda}_{1,\frac{T}{2}}(x),\rho_1(x))$, $x\in[0,\omega]$ and the following inequalities are executed:

1) $$\left\|\left[\frac{\partial}{\partial\lambda}Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)\right]^{-1}\right\|\le\gamma_1(x,\frac{T}{2}),$$

2) $$[(c_1(x)+1)c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int_0^x c(\xi)e^{\int_0^\xi c(\xi_1)d\xi_1}\times$$
$$\times\int_0^\xi\max\{1,L_1(\xi_1)\}[(c_1(\xi_1)+1)c_0(\xi_1)+1]\gamma_1(\xi_1,\frac{T}{2})\times$$
$$\times\left\|Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\tilde{\lambda}_{1,\frac{T}{2}}(\xi)d\xi,0,\tilde{\lambda}_{1,\frac{T}{2}}(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi\tilde{\lambda}_{1,\frac{T}{2}}(\xi_1)d\xi_1,\tilde{\lambda}_{1,\frac{T}{2}}(\xi),\dot{\psi}([\cdot]))d\xi)\right\|d\xi_1 d\xi+[c_0(x)c_1(x)+$$
$$+1]\gamma_1(x,\frac{T}{2})\left\|Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\tilde{\lambda}_{1,\frac{T}{2}}(\xi)d\xi,0,\tilde{\lambda}_{1,\frac{T}{2}}(x),\dot{\psi}([\cdot])+\right.$$
$$\left.+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi\tilde{\lambda}_{1,\frac{T}{2}}(\xi_1)d\xi_1,\tilde{\lambda}_{1,\frac{T}{2}}(\xi),\dot{\psi}([\cdot]))d\xi)\right\|<\rho_1(x).$$

Then there is a number $\alpha\ge 1$ such that the sequence $\lambda_{1,\frac{T}{2}}^{(m+1)}(x)$, $m=0,1,2,...,$ is determined by the iterative process:

$$\lambda_{1,\frac{T}{2}}^{(0)}(x)=\tilde{\lambda}_{1,\frac{T}{2}}(x),$$
$$\lambda_{1,\frac{T}{2}}^{(m+1)}(x)=\lambda_{1,\frac{T}{2}}^{(m)}(x)-$$
$$-\frac{1}{\alpha}\left[\frac{\partial}{\partial\lambda}Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\lambda_{1,\frac{T}{2}}^{(m)}(\xi)d\xi,0,\lambda_{1,\frac{T}{2}}^{(m)}(x),\dot{\psi}([\cdot])+\right.$$
$$\left.+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi\lambda_{1,\frac{T}{2}}^{(m)}(\xi_1)d\xi_1,\lambda_{1,\frac{T}{2}}^{(m)}(\xi),\dot{\psi}([\cdot]))d\xi\right]^{-1}\times$$
$$\times Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x\lambda_{1,\frac{T}{2}}^{(m)}(\xi)d\xi,0,\lambda_{1,\frac{T}{2}}^{(m)}(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+$$

$$+\int_0^{\xi}\lambda^{(m)}_{1,\frac{T}{2}}(\xi_1)d\xi_1, \lambda^{(m)}_{1,\frac{T}{2}}(\xi), \dot{\psi}([\,\cdot\,]))d\xi), \quad (2.68)$$

$m = 0,1,2,...,$ converges to $\lambda_{1,\frac{T}{2}}(x)$ - isolated solution of the system of equations

$$Q_{1,\frac{T}{2}}(x,\psi[\,\cdot\,] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\,\cdot\,]) + \int_0^x f(\xi,[\,\cdot\,],\psi([\,\cdot\,]) +$$

$$+\int_0^{\xi}\lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\,\cdot\,]))d\xi) = 0$$

at $S(\tilde{\lambda}_{1,\frac{T}{2}}(x), \rho_1(x))$ and inequality takes place

$$\left\|\lambda_{1,\frac{T}{2}}(x) - \tilde{\lambda}_{1,\frac{T}{2}}(x)\right\| \le \gamma_1(x,\frac{T}{2})\frac{T}{2}L_1(x)M(x), \quad (2.69)$$

where

$$M(x) = \left\|\int_0^{\frac{T}{2}} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,T}(\xi)d\xi, \lambda_{1,T}(x), \dot{\psi}(\tau) +\right.$$

$$\left. + \int_0^x f(\xi,\tau,\psi(\tau) + \int_0^{\xi}\lambda_{1,T}(\xi_1)d\xi_1, \lambda_{1,T}(\xi), \dot{\psi}(\tau))d\xi)d\tau\right\|.$$

