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4*e*-transport of Josephson current in weak links and anharmonic dependence by a current-phase

4*е*-транспорт джозефсоновского тока в слабых связях и ангармонизм ток-фазовой зависимости

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Бозе-Эйнштейн конденсациясы моделінің аясында экзотикалық асқын өткізгіштер негізінде жасалған джозефсондық әлсіз байланыстар арқылы заряды 4е бозондардың (4е-бозондар) туннелденуі қарастырылған. Джозефсон өтпелі арқылы 4е-бозондардың туннелденуінен асқын өткізгіш токтың фазаға стандартты гармоникалық (синусоидалық) тәуелділігінде ауытқулар пайда болу мүмкіндігі көрсетілген. Асқын өткізгіш токтың 4е-транспорты гипотезасының негізінде ток-фаза тәуелділігінің ангармонизмі байқалатын джозефсон өтпелінің вольт-амперлік сипаттамасында кернеудің бөлшек мәндерінде Шапиро сатылырының пайда болуы, «асқын өткізгіш ток–магнит ағымы» тәуелділігінде магнит ағымының квантына Φ_0 нормаланған сыртқы магнит ағымының Φ бөлшек мәндерінде псевдоминимумдардың пайда болуы интерпретацияланған.

Рассмотрена возможность туннеллирования (просачивания) бозонов с зарядом 4е (4е-бозоны) через экзотические сверхпроводящие слабые связи джозефсоновского типа с учетом модели бозе-эйнштейновкой конденсации. Показано, что туннелирование 4е-бозонов через джозефсоновский переход приводит к отклонению сверхпроводящего тока от стандартной гармонической (синусоидальной) зависимости. На основе гипотезы 4е-транспорта сверхтока интерпретируется возникновение полуцелых ступеней Шапиро на вольт-амперной характеристике джозефсоновского перехода с ангармонической ток-фазовой зависимостью под влиянием высокочастотного сигнала, появление псевдоминимумов в зависимости «сверхток-магнитный поток» при полуцелых значениях внешнего магнитного потока Φ, нормированного на квант магнитного потока Φ_0.

1. Introduction

It is known that the cuprate high-temperature SC (HTSC) have the granular structure and owning to midget of a coherence length $\xi \sim 1\ nm$ in HTSC grains form JWL in them [1]. (Defects in crystalline structure HTSC can operate as weak links too). Through such JWL both usual transport of Cooper pairs (2*e*), and unusual 4*e*-transport of a supercurrent can be carried out [2], (here e — an electron charge). An unusual 4*e*-transport of a supercurrent in the core is carried out in the exotic SC in which the coherence length (the size of Cooper pairs) $\xi \sim \hbar v_F / k_B T_c$ is comparable or less than medial distance between Cooper pairs s (here $\hbar$ — Planck constant, k_B – Boltzmann constant, v_F — Fermi velocity of an electron, T_c — critical temperature): $\xi \le s$, when in BCS-SC $\xi \gg s$. Small coherence length ξ, interelectronic distance s and $\xi \le s$, shows that the suitable theory for the description of the exotic SC is the theory (model) BEC, i.e. the description by means of condensation of bosons with a charge 2*e* (2*e*-bosons) [3].

However, in the model BEC the existence of 4*e*-bosons (bi-Cooper pairs) is possible, for example, in the work [4] the microscopic theory of a superfluidity and the SC on the basis of two-stage Fermi-Bose liquid is constructed and it is shown that the phase transition is accompanied by linking of bosons, i.e. the formation of a pair Bose condensates with a 4*e* charge from the single particle with a 2*e* charge.

2. Anharmonicity of a supercurrent dependence on the phase difference

In the BEC model it is supposed that 2*e*-bosons appear as a result of the Cooper interaction, and their relative quantity is spotted Boltzmann factor $\exp(-\Delta/T)$, where Δ — the characteristic energy of superconducting interaction. In the low-temperature field $T \ll \Delta$ the concentration of electronic pairs is exponentially incremented, and, on the contrary, at $T \gg \Delta$ exponentially decreases. According to the two-liquid model we present an electronic liquid to the SC state in the form of the sum of normal and SC components: $n = n_s + n_n$, where n_s, n_n — concentration of SC and normal electrons. For the exotic SC n_s represents the mixture consisting of the concentration of 2*e*- and 4*e*-bosons (n_{2e}, n_{4e}):

$$n_s = n_{2e} + n_{4e}. \qquad (1)$$

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Having admitted the existence of 4*e*-bosons in the SC, it is natural to guess the possibility of their (partial) tunneling through JJ (4*e*-transport of a supercurrent) besides an usual tunneling of 2*e*-bosons. If a standard dependence of a supercurrent through JJ on a phase difference between wave functions on the different sides of a barrier is featured by a sinusoidal function [5]:

