Auxiliary boundary value problem (BVP). Let $\Omega_1 = \{x, y: -\pi < x, y < \pi\}$ and $Q_1 = \Omega_1 \times \{t > 0\}$.

$$z_t - \Delta z + \alpha z(0, y, t) + \beta z(x, 0, t) = 0, \quad \{x, y, t\} \in Q_1, \tag{5}$$

$$z(x, y, 0) = z_0(x, y), \quad \{x, y\} \in \Omega_1, \tag{6}$$

$$\frac{\partial^j z(-\pi, y, t)}{\partial x^j} = \frac{\partial^j z(\pi, y, t)}{\partial x^j}, \quad \{y, t\} \in (-\pi, \pi) \times \{t > 0\},$$

$$\frac{\partial^j z(x, -\pi, t)}{\partial y^j} = \frac{\partial^j z(x, \pi, t)}{\partial y^j}, \quad \{x, t\} \in (-\pi, \pi) \times \{t > 0\}, \quad j = 0,1. \tag{7}$$

The problem is to find an initial function $z_0(x, y)$ such that a solution of the BVP (5)–(7) satisfies the inequality

$$\|z(x, y, t)\|_{L_2(\Omega_1)} \le C_0 e^{-\sigma t}, \quad \sigma > 0, \quad t > 0. \tag{8}$$

We recall, as we indicated above, that here σ is a given constant and C_0 is an arbitrary bounded constant.

We will define the function $z_0(x, y)$ as a continuation of the function $u_0(x, y)$, which was given in the original domain Ω. Thus in the auxiliary boundary value problem (5)–(7) it is needed to find the function $z_0(x, y)$ on the square Ω_1, so that the requirement (8) is satisfied for a solution $z(x, y, t)$ of the problem (5)–(7). In this case the condition (4) holds for restriction $u(x, y, t)$ of $z(x, y, t)$ too and a required boundary control $p(x, y, t)$, $\{x, y\} \in \Sigma$ is defined as trace of function $z(x, y, t)$ for $\{x, y, t\} \in \Sigma$.

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A NONLOCAL PROBLEM FOR ESSENTIALLY LOADED DIFFERENTIAL EQUATIONS WITH INTEGRAL CONDITIONS

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We consider the following linear boundary value problem for systems of essentially loaded differential equations with integral conditions:

$$\frac{dx}{dt} = A_0(t)x + \sum_{i=1}^{m} A_i(t)x(\theta_i) + \sum_{i=1}^{m} M_i(t)\dot{x}(\theta_i) + f(t), \quad t \in (0,T), \quad (1)$$

$$\sum_{j=0}^{m+1} \int_{\theta_{j-1}}^{\theta_j} B_j(t)x(t)\,dt = d, \quad d \in R^n, \quad x \in R^n, \quad (2)$$

where $(n \times n)$-matrices $A_k(t)$, $(k = \overline{0,m})$,$M_i(t)$, $(i = \overline{1,m})$,$B_j(t)$, $(j = \overline{0,m+1})$,and n-vector-function $f(t)$are continuous on $[0,T]$; and $0 = \theta_0 < \theta_1 < \cdots < \theta_m < \theta_{m+1} = T$, $\|x\| = \max_{i=\overline{1,n}} |x_i|$.

Let $C([0,T],R^n)$ denote the space of continuous functions$x$: $[0,T] \to R^n$ with the norm $\|x\|_1 = \max_{t \in [0,T]} \|x(t)\|$.

A solution to problem (1), (2) is a continuously differentiable on $(0,T)$ function $x(t) \in C([0,T],R^n)$satisfying the system of essentially loaded differential equations (1) and the integral conditions (2).

In recent years the theory of problems for loaded differential equations has been advanced. Various important problems of mathematical physics and mathematical biology lead to boundary value problems for loaded differential equations [1, 2]. Different problems for loaded differential equations with integral conditions and methods for finding their solutions are considered in [3-6].

In this paper we use the approach offered in [7-9] to solve the boundary value problem for systems of essentially loaded differential equations with integral conditions (1), (2). This approach based on the algorithms of the Dzhumabaev parameterization method [10] and numerical methods for solving Cauchy problems for ordinary differential equations.Dzhumabaev parameterization method was previously developed for boundary value problems for loaded differential equations [6]. Conditions for the unique solvability of the investigating problems were established and algorithms for finding approximate solutions were constructed [7].

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