Really, as operator

$$Q_{1,\frac{T}{2}}(x,\psi[\,\cdot\,] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\,\cdot\,]) +$$

$$+\int_0^x f(\xi,[\,\cdot\,],\psi([\,\cdot\,]) + \int_0^{\xi}\lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\,\cdot\,]))d\xi) = 0$$

in $S(\tilde{\lambda}_{1,\frac{T}{2}}(x), \rho_1(x))$ satisfies all the assumptions of Theorem 1 of [87], we obtain the existence $\tilde{\lambda}_{1,\frac{T}{2}}(x)$ - of an isolated solution of equations (2.66), (2.67) and the estimate

$$\left\|\lambda_{1,\frac{T}{2}}(x) - \tilde{\lambda}_{1,\frac{T}{2}}(x)\right\| \le$$

$$\le \max_{x\in[0,\omega]}\gamma_1\left(x,\frac{T}{2}\right)\left\|Q_{1,\frac{T}{2}}(x,\psi[\,\cdot\,] + \int_0^x \tilde{\lambda}_{1,\frac{T}{2}}(\xi)d\xi, 0, \tilde{\lambda}_{1,\frac{T}{2}}(x), \dot{\psi}([\,\cdot\,]) +\right.$$

$$\left. + \int_0^x f(\xi,[\,\cdot\,],\psi([\,\cdot\,]) + \int_0^{\xi}\tilde{\lambda}_{1,\frac{T}{2}}(\xi_1)d\xi_1, \tilde{\lambda}_{1,\frac{T}{2}}(\xi), \dot{\psi}([\,\cdot\,]))d\xi)\right\|. \quad (2.70)$$

Let's estimate norm of the operator

$$Q_{1,\frac{T}{2}}(x,\psi[\,\cdot\,] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\,\cdot\,]) +$$

$$+\int_0^x f(\xi,[\,\cdot\,],\psi([\,\cdot\,]) + \int_0^{\xi}\lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\,\cdot\,]))d\xi):$$

$$\left\|Q_{1,\frac{T}{2}}(x,\psi[\,\cdot\,] + \int_0^x \tilde{\lambda}_{1,\frac{T}{2}}(\xi)d\xi, 0, \tilde{\lambda}_{1,\frac{T}{2}}(x), \dot{\psi}([\,\cdot\,]) +\right.$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \tilde{\lambda}_{1,\frac{T}{2}}(\xi_1)d\xi_1, \tilde{\lambda}_{1,\frac{T}{2}}(\xi), \dot{\psi}([\cdot]))d\xi)\Bigg\| \le$$

$$\le L_1(x)\frac{T}{2}\Bigg\|\int_0^{\frac{T}{2}} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,T}(\xi)d\xi, \lambda_{1,T}(x), \dot{\psi}(\tau) +$$

$$+\int_0^x f(\xi,\tau,\psi(\tau) + \int_0^\xi \lambda_{1,T}(\xi_1)d\xi_1, \lambda_{1,T}(\xi), \dot{\psi}(\tau))d\xi)d\tau\Bigg\|. \quad (2.71)$$

The right part (2.71) substituting in (2.70) we obtain the validity estimates (2.69). Let $\lambda_{1,\frac{T}{2^{k-1}}} \in C([0,\omega], R^{n2^{k-1}})$ - the solution of the system of equations

$$Q_{1,\frac{T}{2^{k-1}}}(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) +$$

$$+\int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) = 0,$$

at $\nu = 1,$ $N = 2^{k-1}$ and there is an inequality

$$\left\|\lambda_{1,\frac{T}{2^{k-1}}}(x) - \tilde{\lambda}_{1,\frac{T}{2^{k-1}}}(x)\right\| \le \max_{x\in[0,\omega]} \gamma_2(x,\frac{T}{2^{k-1}})\frac{T}{2^{k-1}}L_1(x)M(x),$$

$$M(x) = \max_{r=1,2^{k-2}}\Bigg\|\int_{(2r-2)\frac{T}{2^{k-1}}}^{(2r-1)\frac{T}{2^{k-1}}} f(x,\tau,\psi(\tau) +$$

$$+\int_0^x \lambda_{1,\frac{T}{2^{k-2}},r}(\xi)d\xi, \lambda_{1,\frac{T}{2^{k-2}},r}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+\int_0^\xi \lambda_{1,\frac{T}{2^{k-2}},r}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^{k-2}},r}(\xi), \dot{\psi}(\tau))d\xi)d\tau\Bigg\|.$$

Find $\lambda_{1,\frac{T}{2^k}}(x)$ - solution of the equation

$$Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) +$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) = 0,$$

at $\nu = 1,$ $N = 2^k$, which is in vector-coordinate form is written as

$$\lambda_{1,\frac{T}{2^k},1}(x) - \lambda_{1,\frac{T}{2^k},2^k}(x) - \int_{\frac{T}{2}}^{T} f(x,\tau,\psi(\tau) +$$

$$+\int_0^x \lambda_{1,\frac{T}{2^k},2^k}(\xi)d\xi, \lambda_{1,\frac{T}{2^k},2^k}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+\int_0^\xi \lambda_{1,\frac{T}{2^k},2^k}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k},2^k}(\xi), \dot{\psi}(\tau))d\xi)d\tau = 0, \quad (2.72)$$

$$\lambda_{1,\frac{T}{2^k},s}(x) + \int\limits_{(s-1)\frac{T}{2^k}}^{s\frac{T}{2^k}} f(x,\tau,\psi(\tau) +$$

