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ON SOME PROPERTIES OF TOPOLOGICAL QUASIVARIETIES GENERATED BY CERTAINFINITE MODULAR LATTICES

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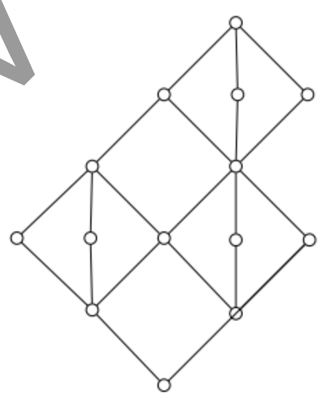
The present work considers certain finite modular lattices and topologicalquasivarieties generated by these lattices, investigates their properties.

Quasivariety is a class of algebras of the same type that is closed with respect to subalgebras, direct products (including the direct product of an empty family), and ultraproducts. The smallest quasivariety containing a class K is denoted by Q(K). If K is a finite family of finite algebras then Q(K) is called finitely generated. In the case K={A} we write Q(A) instead of Q({A}).

A finite algebra A with discrete topology τ generates a topological quasivariety Qτ(A) consisting of all topologically closed subalgebras of non-zero direct powers of A endowed with the product topology. Profinite algebras are exactly those that are isomorphic to inverse limits of finite algebras. Such algebras are naturally equipped with Boolean topologies. A topology τ is Boolean if it is compact, Hausdorff, and totally disconnected. A topological quasivariety Qτ(A) is standard if every Boolean topological algebra with the algebraic reduct in Q(A) is profinite. In this case we say that algebra A generates a standard topological quasivariety [1].

We construct the finite modular lattice T (see Figure 1) that does not satisfy one of Tumanov's conditions [2] but quasivariety Q(T) generated by this lattice is not finitely based. It has no finite basis of quasi-identities.We investigate the topological quasivariety generated by the lattice T and prove that it is not standard. And we would like to note that there is an infinite number of lattices similar to the lattice T.



T

Figure 1. Lattice T.

The main result of this work is as follows.

Theorem. The topological quasivariety generated by the lattice T is not standard.

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ON SMOOTHLY APPROXIMABLE ACYCLIC GRAPHS

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We deal with pseudofinite countably categorical structures [1]-[4], in particular, countable acyclic graphs [6,7].

Theorem 1. [7] Let Γ be an arbitrary countable graph in which each component contains a finite number of cycles. Then Γ is countably categorical if and only if Γ is bounded and finitely many 1-types are realized in it.

A. Lachlan introduced the concept of *smoothly approximable* structures to change the direction of analysis from finite to infinite, that is, to classify large finite structures that appear to be *smooth approximations* to an infinite limit. A more general approach is developed in [8].

Definition. [2] Let L be a countable language and let M be a countable and ω-categorical L-structure. L-structure M (or $Th(M)$) is said to be *smoothly approximable* if there is an ascending chain of finite substructures $A_0 \subseteq A_1 \subseteq \cdots \subseteq M$ such that $i \in \omega A_i = M$ and for every i, and for every $\acute{a}, \acute{b} \in A_i$ if $tp_M(\acute{a}) = tp_M(\acute{b})$, then there is an automorphism σ of M such that $\sigma(\acute{a}) = \acute{b}$ and $\sigma(A_i) = A_i$, or equivalently, if it is the union of an ω-chain of finite homogeneous substructures; or equivalently, if any sentence in $Th(M)$ is true of some finite homogeneous substructure of M.

Smoothly approximated structures were first examined in generality in [2], subsequently in [4]. The model theory of smoothly approximable structures has been developed very much further by G. Cherlin and E. Hrushovski [1].

Recall the following class defined in [5] for acyclic graphs.

Let $\mathrm{G_{fin}}(\lambda)$, for arbitrary cardinality λ, be the family of all infinite acyclic graphs consisting of λ connected components of bounded in aggregate diameters.

Theorem 2. [5] Theory T of any infinite acyclic graph Γ from the class $\mathrm{G_{fin}}(\lambda)$, for arbitrary cardinality λ, is pseudofinite.

Let us distinguish a subclass $\mathrm{G_{cc}}(\lambda)$ of the class $\mathrm{G_{fin}}(\lambda)$ as the class of all countably categorical acyclic graphs.

Theorem 3**.** Any theory Th(Γ), $\Gamma \in \mathrm{G_{cc}}(\lambda)$ is smoothly approximable.

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