МАТЕМАТИКА

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Recurrence relation generalized calculation of heat transfer coefficient

In this work, studying the task of distribution of heat in soil, the mathematical model of unidimensional task is offered. The measured value of temperature of soil of earth and temperature of air are set on a terrene. An inverse boundary value problem is considered, the adjoint problem is built, the iterative method of calculation of coefficient of heat emission of soil is obtained in an environment.

Key words: a beginning – regional task, the attended task, functional, the iterative method.

Репозиторий КарГУ

It is determined the following task [1] is

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta}{\partial z}\right),\ z\in(0,H),\quad t\in(0,t_{\max}); \tag{1}$$

$$\theta\big|_{t=0}=\varphi(x),\ \theta\big|_{z=0}=T_1; \tag{2}$$

$$\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=H}=-N(t)\left(\theta\big|_{z=H}-T_0(t)\right). \tag{3}$$

We'll look $N(t)$ for the generalized coefficient of heat exchange. The axis z directs towards, the beginning of coordinates is on the unchanging layer of soil temperature. Additionally the measured value of temperature of soil on a terrene $T_g(t)$ and temperature of air are set on a terrene.

We pay attention to the private case, when $T_0(t)=T_b(t)$. In this way, having the measures $C,\ \lambda,\ \varphi(x),\ T_1,\ T_b(t)$ and $T_g(t)$, it is required to define $N(t)$ and $\theta(z,t)$. The task is determined by the iterative method.

This n is the iterative parameter. In this way $N(t)$ is determined by the iterative $N(t,n),\ n=0,1,...$

In this case, the solution of the task (1)–(3) will also depend from n and $\theta(z,t)\to\theta(z,t,n)=\theta_n(z,t)$. We take the beginning meaning $N(t,0)$ and the following meaning $N(t,n)$ is determined from the condition of monotony of functional [2, 3]

$$J(N)=\int_0^{t_{\max}}(\theta(H,t)-T_g(t))^2\,dt. \tag{4}$$

For two following meaning n and $n+1$ of the iterative parameter the task (1)–(3) is written in such a way

$$C\frac{\partial\theta_{n+1}}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta_{n+1}}{\partial z}\right),\ C\frac{\partial\theta_n}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta_n}{\partial z}\right);$$

$$\theta_{n+1}\big|_{t=0}=\theta_0(z),\ \theta_n(z,t)\big|_{t=0}=\theta_0(z);$$

$$\theta_{n+1}\big|_{z=0}=T_1(t),\ \theta_n\big|_{z=0}=T_1(t);$$

$$\lambda\frac{\partial\theta_{n+1}}{\partial z}\bigg|_{z=H}=-N(t,n+1)\left(\theta_{n+1}-T_b(t)\right)_{z=H},\ \lambda\frac{\partial\theta_n}{\partial z}\bigg|_{z=H}=-N(t,n)\left(\theta_n-T_b(t)\right)_{z=H}.$$

Then for a difference

$$\Delta\theta(z,t)=\theta_{n+1}(z,t)-\theta_n(z,t)$$

an auxiliary task is made

$$C\frac{\Delta\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\Delta\theta}{\partial z}\right); \tag{5}$$

$$\Delta\theta\big|_{t=0}=0,\ \Delta\theta\big|_{z=0}=0; \tag{6}$$

$$\lambda\frac{\partial\Delta\theta}{\partial z}\bigg|_{z=H}+N_n\Delta\theta\big|_{z=H}=-\Delta N(\theta_{n+1}-T_b(t))_{z=H}\,. \tag{7}$$

We multiply (5) by an arbitrary function $\psi(z,t)$ and integrate on z from 0 till H, on t from 0 till $t_{\max}$. After single integration by installments on variables z and t we get equality

$$\int\limits_0^H (C\Delta\theta\cdot\psi)\Big|_{t=0}^{t=t_{\max}}dz-\int\limits_0^H\int\limits_0^{t_{\max}}\Delta\theta\cdot C\frac{\partial\psi}{\partial t}dtdz=$$

$$=\int\limits_{t=0}^{t=t_{\max}}\left(\lambda\frac{\partial\Delta\theta}{\partial z}\cdot\psi\right)_{z=0}^{z-H}-\int\limits_0^{T_{\max}}\int\limits_0^{k}\frac{\partial\Delta\theta}{\partial z}\cdot\lambda\frac{\partial\psi}{\partial z}dzdt.$$