$$+ \int\limits_0^x \lambda_{1,\frac{T}{2^k},s}(\xi)d\xi, \lambda_{1,\frac{T}{2^k},s}(x), \dot{\psi}(\tau) + \int\limits_0^x f(\xi,\tau,\psi(\tau) +$$

$$+ \int\limits_0^{\xi} \lambda_{1,\frac{T}{2^k},s}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k},s}(\xi), \dot{\psi}(\tau))d\xi)d\tau - \lambda_{1,\frac{T}{2^k},s+1}(x) = 0, \quad (2.73)$$

where $x \in [0,\omega]$, $s = \overline{1, 2^k - 1}$. Initial approximation systems (2.72), (2.73) vector-function

$$\tilde{\lambda}_{1,\frac{T}{2^k}}(x) = (\tilde{\lambda}_{1,\frac{T}{2^k},1}(x), \tilde{\lambda}_{1,\frac{T}{2^k},2}(x), \ldots, \tilde{\lambda}_{1,\frac{T}{2^k},2^k}(x))' \in C([0,\omega], R^{n2^k})$$

we define as follows:

$$\tilde{\lambda}_{1,\frac{T}{2^k},2r-1}(x) = \lambda_{1,\frac{T}{2^{k-1}},r}(x),\ r = \overline{1, 2^{k-1}},$$

$$\tilde{\lambda}_{1,\frac{T}{2^k},2r}(x) = \lambda_{1,\frac{T}{2^{k-1}},r}(x) +$$

$$+ \int\limits_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f(x,\tau,\psi(\tau) + \int\limits_0^x \lambda_{1,\frac{T}{2^{k-1}},r}(\xi)d\xi, \lambda_{1,\frac{T}{2^{k-1}},r}(x), \dot{\psi}(\tau) +$$

$$+ \int\limits_0^x f(\xi,\tau,\psi(\tau) + \int\limits_0^{\xi} \lambda_{1,\frac{T}{2^k-1},r}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k-1},r}(\xi), \dot{\psi}(\tau))d\xi)d\tau.$$

Take $\tilde{\lambda}_{1,\frac{T}{2^k}}(x)$, the function $\rho_k(x) > 0$ and construct sets:

$$S(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_k(x)) =$$

$$= \{\lambda_{1,\frac{T}{2^k}}(x) = (\lambda_{1,\frac{T}{2^k},1}(x), \lambda_{1,\frac{T}{2^k},2}(x), \ldots, \lambda_{1,\frac{T}{2^k},2^k}(x))' \in C([0,\omega], R^{n2^k}):$$

$$\left\| \lambda_{1,\frac{T}{2^k},r}(x) - \tilde{\lambda}_{1,\frac{T}{2^k},r}(x) \right\| < \rho_k(x),\ r = \overline{1, 2^{k-1}}\}.$$

$$G_1(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_k(x)) = \{(x,t,u,v,w) : (x,t) \in \overline{\Omega},$$

$$\left\| u - \tilde{\lambda}_{1,\frac{T}{2^k},s} \right\| < \rho_k(x), \quad \left\| v - \tilde{\lambda}_{1,\frac{T}{2^k},s} \right\| < \rho_k(x),$$

$$\left\| w - \tilde{\lambda}_{1,\frac{T}{2^k},s} \right\| < \rho_k(x), \quad x \in [0,\omega], t \in \left[(s-1)\frac{T}{2^k}, s\frac{T}{2^k} \right), s = \overline{1,2^k} \Big\}.$$

Statement k. Let function $f(x,t,u,v,w)$ *at* $G_1(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_k(x))$ has uniformly continuous partial derivatives $f_u'(x,t,u,v,w)$, $f_v'(x,t,u,v,w)$, $f_w'(x,t,u,v,w)$ and matrix Jacobi $\frac{\partial}{\partial\lambda} Q_{1,\frac{T}{2^k}}(x, \psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi)$ is invertible for all $\lambda(x) \in S(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_1(x))$, $x \in [0,\omega]$ and the following inequalities are executed:

1) $$\left\| \left[\frac{\partial}{\partial\lambda} Q_{1,\frac{T}{2^k}}(x, \psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) \right]^{-1} \right\| \le \gamma_1(x, \frac{T}{2^k}),$$

2) $$[(c_1(x)+1)c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int_0^x c(\xi)e^{\int_0^\xi c(\xi_1)d\xi_1} \times$$

$$\times \int_0^\xi \max\{1, L_1(\xi_1)\}[(c_1(\xi_1)+1)c_0(\xi_1)+1]\gamma_1(\xi_1, \frac{T}{2^k}) \times$$

$$\times \left\| Q_{1,\frac{T}{2^k}}(x, \psi[\cdot] + \int_0^x \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi)d\xi, 0, \tilde{\lambda}_{1,\frac{T}{2^k}}(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \right.$$

$$\left. + \int_0^\xi \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi_1)d\xi_1, \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi), \dot{\psi}([\cdot]))d\xi) \right\| d\xi_1 d\xi + [c_0(x)c_1(x) +$$

$$+1]\gamma_1(x, \frac{T}{2^k}) \left\| Q_{1,\frac{T}{2^k}}(x, \psi[\cdot] + \int_0^x \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi)d\xi, 0, \tilde{\lambda}_{1,\frac{T}{2^k}}(x), \dot{\psi}([\cdot]) + \right.$$

$$\left. + \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi_1)d\xi_1, \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi), \dot{\psi}([\cdot]))d\xi) \right\| < \rho_1(x).$$

