$$\begin{cases} \dot{a}_n = a_n(b_n - b_{n+1}) + a_n \sum_{i=1}^{N} \left((g_n^i)^2 - (g_{n+1}^i)^2 \right), \\ \dot{b}_n = 2(a_{n-1}^2 - a_n^2) - 2\sum_{i=1}^{N} g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k,\ k = 1,\dots,N,\ n = 0,1,\dots,N-1 \end{cases} \quad (1)$$

with the boundary conditions

$$a_{-1} = a_{N-1} = 0. \quad (2)$$

The system (1),(2) is considered subject to the initial conditions

$$a_n(0) = a_n^0,\ b_n(0) = b_n^0, n = 0,1,\dots,N-1, \quad (3)$$

where a_n^0, b_n^0 are given complex numbers such that $a_n^0 \neq 0$ $(n = 0,1,\dots,N-2)$, $a_{N-1}^0 = 0$.

The main aim of this work is to derive representations for the solutions $a_n(t), b_n(t)$, $g_n^1(t), g_n^2(t),\dots, g_n^N(t)$, $n = 0,1,\dots,N-1$ of the finite complex Toda lattice (1) with a self-consistent source by means of the inverse spectral problem for the complex Jacobi matrices. For this goal spectral data of the complex Jacobi matrices are introduced and an inverse spectral problem from the spectral data is solved. The time evolution of the spectral data for the Jacobi matrix associated with the solution of the Toda lattice is computed. Using the solution of the inverse spectral problem with respect to the time-dependent spectral data we reconstruct the time-dependent Jacobi matrix and hence the desired solution of the finite complex Toda lattice with self-consistent source.

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A PROBLEM FOR LOADED DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT OF GENERALIZED TYPE

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We consider the following linear three-point boundary-value problem for the system of loaded differential equations with piecewise constant argument of generalized type:

$$\frac{dx}{dt} = A_0(t)x + K(t)x(\gamma(t)) + \sum_{i=1}^{m+1} A_i(t)x(\theta_{i-1}) + f(t), \quad t \in (0,T), \quad (1)$$

$$Bx(0) + Dx(\theta_1) + Cx(T) = d, \quad d \in R^n, \quad x \in R^n, \quad (2)$$

where $(n \times n)$-matrices $A_j(t)$, $(j = \overline{0, m+1})$, $K(t)$are continuous on $[0,T]$,and n-vector-function $f(t)$ is piecewise continuous on $[0,T]$ with possible discontinuities of the first kind at the points

$t = \theta_i,\ i = \overline{1,m}$; B, C and $D$are constant $(n \times n)$-matrices, and $0 = \theta_0 < \theta_1 < \cdots < \theta_m < \theta_{m+1} = T$, $\|x\| = \max_{i=\overline{1,n}} |x_i|$.

The argument $\gamma(t)$ is a step function defined as $\gamma(t) = \xi_{i-1}$if $t \in [\theta_{i-1}, \theta_i)$, $i = \overline{1, m+1}$; $\theta_{i-1} < \xi_{i-1} < \theta_i$for all $i = \overline{1, m+1}$; where$0 = \theta_0 < \theta_1 < \cdots < \theta_m < \theta_{m+1} = T$.

A function$x(t)$ is called a solution to problem (1), (2) if:

(i) $x(t)$is continuous on$[0, T]$;

(ii) $x(t)$is differentiable on $[0, T]$with the possible exception of the points θ_j, $j = \overline{0, m}$, where the one-sided derivatives exist;

(iii) $x(t)$satisfies(1) on each interval (θ_{i-1}, θ_i), $i = \overline{1, m+1}$; at the points θ_j, $j = \overline{0, m}$, Eq. (1) is satisfied by the right-hand derivatives of $x(t)$;

(iv) $x(t)$satisfies the boundary condition (2).

Boundary-value problems for differential equations with piecewise constant argument have been under intensive investigation of researchers in mathematics, biology, engineering, and other fields for the last 30 years [1-3]. Many problems for loaded differential equations and methods for solving them were investigated in [4-8] and the references therewith.

Main goal in this paper is to extend the Dzhumabaev parametrization method [9] to loaded differential equations with piecewise constant argument of generalized type. For this purpose, we have developed computational method solving a boundary-value problem for loaded differential equations with piecewise constant argument of generalized type.

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INTEGRATION OF THE LOADED SINE-GORDON EQUATION WITH SOURCE OF THE INTEGRAL TYPE.

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The sine-Gordon equation arises in applications as diverse as the description of surfaces of constant mean curvature.

In the work [1],[2] were shown that the sine-Gordon equation with a self-consistent source and loaded sine-Gordon equation with a self-consistent source can be solved using the inverse scattering problem method.