Proposing that $\psi(z,t_{\max})=0,\ \psi(0,t)=0$ and accounting the conditions (6) and (7), we have

$$-\int\limits_0^H\int\limits_0^{t_{\max}}\Delta\theta\cdot C\frac{\partial\psi}{\partial t}dtdz=-\int\limits_0^{t_{\max}}\left[N_n\Delta\theta+\Delta N(\theta_{n+1}-T_b(t))\right]_{z=H}\psi(\mathrm{H},t)dt-$$

$$-\int\limits_0^{t_{\max}}\int\limits_0^{H}\frac{\partial\Delta\theta}{\partial z}\cdot\lambda\frac{\partial\psi}{\partial z}dzdt.$$

The second integral in the right part of the equality is integrated by installments on variables z. In that case we obtain

$$-\int\limits_0^H\int\limits_0^{t_{\max}}\Delta\theta\cdot C\cdot\frac{\partial\psi}{\partial t}dtdz=-\int\limits_0^{t_{\max}}\left[N_n\Delta\theta+\Delta N(\theta_{n+1}-T_b(t)\right]_{z=H}\psi(\mathrm{H},t)dt-$$

$$-\int\limits_0^{t_{\max}}\left(\Delta\theta\cdot\lambda\frac{\partial\psi}{\partial z}\right)_{z=0}^{z=H}dt+\int\limits_0^{t_{\max}}\int\limits_0^{H}\Delta\theta\cdot\frac{\partial}{\partial z}\left(\lambda\frac{\partial\psi}{\partial z}\right)dzdt.$$

Summarizing the equal number and taking attention to the boundary condition (6)–(7) we come to the conclusion

$$\int\limits_0^{t_{\max}}\int\limits_0^{H}\Delta\theta\left[C\cdot\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial z}\left(\lambda\frac{\partial\psi}{\partial z}\right)\right]dzdt+$$

$$+\int\limits_0^{t_{\max}}\Delta\theta(k,z)\cdot\left[\lambda\frac{\partial\psi}{\partial z}+N_n\psi\right]_{z=H}dt=-\int\limits_0^{t_{\max}}\Delta N\cdot(\theta_{n+1}-T_b(t)\big|_{z=H}\psi(H,t)dt.$$

This is

$$C\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial z}\left(\lambda\frac{\partial\psi}{\partial z}\right)=0;$$

$$\left(\lambda\frac{\partial\psi}{\partial z}+N_n\psi\right)_{z=H}=2\big(\theta(z,t)-T_g(t)\big)_{z=H}\,.$$

Then we have

$$2\int\limits_0^{t_{\max}}\Delta\theta\big(H,t\big)\cdot(\theta(H,t)-T_g(t))dt=$$

$$=-\int\limits_0^{t_{\max}}\Delta N\cdot\big(\theta_n-T_b(t)\big)_{z=H}\psi(k,t)dt-\int\limits_0^{t_{\max}}\big(\Delta N\cdot\Delta\theta\cdot\psi\big)_{z=H}dt. \tag{8}$$

Репозиторий КарГУ

In the process of calculation the adjoint task is received

$$C\frac{\partial\psi}{\partial t}+\frac{\partial}{\partial z}\left(\lambda\frac{\partial\psi}{\partial z}\right)=0,\ \psi\big|_{t=t_{\max}}=0; \tag{9}$$

$$\psi\big|_{z=0}=0,\ \left(\lambda\frac{\partial\psi}{\partial z}+N_n(t)\psi\right)_{z=H}=2\left(\theta-T_g(t)\right)_{z=H}. \tag{10}$$

Using a functional from equality (4), correlation is made

$$\begin{gathered}J\left(N(n+1)\right)-J\left(N(n)\right)=\\=\int_0^{t_{\max}}\left(\theta(H,t;n+1)-T_g(t)\right)^2dt-\int_0^{t_{\max}}\left(\theta(H,t;n)-T_g(t)\right)^2dt=\\=2\int_0^{t_{\max}}\Delta\theta(H,t)\left(\theta(H,t;n)-T_g(t)\right)dt=\\=-\int_0^{t_{\max}}\Delta N\left(\theta(H,t;n)-T_g(t)\right)\psi(H,t)dt-\int_0^{t_{\max}}\Delta N(\Delta\theta\cdot\psi)_{z=H}\,dt.\end{gathered}$$

To minimize a functional (4) we put, that

$$\Delta N=\beta(n)\left(\theta(H,t;n)-T_g(t)\right)\cdot\psi(H,z).$$

Then the increase of functional is written down in a kind

$$\begin{gathered}J\left(N(n+1)\right)-J\left(N(n)\right)-\beta(n)\int_0^{t_{\max}}\left(\theta(H,t;n)-T_g(t)\right)^2\psi^2(H,t)dt-\\-\beta(n)\int_0^{t_{\max}}\left(\theta(H,t;n)-T_g(t)\right)\cdot\psi^2(H,t)\Delta\theta(H,t)dt.\end{gathered} \tag{11}$$