Then there is a number $\alpha \ge 1$ such that the sequence $\lambda^{(m+1)}_{1,\frac{T}{2^k}}(x)$, $m = 0,1,2,...,$ is determined by the iterative process:

$$\lambda^{(0)}_{1,\frac{T}{2^k}}(x) = \tilde{\lambda}_{1,\frac{T}{2^k}}(x),$$

$$\lambda^{(m+1)}_{1,\frac{T}{2^k}}(x) = \lambda^{(m)}_{1,\frac{T}{2^k}}(x) -$$

$$- \frac{1}{\alpha}\left[\frac{\partial}{\partial\lambda} Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi)d\xi, 0, \lambda^{(m)}_{1,\frac{T}{2^k}}(x), \dot{\psi}([\cdot]) + \right.$$

$$\left. + \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi_1)d\xi_1, \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi), \dot{\psi}([\cdot]))d\xi\right]^{-1} \times$$

$$\times Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi)d\xi, 0, \lambda^{(m)}_{1,\frac{T}{2^k}}(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) +$$

$$+ \int_0^\xi \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi_1)d\xi_1, \lambda^{(m)}_{1,\frac{T}{2^k}}(\xi), \dot{\psi}([\cdot]))d\xi), \qquad (2.74)$$

$m = 0,1,2,...,$ converges to $\lambda_{1,\frac{T}{2^k}}(x)$ - isolated solution of the system of equations

$$Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) + \int_0^x f(\xi,[\cdot],\psi([\cdot]) +$$

$$+ \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) = 0$$

at $S(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_1(x))$ and inequality takes place

$$\left\|\lambda_{1,\frac{T}{2^k}}(x) - \tilde{\lambda}_{1,\frac{T}{2^k}}(x)\right\| \le \gamma_1(x,\frac{T}{2^k})\frac{T}{2^k}L_1(x)M(x), \qquad (2.75)$$

where

$$M(x) = \max_{r=1,2^{k-1}}\left\|\int_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f(x,\tau,\psi(\tau) + \int_0^x \lambda_{1,\frac{T}{2^{k-1}},r}(\xi)d\xi, \lambda_{1,\frac{T}{2^{k-1}},r}(x), \dot{\psi}(\tau) + \right.$$

$$\left. + \int_0^x f(\xi,\tau,\psi(\tau) + \int_0^\xi \lambda_{1,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^{k-1}},r}(\xi), \dot{\psi}(\tau))d\xi)d\tau\right\|.$$

Really, as operator

$$Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) +$$

$$+ \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) = 0$$

in $S(\tilde{\lambda}_{1,\frac{T}{2^k}}(x), \rho_k(x))$ satisfies all the assumptions of Theorem 1 of [87], we obtain the existence $\tilde{\lambda}_{1,\frac{T}{2^k}}(x)$ - of an isolated solution of equations (2.72), (2.73) and the estimate

$$\left\|\lambda_{1,\frac{T}{2^k}}(x) - \tilde{\lambda}_{1,\frac{T}{2^k}}(x)\right\| \le$$

$$\leq \max_{x\in[0,\omega]} \gamma_1\left(x,\frac{T}{2^k}\right)\Big\| Q_{1,\frac{T}{2^k}}(x,\psi[\cdot]+\int_0^x \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi)d\xi,0,\tilde{\lambda}_{1,\frac{T}{2^k}}(x),\dot{\psi}([\cdot])+$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \tilde{\lambda}_{1,\frac{T}{2^k}}(\xi_1)d\xi_1,\tilde{\lambda}_{1,\frac{T}{2^k}}(\xi),\dot{\psi}([\cdot]))d\xi)\Big\|. \quad (2.76)$$

Let's estimate norm of the operator

$$Q_{1,\frac{T}{2^k}}(x,\psi[\cdot]+\int_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi):$$

$$\Big\| Q_{1,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x \tilde{\lambda}_{1,\frac{T}{2}}(\xi)d\xi,0,\tilde{\lambda}_{1,\frac{T}{2}}(x),\dot{\psi}([\cdot])+$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \tilde{\lambda}_{1,\frac{T}{2}}(\xi_1)d\xi_1,\tilde{\lambda}_{1,\frac{T}{2}}(\xi),\dot{\psi}([\cdot]))d\xi)\Big\| \leq$$

$$\leq L_1(x)\frac{T}{2^k}\max_{r=1,2^{k-1}}\Big\| \int_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f(x,\tau,\psi(\tau)+$$

$$+\int_0^x \lambda_{1,\frac{T}{2^{k-1}},r}(\xi)d\xi,\lambda_{1,\frac{T}{2^{k-1}},r}(x),\dot{\psi}(\tau)+$$

$$+\int_0^x f(\xi,\tau,\psi(\tau)+\int_0^\xi \lambda_{1,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1,\lambda_{1,\frac{T}{2^{k-1}},r}(\xi),\dot{\psi}(\tau))d\xi)d\tau\Big\|. \quad (2.77)$$