$$I_s(\varphi) = I_c \sin\varphi, \tag{2}$$

(where I_c — critical current, φ — phase difference), does the presence of 4*e*-bosons in the SC influence the supercurrent magnitude (1) proceeding through JJ? For the determining of a supercurrent within the limits of the semiclassical theory of the SC it is sufficient to know the change of electrons SC density in the system with the velocity dn_s/dt [6], i.e. $I_s \sim dn_s/dt$. Taking into consideration (1) in I_s we have:

$$I_s \sim \frac{dn_{2e}}{dt} + \frac{dn_{4e}}{dt} = I_{2e} + I_{4e}. \tag{3}$$

The 4*e*-condensate is characterised by the presence of the quasilong-range order in correlators $\exp\left[2i(\chi_1 - \chi_2)\right]$, where χ_1, χ_2 — phases of order parametres of two SC [7]. In the usual superconducting correlator the quasilong-range order misses. Considering the quasilong-range order for the 4*e*-condensate the expression (3) can be written as the following:

$$I_s \sim I_1\left(\exp[i\varphi] - \exp[-i\varphi]\right) + I_2\left(\exp[2i\varphi] - \exp[-2i\varphi]\right), \tag{4}$$

where $\varphi = \chi_1 - \chi_2$ — phase difference, I_1, I_2 — constant components of a current.

Using property of a hyperbolic sine $\mathrm{sh}ix = i\sin x$ and having designated $I_{c1} = 2iI_1$, $I_{c2} = 2iI_2$ from (4) we have:

$$I_s(\varphi) = I_{c1}\sin\varphi + I_{c2}\sin 2\varphi, \tag{5}$$

where I_{c1}, I_{c2} — critical currents of the first and second harmonics. From (4), (5) it is obvious that a supercurrent dependence on the phase difference is deviated from the standard sinusoidal shape (2), i.e. becomes anharmonic (as well as in [8]). For an estimate of influence of the second harmonic on RCP we input coefficient of an anharmonicity $k = I_{c2}/I_{c1}$ and representing the value I_{c2} in (5) through I_{c1} we have:

$$I_s(\varphi) = I_{c1}\sin\varphi + kI_{c1}\sin 2\varphi. \tag{6}$$

In our case the influence of anharmonicity RCP on JJ at $k = 0{,}1 \div 0{,}8$ is studied, relying on the experimental data where anharmonicity RCP is shown by the presence of the second harmonic [9].

3. Influence of the 4e-transport of a supercurrent on the properties of Josephson weak links

It is known that on CVC JJ with anharmonic RCP under the influence of exterior HF radiations are observed half-integer steps of Shapiro at voltages $V_m = \hbar\omega m/4e$, besides usual integer which arise in JJ with harmonious RCP at voltages $V_m = \hbar\omega m/2e$, where ω — frequency of external HF signal, m — integer (fig. 1). In the work [10] are considered various reasons of occurrence of half-integer steps in CVC JJ related to terminal capacity JJ, large width of junction and nonsinusoidal RCP, in the work [11] the version of one of the reasons of a half-integer step formation with the increase of plasma frequency JJ under the influence of an anharmonicity has been offered. On the fig. 1 the results of the numerical modelling are presented taking into account an anharmonicity of a view (6) within the limits of resistive model JJ [12] (a continuous curve 1) and experiment (a dotted curve 2) at an exterior irradiation on frequency of 400 GHz [10].

We anharmonicity RCP which has arisen owing to 4*e*-transport of a supercurrent, in similar systems occurrence of half-integer steps it is possible to explain with the presence of 4*e*-bosons. In an usual 2*e*-mode, Shapiro steps arise in the result of influence on them by exterior HF signal with the frequency multiple to $f = \omega/2\pi$ at voltages [13]:

$$V_{2e} = \frac{m\hbar}{2e}\omega = \frac{m\hbar}{2e}\cdot 2\pi f = m\Phi_0 f, \tag{7}$$

where Ô$_0$ — magnetic flux quantum (fluxoid). In a mode of 4*e*-transport of a current in the formula (7) we replace $2e \to 4e$, Shapiro steps should appear twice more often (fig. 1, vertical dashed lines — for 4*e*-mode, and a dash-point line — for 2*e*-mode):

$$V_{4e} = \frac{m\hbar}{4e}\omega = \frac{m\hbar}{4e}\cdot 2\pi f = \frac{m\Phi_0}{2} f = \frac{V_{2e}}{2}. \tag{8}$$

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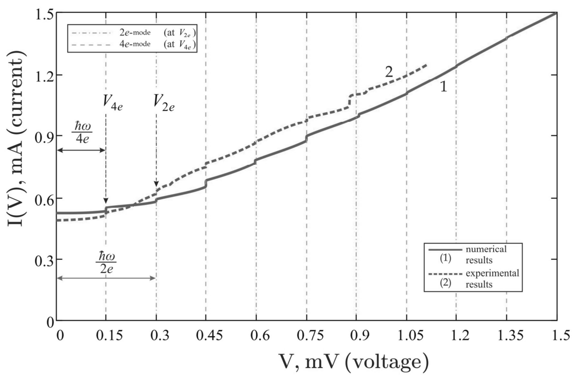

Figure 1. Half-integer steps of Shapiro on CVC JJ.