Thus for the calculation of the generalized coefficient of heat emission an iterative formula is accepted

$$N(t;n+1)=N(t;n)+\beta(n)\left(\theta(H,t;n)-T_g(t)\right)\cdot\psi(H,t). \tag{12}$$

The generalized coefficient of heat emission can receive the different meanings. We try to determine the case, when the coefficient is $N(t)=N=const$. In this case from (12) a formula is made

$$N(n+1)=N(n)+\beta(n)\int_0^{T_{\max}}(\theta(H,t;n)-T_g(t))\psi(H,t)dt$$

and the equality (11) is written in this kind

$$\begin{gathered}J\left(N(n+1)\right)-J\left(N(n)\right)=-\beta(n)\left(\int_0^{t_{\max}}\left(\theta(H,t;n)-T_g(t)\right)\psi(H,t)dt\right)^2-\\-\beta(n)\int_0^{t_{\max}}\left(\theta(H,t;n)-T_g(t)\right)\psi(H,t)dt\cdot\int_0^{t_{\max}}\psi(H,t)\Delta\theta(H,t)dt.\end{gathered}$$

References

1 *Rysbaiuly B.* Newton's method to solve the problem of heat transfer in the freezing soil // Pensee J. — Vol. 76. — Is. 1. — p. 261–275.

2 *Hasanov A.* Simultaneous determination of source terms in a linear parabolic problem from the final overdetermination: Weak solution approach // J. Mathematical Analysis and Applications. — 2007. — 330. — p. 766–779.

3 *Rysbaiuly B., Baiymankulov A.* Development and justification of the method of calculation the capillary diffusion of the soil // Wulfenia Journal, 2014, March — Vol. 20. — Is. 12. — p. 483–500.

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Жалпыланған жылу беру коэффициентін есептеудің рекурренттік арақатынасы

Мақалада автормен топырақта жылудың таратылу есебі қарастырыла отырып, бірөлшемді есептің математикалық моделі ұсынылды. Жер топырағы температурасының өлшенген мәні мен жер бетіндегі ауа температурасы берілді. Кері бастапқы-аймақтық есеп қарастырылып, қосалқы есеп шығарылды, топырақтың қоршаған ортаға жылу беру коэффициентін есептеу үшін итерациялық әдіс анықталды.

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Рекуррентное соотношение расчета обобщенного коэффициента теплообмена

Автором в результате изучения задачи распространения тепла в грунте предложена математическая модель одномерной задачи. Заданы измеренное значение температуры грунта земли и температура воздуха на поверхности земли. Рассмотрена обратная начально-краевая задача, построена сопряженная задача, выведен итерационный метод расчета коэффициента теплоотдачи почвы в окружающую среду.

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Priori estimates for the solution of direct and adjoint problems

In this work the reverse coefficient task is considered. The system of joint equalizations of transfer of heat, in an array «underground — under earth layer of atmospheres — the active layer of soil» in soil is described by nonlinear differential equation of the second order. A priori estimates for decisions direct and adjoint tasks for the case, when the generalized coefficient of heat emission is equal to the permanent size, are concluded.

Key words: the coefficient of heate mission, direct an dattended tasks, apriories timations, Koshi's in equality.

It is decided the task [1] is:

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial\theta}{\partial z}\right),\ z\in(0,H),\quad t\in(0,t_{\max}); \tag{1}$$

$$\theta\big|_{t=0}=\theta_0(z),\ \theta\big|_{z=0}=T_1; \tag{2}$$

$$\lambda\frac{\partial\theta}{\partial z}\bigg|_{z=H}=-N(t)\left(\theta\big|_{z=H}-T_0(t)\right). \tag{3}$$

It is required to define the meaning $N(t)$ that is the generalized coefficient of heat exchange. The axis z directs upwards, the beginning of coordinates is on the unchanging layer of temperature of soil. In the capacity of additional entered basis it is given the air temperature and measured value of temperature of soil on a surface. It is considered the particular case, when $T_0(t)=T_b(t)$. That is, having the measures C, λ, $\varphi(x)$, T_1, $T_b(t)$ and $T_g(t)$ it is required to define $N(t)$ and $\theta(z,t)$. Here $T_g(t)$ is the measured value of temperature of soil on the earth surface.

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