The right part (2.77) substituting in (2.76) we obtain the validity estimates (2.75). From the obtained estimate it can be seen that the norm of the difference $\lambda_{2,\frac{T}{2^k}}(x)-\tilde{\lambda}_{2,\frac{T}{2^k}}(x)$ as the number of splits increases N becomes a small value. Here we have advanced in step h proceeding from the solution of the system of equations (2.60) with N = 1, ν = 2. Now we will advance in ν. Let $\lambda_{1,\frac{T}{2^{k-1}}}\in C([0,\omega],R^{n2^{k-1}})$ - the solution of the system of equations

$$Q_{1,\frac{T}{2^{k-1}}}(x,\psi[\cdot]+\int_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int_0^x f(\xi,[\cdot],\psi([\cdot])+$$

$$+\int_0^\xi \lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)=0,$$

at $\nu=1$, $N=2^{k-1}$ and there is an inequality

$$\Big\|\lambda_{1,\frac{T}{2^{k-1}}}(x)-\tilde{\lambda}_{1,\frac{T}{2^{k-1}}}(x)\Big\| \leq \max_{x\in[0,\omega]}\gamma_2\left(x,\frac{T}{2^{k-1}}\right)\frac{T}{2^{k-1}}L_1(x)M(x),$$

$$M(x) = \max_{r=\overline{1,2^{k-2}}} \int\limits_{(2r-2)\frac{T}{2^{k-1}}}^{(2r-1)\frac{T}{2^{k-1}}} f(x,\tau,\psi(\tau) +$$

$$+ \int_0^x \lambda_{1,\frac{T}{2^{k-2}},r}(\xi)d\xi, \lambda_{1,\frac{T}{2^{k-2}},r}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+ \int_0^\xi \lambda_{1,\frac{T}{2^{k-2}},r}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^{k-2}},r}(\xi), \dot{\psi}(\tau))d\xi)d\tau \Big\|.$$

Find $\lambda_{1,\frac{T}{2^k}}(x)$ - solution of the equation

$$Q_{1,\frac{T}{2^k}}(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) +$$

$$+ \int_0^x f(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot]))d\xi) = 0,$$

at $\nu = 1$, $N = 2^k$, which is in vector-coordinate form is written as

$$\lambda_{1,\frac{T}{2^k},1}(x) - \lambda_{1,\frac{T}{2^k},2^k}(x) - \int\limits_{\frac{T}{2}}^{T} f(x,\tau,\psi(\tau) +$$

$$+ \int_0^x \lambda_{1,\frac{T}{2^k},2^k}(\xi)d\xi, \lambda_{1,\frac{T}{2^k},2^k}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+ \int_0^\xi \lambda_{1,\frac{T}{2^k},2^k}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k},2^k}(\xi), \dot{\psi}(\tau))d\xi)d\tau = 0, \quad (2.78)$$

$$\lambda_{1,\frac{T}{2^k},s}(x) + \int\limits_{(s-1)\frac{T}{2^k}}^{s\frac{T}{2^k}} f(x,\tau,\psi(\tau) +$$

$$+ \int_0^x \lambda_{1,\frac{T}{2^k},s}(\xi)d\xi, \lambda_{1,\frac{T}{2^k},s}(x), \dot{\psi}(\tau) + \int_0^x f(\xi,\tau,\psi(\tau) +$$

$$+ \int_0^\xi \lambda_{1,\frac{T}{2^k},s}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k},s}(\xi), \dot{\psi}(\tau))d\xi)d\tau - \lambda_{1,\frac{T}{2^k},s+1}(x) = 0, \quad (2.79)$$

where $x \in [0,\omega]$, $s = \overline{1,2^k - 1}$. Initial approximation systems (2.78), (2.79) vector-function

$$\tilde{\lambda}_{1,\frac{T}{2^k}}(x) = (\tilde{\lambda}_{1,\frac{T}{2^k},1}(x), \tilde{\lambda}_{1,\frac{T}{2^k},2}(x), \ldots, \tilde{\lambda}_{1,\frac{T}{2^k},2^k}(x))' \in C([0,\omega], R^{n2^k})$$

we define as follows: $\tilde{\lambda}_{1,\frac{T}{2^k},2r-1}(x) = \lambda_{1,\frac{T}{2^{k-1}},r}(x), r = \overline{1,2^{k-1}}$,

$$\tilde{\lambda}_{1,\frac{T}{2^k},2r}(x) = \lambda_{1,\frac{T}{2^{k-1}},r}(x) +$$

$$+\int\limits_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f(x,\tau,\psi(\tau)+\int\limits_0^x \lambda_{1,\frac{T}{2^{k-1}},r}(\xi)d\xi, \lambda_{1,\frac{T}{2^{k-1}},r}(x),\dot{\psi}(\tau)+$$

$$+\int\limits_0^x f(\xi,\tau,\psi(\tau)+\int\limits_0^{\xi} \lambda_{1,\frac{T}{2^k-1},r}(\xi_1)d\xi_1, \lambda_{1,\frac{T}{2^k-1},r}(\xi),\dot{\psi}(\tau))d\xi d\tau.$$

