THE BOUNDARY VALUE PROBLEMS IN MATHEMATICAL MODELING OF MECHANICAL PROCCESES

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We shall consider the partial differential equations that describe mathematical models of mechanical and physical phenomena. We often use the second order partial differential equations of hyperbolic type in the problems of oscillation theory and we apply the parabolic equations in problems of mechanics, where the characteristics of the various elements of constructions are investigated under the influence of different temperatures.

Consider the problem of vibrations of the infinite rod [1]

$$u_{xx} - u_{yy} + au_x + bu_y + cu = 0,\ -\infty < x < \infty,\ 0 < y < \infty;\ u(x,0) = \varphi(x),\ u_y(x,0) = g(x).$$

Using the method of the Riemann function, we find

$$u(x,y) = \frac{\varphi(x-y)\cdot e^{-\frac{a-b}{2}y} + \varphi(x+y)\cdot e^{-\frac{a+b}{2}y}}{2} -$$

$$-\frac{1}{2}e^{\frac{b}{2}y}\int_{x-y}^{x+y}\left\{\frac{b}{2}J_0\left(\sqrt{c_1}\sqrt{(x-\xi)^2-y^2}\right) - \sqrt{c_1}\,y\frac{J_1\left(\sqrt{c_1}\sqrt{(x-\xi)^2-y^2}\right)}{\sqrt{(x-\xi)^2-y^2}}\right\}\cdot e^{-\frac{a}{2}(x-\xi)}\varphi(\xi)d\xi +$$

$$+\frac{1}{2}e^{\frac{b}{2}y}\int_{x-y}^{x+y}J_0\left(\sqrt{c_1}\sqrt{(x-\xi)^2-y^2}\right)\cdot e^{-\frac{a}{2}(x-\xi)}g(\xi)d\xi\ /$$

Applying the Laplace transformation to the more general problem for the wave equation [1]

$$u_{tt} = a^2u_{xx} + c^2u + f(x,t),\ -\infty < x < \infty, 0 < t < \infty;\ u(x,0) = \varphi(x),\ u_t(x,0) = g(x),$$

we receive the solution in the analytic form

$$u(x,t) = \frac{1}{2a}\cdot\int_{x-at}^{x+at}g(\xi)I_0\left(c\sqrt{t^2-\frac{(x-\xi)^2}{a^2}}\right)d\xi +$$

$$+\frac{\varphi(x-at)+\varphi(x+at)}{2} + \frac{ct}{2a}\cdot\int_{x-at}^{x+at}\varphi(\xi)\frac{I_1\left(c\sqrt{t^2-\frac{(x-\xi)^2}{a^2}}\right)}{\sqrt{t^2-\frac{(x-\xi)^2}{a^2}}}d\xi +$$

$$+\frac{1}{2a}\cdot\int_0^t d\tau\int_{x-a(t-\tau)}^{x+a(t-\tau)}f(\xi,\tau)I_0\left(c\sqrt{(t-\tau)^2-\frac{(x-\xi)^2}{a^2}}\right)d\xi.$$

The boundary value problem for the heating of the infinite rod [2]

$$\frac{\partial u}{\partial t} = \frac{a^2}{x^\nu}\cdot\frac{\partial}{\partial x}\left(x^\nu\frac{\partial u}{\partial x}\right),\ \ 0 < x < \infty,\ \ 0 < t < \infty;\ u(0,t) = \varphi(t),\ \ u(\infty,t) = 0,\ \ u(x,0) = 0.$$

by applying the mathematical methods has the following solution of this problem

$$u(x,t) = \int_0^t \frac{x^{2\eta}\cdot\varphi(\tau)}{\Gamma(\eta)\cdot\left(4a^2\right)^\eta\cdot(t-\tau)^{1+\eta}}\cdot\exp\left(-\frac{x^2}{4a^2\cdot(t-\tau)}\right)d\tau.$$

References

1. *Vlasenko V.D*. Mathematical modeling in continuum mechanics problems. - Khabarovsk: Publishing house Tikhookean. State. University, 2010. - 103 p.

2. *Shpadi Y.R*. The heat conduction problems in solids with variable section. /Thesis for the degree of Ph.D. - Almaty, 1998. - 140 p.