

Let $0 < p < \infty$. Then $\varphi: (0, \infty) \rightarrow (0, \infty)$ belongs to the class G_p , if φ is essentially nondecreasing and there exist positive constants C, C' such that the inequalities $\varphi(t_1) \leq C\varphi(t_2)$ and $t_2^{\frac{d}{p}}\varphi(t_2) \leq C't_1^{\frac{d}{p}}\varphi(t_1)$ hold for all $0 < t_1 \leq t_2 < \infty$.

Let $M = \{M_j(x)\}_{j=0}^{\infty} \subset L_{\infty}(R)$ be a sequence of functions which can be represented as $M_j(x) = \int_{-\infty}^{x_1} d\mu_j(y)$; $j = 0, 1, 2, \dots$, where the μ_j 's; $j = 0, 1, 2, \dots$; are finite measures with uniformly bounded variation, i.e. $\sup_{j \in \mathbb{N}} \int_R |d\mu_j| < \infty$.

Let $k \geq 0$. Then Bessel potential space $H_2^k(R)$ is the collection of all $L_2(R)$ such that

$$\|f|H_2^k(R)\| := \|F^{-1}[(1+|\xi|^2)^{k/2}Ff(\xi)]|L_2(R)\| < \infty$$

Theorem. Let $1 \leq p < \infty$, $1 \leq q \leq \infty$ and a function $\varphi: (0, \infty) \rightarrow (0, \infty)$, $\varphi \in G_p$. Let $(\Lambda_j)_j$ be a given family of finite and nontrivial intervals. With d_j we denote the length of Λ_j . Let

$k > \frac{1}{2} + \frac{1}{\min(p, q)}$. Then there exists a constant C such that the inequality

$$\left\| \left(\sum_{j=0}^{\infty} \left| \sum_{k=-\infty}^{\infty} M_j(k) c_k(f_j) e^{ikx} \right|^q \right)^{1/q} |M_p^{\varphi}(T, l_q) \right\| \leq C \sup_{j=0, 1, \dots} \|M_j(d_j \cdot)|H_2^k(R)\| \| (f_j)_j |M_p^{\varphi}(T, l_q)\|$$

holds for all sequences of functions $M_j \in H_2^k(R)$ and all sequences $(f_j)_j$ of trigonometric polynomials such that $c_k(f_j) = 0$ if $k \notin \Lambda_j$, $j \in \mathbb{N}_0$. Theorem is a generalization of the corresponding result for $\varphi = |B(0, r)|^{\frac{1}{p}}$, for we refer to [1].

References

1. *Schmeisser H.-J. and Triebel H.* Topics in Fourier analysis and function spaces, Wiley (Chichester, 1987).

SEMIFINITE TRACIAL SUBALGEBRAS

Bekjan T.^{1,2}, Oshanova A.²

Faculty of Mechanics and Mathematics,¹ Xinjiang University (China, Urumqi),

²Gumilyov Eurasian National University (Kazakhstan, Astana)

E-mail: bekjant@yahoo.com

Noncommutative Hardy space theory has received considerable progress since the seminal paper by Arveson [1] in 1967. He introduced the notion of finite, maximal, subdiagonal algebras \mathbf{A} of \mathbf{M} , as noncommutative analogues of weak* Dirichlet algebras. Many classical results of Hardy space have been successfully transferred to the noncommutative setting. The first author [2] obtained that if a tracial subalgebra has L_p -factorization ($0 < p < \infty$), then it is a subdiagonal algebra. In [3], the first author and Ospanov proved that if a tracial subalgebra \mathbf{A} has L_E -factorization, then \mathbf{A} is a subdiagonal algebra, where E is a symmetric quasi Banach space on $[0, 1]$.

We continue this line of investigation. The aim of this talk is to prove some characterizations of subdiagonal algebras of semifinite von Neumann algebras.

Theorem. *Let \mathbf{A} be a tracial subalgebra of \mathbf{M} with respect to \mathbf{D} . Then the following conditions are equivalent:*

1. \mathbf{A} is a subdiagonal algebra of \mathbf{M} .
2. \mathbf{A} is a τ -maximal tracial subalgebra of \mathbf{M} satisfying the unique normal state extension property.
3. \mathbf{A} satisfies L_2 -density and the unique normal state extension property.

References

1. Arveson W.B. Analyticity in operator algebras, Amer. J. Math **89** (1967) 578-642.
2. Bekjan T.N. Characterization of subdiagonal algebras, Proc. Amer. Math. Soc. **139** (2011), 1121-1126.
3. Bekjan T.N. and Osipov K.N. Symmetric space characterization of subdiagonal algebras, **139** (2011), 1121-1126.

НЕРАВЕНСТВО ТИПА ХАРДИ В МАТРИЧНОМ ПРЕДСТАВЛЕНИИ

Билал Ш.

Институт математики и математического моделирования МОН РК, Алматы, Казахстан

E-mail: bilal44@mail.ru

1. Постановка задачи. Пусть $w = \{w_i\}_{i=1}^{\infty}, v = \{v_i\}_{i=1}^{\infty}$ последовательности неотрицательных чисел, $u_k > 0, k \geq 1$. Пусть $f = \{f_i\}_{i=1}^{\infty}$ произвольная последовательность действительных чисел. Положим $K = \{f: f \geq 0\}, K^{\uparrow} = \{f: 0 \leq f^{\uparrow}\}, K^{\downarrow} = \{f: 0 \leq f^{\downarrow}\}, F_k = \sum_{i=1}^k f_i, F_k^* = \sum_{i=k}^{\infty} f_i$ при $k \geq 1$ и $F_0 = 0$ (\uparrow - знак невозрастания, \downarrow - знак неубывания). Рассматривается задача о нахождении величины

$$J_{\infty}(u, v, g, K) = \sup_{f \geq 0} \sum_{i=1}^{\infty} f_i g_i \quad (1)$$

$$\sup_{1 \leq i < \infty} u_i f_i + \sup_{1 \leq i < \infty} v_i F_i$$

для $g \in K^{\downarrow}$ и на основе этого устанавливается неравенство дуальное неравенству вида:

$$\sup w_k (Af)_k \leq C \left[\sup_{1 \leq i < \infty} u_i f_i + \sup_{1 \leq i < \infty} v_i F_i \right], f \geq 0, \quad (2)$$

A – действительный матричный оператор вида $(Af)_k = \sum_{i=1}^k a_{k,i} f_i, k \geq 1$. Для каждого $n \geq 1$ определим

$$\varphi_n = \left\{ \min_{1 \leq k < n} \left[\left(\sum_{i=k}^n u_i^{-1} \right)^{-1} + \sup_{k \leq i < \infty} v_i \right] \right\} \quad \text{и положим } \varphi_0 = 0.$$

Теорема 1. Пусть $g \in K^{\downarrow}$. Тогда $J_{\infty}(u, v, g, K) = \bigcup_{1 \leq i < \infty} \sup g_i (\varphi_i - \varphi_{i-1})$.