Take $\tilde{\lambda}_{1,\frac{T}{2^k}}(x),$ the function $\rho_k(x)>0$ and construct sets:

$$S(\tilde{\lambda}_{1,\frac{T}{2^k}}(x),\rho_k(x))=$$

$$=\{\lambda(x)=(\lambda_{1,\frac{T}{2^k},1}(x),\lambda_{1,\frac{T}{2^k},2}(x),\ldots,\lambda_{1,\frac{T}{2^k},2^k}(x))'\in C([0,\omega],R^{n2^k}):$$

$$\left\|\lambda_{1,\frac{T}{2^k},r}(x)-\tilde{\lambda}_{1,\frac{T}{2^k},r}(x)\right\|<\rho_k(x), r=\overline{1,2^{k-1}}\}.$$

$$G_1(\tilde{\lambda}_{1,\frac{T}{2^k}}(x),\rho_k(x))=\{(x,t,u,v,w):(x,t)\in\overline{\Omega},$$

$$\left\|u-\tilde{\lambda}_{1,\frac{T}{2^k},s}\right\|<\rho_k(x), \quad \left\|v-\tilde{\lambda}_{1,\frac{T}{2^k},s}\right\|<\rho_k(x),$$

$$\left\|w-\tilde{\lambda}_{1,\frac{T}{2^k},s}\right\|<\rho_k(x), \quad x\in[0,\omega], t\in\left[(s-1)\frac{T}{2^k}, s\frac{T}{2^k}\right), s=\overline{1,2^k}\right\}.$$

Statement (ν, k). Let function $f(x,t,u,v,w)$ *at* $G_1(\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),\rho_{\nu,k}(x))$ has uniformly continuous partial derivatives $f_u'(x,t,u,v,w)$, $f_v'(x,t,u,v,w)$, $f_w'(x,t,u,v,w)$ and matrix Jacobi $\frac{\partial}{\partial\lambda}Q_{\nu,\frac{T}{2^k}}(x,\psi[\cdot]+\int\limits_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot])+\int\limits_0^x f(\xi,[\cdot],\psi([\cdot])+$ $+\int\limits_0^{\xi}\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)$ is invertible for all $\lambda(x)\in S(\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),\rho_{\nu,k}(x))$, $x\in[0,\omega]$ and the following inequalities are executed:

1) $$\left\|\left[\frac{\partial}{\partial\lambda}Q_{\nu,\frac{T}{2^k}}(x,\psi[\cdot]+\int\limits_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+\int\limits_0^x f(\xi,[\cdot],\psi([\cdot])+\right.\right.$$

$$\left.\left.+\int\limits_0^{\xi}\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi)\right]^{-1}\right\|\le\gamma_\nu(x,\frac{T}{2^k}),$$

2) $$[(c_1(x)+1)c_0(x)+1]c_2(x)[L_2(x)+L_3(x)]\int\limits_0^x \alpha(\xi)e^{\int\limits_0^{\xi}\alpha(\xi_1)d\xi_1}\times$$

$$\times\int\limits_0^{\xi}\max\{1,L_1(\xi_1)\}[(c_1(\xi_1)+1)c_0(\xi_1)+1]\gamma_\nu(\xi_1,\frac{T}{2^k})\times$$

$$\times\left\| Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot]+\int_0^x \tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi)d\xi,0,\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),\dot{\psi}([\cdot])+\int_0^x f\Big(\xi,[\cdot],\psi([\cdot])+\right.$$

$$\left.+\int_0^{\xi}\tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi_1)d\xi_1,\tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi),\dot{\psi}([\cdot])\Big)d\xi\Big)\right\| d\xi_1 d\xi+[c_0(x)c_1(x)+$$

$$+1]\gamma_\nu\left(x,\frac{T}{2^k}\right)\left\| Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot]+\int_0^x \tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi)d\xi,0,\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),\dot{\psi}([\cdot])+\right.$$

$$\left.+\int_0^x f\Big(\xi,[\cdot],\psi([\cdot])+\int_0^{\xi}\tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi_1)d\xi_1,\tilde{\lambda}_{\nu,\frac{T}{2^k}}(\xi),\dot{\psi}([\cdot])\Big)d\xi\Big)\right\| < \rho_1(x).$$