So, in the combined mode from 2*e* and 4*e* modes we can observe at voltages V_{2e} usual integer (7), and at V_{4e} — half-integer of Shapiro steps (8).

One more feature of JJ with anharmonicity RCP is the display of minimums depending from a supercurrent on the magnetic flux $I_s(\Phi)$ at half-integer values of the normalised on fluxoid an exterior magnetic flux [14].

Let's consider the magnetic flux influence on JJ taking into account 4*e*-transport of a supercurrent. As in a classical variant [15], let the axis x lies in the plane JJ, and the magnetic field is guided along an axis z, and along an axis y the field with a size d is shaped, where there is a magnetic field.

At such standing JJ using the modernised equation of Ferrell-Prange we find $\varphi(x)$, interposing $\varphi(x)$ into expression of a current density j_s according to (6), having integrated j_s on x along junction and after simple transformations we have the formula for the maximal value of a supercurrent:

$$I_{\max} = I_c \left| \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0} + \frac{k\sin(2\pi\Phi/\Phi_0)}{2\pi\Phi/\Phi_0} \right|, \qquad (9)$$

Dependence of the maximal supercurrent (9) at various values k is represented on the fig. 2 (a-d). As it is seen, in the case of anharmonicity besides «traditional» minimums at the integer values $\Phi/\Phi_0 = 0$; 1; 2; …, at half-integer values $\Phi/\Phi_0 = 0{,}5$; 1,5; 2,5; … are seen minimums (fig. 2, vertical dashed lines).

In our case the occurrence of minimums is caused with the presence and transport of 4*e*-bosons: $\Phi = m\Phi_0/2 = mh/4e$, and standard minimums with 2*e*-transport of a supercurrent: $\Phi = m\Phi_0 = mh/2e$, where m – integer.

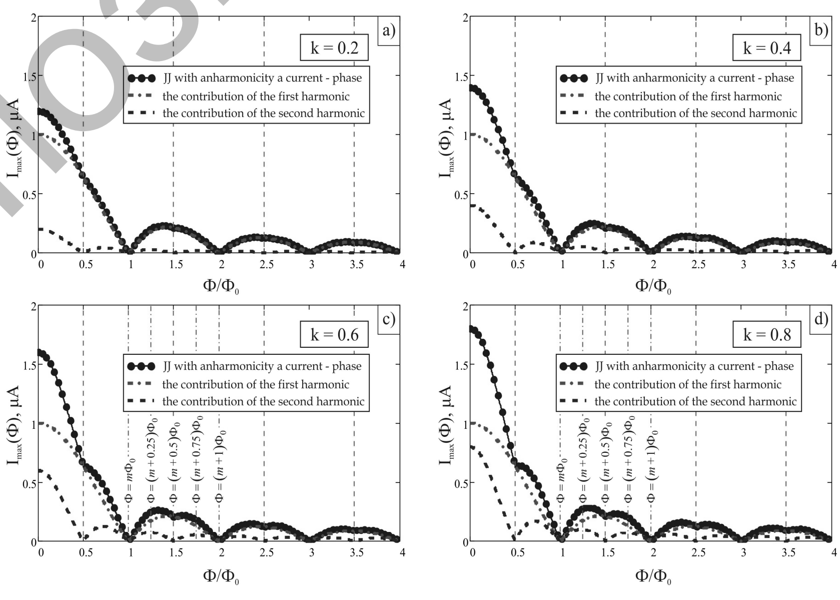

Figure 2. Dependence of a supercurrent on the magnetic flux at $k = 0{,}2$ (a); $0{,}4$ (b); $0{,}6$ (c); $0{,}8$ (d).

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4. Conclusion

Thus, it is ascertained, that in JJ on the basis of the SC described by the BEC model where probably 4*e*-transport of a supercurrent because of the presence of 4*e*-bosons is observed anharmonicity RCP (a diversion of a supercurrent magnitude from the standard sinusoidal function). In similar systems the appearance of the following phenomena is possible: occurrence of half-integer Shapiro steps on CVC at a voltage V_{4e} (8); if in the JJ with sinusoidal RCP at the integer values Φ/Φ_0 the mixed state is absolutely labile, and is stable only in half-integer values Φ/Φ_0, while in the JJ with anharmonic RCP the relative instability of the mixed state is observed both at the integer and half-integer values Φ/Φ_0; at values $\Phi = (m + 0,25)\Phi_0$ and $(m + 0,75)\Phi_0$ the maximal stability of the mixed state is developing and the increase of k stimulates the stability of the mixed state in these points (fig. 2 c, d).

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