Then there is a number $\alpha \geq 1$ such that the sequence $\lambda^{(m+1)}_{\nu,\frac{T}{2^k}}(x)$, $m = 0,1,2,...,$ is determined by the iterative process:

$$\lambda^{(0)}_{\nu,\frac{T}{2^k}}(x) = \tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),$$

$$\lambda^{(m+1)}_{\nu,\frac{T}{2^k}}(x) = \lambda^{(m)}_{\nu,\frac{T}{2^k}}(x) -$$

$$-\frac{1}{\alpha}\left[\frac{\partial}{\partial\lambda}Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot]+\int_0^x \lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi)d\xi,0,\lambda^{(m)}_{\nu,\frac{T}{2^k}}(x),\dot{\psi}([\cdot])+\right.$$

$$\left.+\int_0^x f\Big(\xi,[\cdot],\psi([\cdot])+\int_0^{\xi}\lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi_1)d\xi_1,\lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi),\dot{\psi}([\cdot])\Big)d\xi\Big)\right]^{-1}\times$$

$$\times Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot]+\int_0^x \lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi)d\xi,0,\lambda^{(m)}_{\nu,\frac{T}{2^k}}(x),\dot{\psi}([\cdot])+\int_0^x f\Big(\xi,[\cdot],\psi([\cdot])+$$

$$+\int_0^{\xi}\lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi_1)d\xi_1,\lambda^{(m)}_{\nu,\frac{T}{2^k}}(\xi),\dot{\psi}([\cdot])\Big)d\xi\Big),$$

converges to $\lambda_{\nu,\frac{T}{2^k}}(x)$ - isolated solution of the system of equations

$$Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot]+\int_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+$$

$$+\int_0^x f\Big(\xi,[\cdot],\psi([\cdot])+\int_0^{\xi}\lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot])\Big)d\xi\Big)=0$$

in $S(\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x),\rho_{\nu,k}(x))$ and inequality takes place

$$\left\|\lambda_{\nu,\frac{T}{2^k}}(x)-\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x)\right\| \leq \gamma_2\left(x,\frac{T}{2^k}\right)\frac{T}{2^k}L_1(x)M(x), \qquad (2.80)$$

where

$$M(x) = \max_{r=1,2^{k-1}} \Bigg\| \int\limits_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f\Big(x,\tau_1,\psi(\tau_1) +$$

$$+ \int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi, \int\limits_{(2r-2)\frac{T}{2^k}}^{\tau_1} f\Big(x,\tau_2,\psi(\tau_2) +$$

$$+ \int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi, \ldots \int\limits_{(2r-2)\frac{T}{2^k}}^{\tau_{\nu-1}} f\Big(x,\tau_\nu,\psi(\tau_\nu) +$$

$$+ \int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi, \lambda_{\nu,\frac{T}{2^{k-1}},r}(x), \dot{\psi}(\tau_\nu) + \int_0^x f\Big(\xi,\tau_\nu,\psi(\tau_\nu) +$$

$$+ \int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1, \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi), \dot{\psi}(\tau_\nu)\Big)d\xi\Big)d\tau_\nu \ldots, \dot{\psi}(\tau_2) +$$

$$+ \int_0^x f\Big(\xi,\tau_2,\psi(\tau_2) + \int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1, \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi), \dot{\psi}(\tau_2)\Big)d\xi\Big)d\tau_2, \dot{\psi}(\tau_1) +$$

$$+ \int_0^x f\Big(\xi,\tau_1,\psi(\tau_1) + \int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1, \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi), \dot{\psi}(\tau_1)\Big)d\xi\Big)d\tau_1 \Bigg\|.$$

Really, as operator

$$Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) +$$
$$+ \int_0^x f\Big(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot])\Big)d\xi\Big) = 0$$

in $S(\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x), \rho_{\nu,k}(x))$ satisfies all the assumptions of Theorem 1 of [87], then we obtain the existence of $\tilde{\lambda}_{\nu,\frac{T}{2^k}}(x)$ - isolated solution of equations

$$Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot] + \int_0^x \lambda(\xi)d\xi, 0, \lambda(x), \dot{\psi}([\cdot]) +$$
$$+ \int_0^x f\Big(\xi,[\cdot],\psi([\cdot]) + \int_0^\xi \lambda(\xi_1)d\xi_1, \lambda(\xi), \dot{\psi}([\cdot])\Big)d\xi\Big) = 0$$

and the estimate

$$\left\| \lambda_{\nu,\frac{T}{2^k}}(x) - \tilde{\lambda}_{\nu,\frac{T}{2^k}}(x) \right\| \le$$

$$\le \max_{x\in[0,\omega]} \gamma_\nu\Big(x,\frac{T}{2^k}\Big) \Bigg\| Q_{\nu,\frac{T}{2^k}}\Big(x,\psi[\cdot] + \int_0^x \tilde{\lambda}_{\nu,\frac{T}{2}}(\xi)d\xi, 0, \tilde{\lambda}_{\nu,\frac{T}{2}}(x), \dot{\psi}([\cdot]) +$$

$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \tilde{\lambda}_{\nu,\frac{T}{2}}(\xi_1)d\xi_1,\tilde{\lambda}_{\nu,\frac{T}{2}}(\xi),\dot{\psi}([\cdot]))d\xi\Bigg\|. \quad (2.81)$$

Let's estimate norm of the operator

$$Q_{\nu,\frac{T}{2^k}}(x,\psi[\cdot]+\int_0^x \lambda(\xi)d\xi,0,\lambda(x),\dot{\psi}([\cdot])+$$
$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \lambda(\xi_1)d\xi_1,\lambda(\xi),\dot{\psi}([\cdot]))d\xi):$$

$$\Bigg\|Q_{\nu,\frac{T}{2}}(x,\psi[\cdot]+\int_0^x \tilde{\lambda}_{\nu,\frac{T}{2}}(\xi)d\xi,0,\tilde{\lambda}_{\nu,\frac{T}{2}}(x),\dot{\psi}([\cdot])+$$
$$+\int_0^x f(\xi,[\cdot],\psi([\cdot])+\int_0^\xi \tilde{\lambda}_{\nu,\frac{T}{2}}(\xi_1)d\xi_1,\tilde{\lambda}_{\nu,\frac{T}{2}}(\xi),\dot{\psi}([\cdot]))d\xi)\Bigg\|\le$$

$$\le L_1(x)\frac{T}{2^k}\max_{r=\overline{1,2^{k-1}}}\Bigg\|\int_{(2r-2)\frac{T}{2^k}}^{(2r-1)\frac{T}{2^k}} f(x,\tau_1,\psi(\tau_1)+$$

$$+\int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi,\ \int_{(2r-2)\frac{T}{2^k}}^{\tau_1} f(x,\tau_2,\psi(\tau_2)+\int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi,\ldots$$

$$\ldots\int_{(2r-2)\frac{T}{2^k}}^{\tau_{\nu-1}} f(x,\tau_\nu,\psi(\tau_\nu)+\int_0^x \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi)d\xi,\lambda_{\nu,\frac{T}{2^{k-1}},r}(x),\dot{\psi}(\tau_\nu)+$$

$$+\int_0^x f(\xi,\tau_\nu,\psi(\tau_\nu)+$$

$$+\int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1,\lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi),\dot{\psi}(\tau_\nu))d\xi)d\tau_\nu\ldots,\dot{\psi}(\tau_2)+$$

$$+\int_0^x f(\xi,\tau_2,\psi(\tau_2)+$$

$$+\int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1,\lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi),\dot{\psi}(\tau_2))d\xi)d\tau_2,\dot{\psi}(\tau_1)+$$

$$+\int_0^x f(\xi,\tau_1,\psi(\tau_1)+\int_0^\xi \lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi_1)d\xi_1,\lambda_{\nu,\frac{T}{2^{k-1}},r}(\xi),\dot{\psi}(\tau_1))d\xi)d\tau_1\Bigg\|.$$

Substituting in (2.80) inequality (2.81) we obtain the validity of the estimate (2.79). Thus, the introduction of additional parameters and the consideration of an equivalent problem with parameter (2.37)-(2.41) allow you to reduce the problem of the initial approximation for the nonlinear semi-periodic boundary value problem (2.1)-(2.3) to finding $\lambda^{(0)}(x)\in C([0,\omega],R^{n2^k})$ - solutions system of equations (2.60). Promotion by number of substitutions $\nu\in N$ and in step $h>0: Nh=T$ carried out based on iterative processes where as an initial approximation the solution of equation (2.60) is taken with the previous parameters ν and h.

Conclusion

This work is devoted to the study of the solvability of semi-periodic boundary value problems for systems of hyperbolic equations with mixed derivatives and the construction of constructive algorithms for finding approximate solutions.

Before, using the method of introducing functional parameters for nonlocal boundary value problems of hyperbolic mixed derivative equations, two-parameter families of algorithms for finding solutions to nonlocal boundary value problems were constructed, at each step of which Goursat problems are solved. On the basis of this algorithm, necessary and sufficient conditions for the unique solvability of nonlocal boundary value problems for linear hyperbolic equations with a mixed derivative are established, and for non-local boundary value problems for non-linear hyperbolic equations - sufficient conditions for the existence of a solution. In this regard, it is important to find the necessary and sufficient conditions for the existence of an "isolated" solution of nonlocal boundary value problems for systems of nonlinear hyperbolic equations and the construction of systems of equations that allow one to determine the initial approximations of solutions. Therefore, along with these questions, we study the construction of constructive algorithms for finding solutions and the establishment of new criteria of the unique solvability of the semi-periodic boundary value problem for systems of linear hyperbolic equations with a mixed derivative.

In the first section of work, a linear semi-periodic boundary value problem for systems of hyperbolic equations with a mixed derivative is investigated. The studied boundary value problems are reduced to families of periodic boundary value problems for ordinary differential equations and functional relations. In terms of the initial data on the basis of a method of parametrization, constructive algorithms for finding approximate solutions to the problems under study are proposed, at each step of which there is no need to solve the problems of Goursat, and the coefficient criteria of unique solvability are obtained.

In the second section for the semi-periodic boundary value problem of systems of nonlinear hyperbolic equations with a mixed derivative in terms of the initial data, an algorithm for finding an approximate solution is proposed and sufficient convergence conditions are obtained.

One of the difficult problems of the theory of nonlinear boundary value problems is the choice of the initial approximation to the solution. In order to find the initial values of the functional parameters, systems of nonlinear equations are constructed. They are determined by the initial data of the problem and differ in the choice of the partition step and the number of substitutions [91-115].

The results obtained in this work are theoretical and can be used in the construction of computational algorithms for solving semi-periodic boundary value problems for systems of hyperbolic equations, as well as in the reading of special courses at the mathematical faculties of universities.

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