DOI 10.31489/2022PH2/37-48

UDC 530.1;53.04

G.S. Altayeva[1], O.V. Razina[1], P.Yu. Tsyba[1]

[1]L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan
(E-mail.ru: olvikraz@mail.ru)

Cosmography in the multifield cosmological model

This paper analyzes a cosmological model containing the fermion field, scalar field and vector field with Yukawa interaction. Such a model allows one to research the contribution of various types of matter to the dynamics of the universe. In flat, homogeneous, and isotropic space-time, this coupling can provide the acceleration expansion of the universe. Cosmological reconstruction of dynamical equations is obtained using hybrid solution. This solution is researched by cosmography and energy condition. In the model under study, a zero energy condition, a strong energy condition, and a dominant energy condition are satisfied, and a weak energy condition, which is not mandatory, is not satisfied. It is shown how the cosmographic parameters – the parameters of deceleration q, jerk j, and snap s – can be related to the hybrid value of the scale factor. The resulting analysis makes it possible to relate the model-independent results obtained from cosmography to theoretically substantiated assumptions of gravity. The total density and pressure of the energy of the gravitational field are found in the form of the sum of contributions, which are associated with the bosonic, fermionic, vector fields, as well as the Yukawa type potential. In the model under study, in the early epoch, the bosonic field is responsible for the accelerated regime. Fermionic and vector fields have a positive pressure value, and therefore slow down the accelerated expansion of the universe. At a later time, a transition to a slow mode occurs, as the total pressure tends to zero.

Keywords: scalar field, fermionic field, Yukawa-type interaction, vector field, cosmography, deceleration parameter.

Introduction

The accelerated expansion of the universe was discovered in 1998 based on an SN Ia type brightness curve and its luminosity at maximum. This phenomenon is supported by data of other cosmological observations, such as measurements of the temperature anisotropy of the CMB and the polarization of the cosmic microwave background and large-scale structure in the [1–4]. There are a large of number theoretical models capable of explaining the acceleration expansion and the most popular model assumes that a considerable part of the universe is in the form of dark energy or dark matter [5–17]. An unusual property of dark energy is that it exerts negative pressure on space. Understanding the nature and origin of dark energy is an important question and still an unsolved problem of modern cosmology.

The Standard Model is a successful model of the Big Bang theory. His predictions were confirmed by observations of [18–23], in particular by the expansion of the universe and the existence of relic radiation. Its success is associated with the explanation of the synthesis of light elements and the model of the early universe. The standard model is homogeneous and isotropic at large scales, as evidenced by observations.

The search for the responsible elements for accelerated periods in the evolution of the universe is fundamental in cosmology. Several candidates have been proposed describing both the inflationary period and the modern accelerated epoch: scalar fields, exotic equations of state, and the cosmological constant.

Another way is to consider the fermion field as a gravity sourse in expansion of the universe [24–26]. In [27–33], gravitation models were researched using multiple sources. The result includes exact solutions, anisotropy to isotropy transition scenarios, and cyclic cosmology.

At considering the fermion field as responsible for acceleration expansion of the universe then different regimes arise. The fermion field rapidly increases and matter is created until it begins to dominate, and as a result, the initial accelerated expansion slows down. When the universe enters the area of dominance of matter, then the fermion field again prevails, which leads to an era of accelerated growth rates of the scale factor. In this case, the fermion field is responsible for inflation in the early Universe and dark energy for the late Universe, without the need for a cosmological study of the constant terms or the scalar field. In the late Universe, energy begins to dominate again and a gradual transition to dark energy occurs, the so-called fermionic energy period, in which the accelerated regime begins and continues in the modern era.

Thus, the purpose of our work is to study the influence of scalar, fermion and massive vector fields and their interaction on the dynamics and evolution of cosmological regimes in a homogeneous and isotropic spatially flat universe. To achieve the goal and for the selected action, we will perform the following tasks we will find the equations of motion and then construct a solution to these equations using a hybrid scaling factor.

Experimental

Let us consider the general action in the form of

$$S=\int\sqrt{-g}d^4x\{\frac{R}{2}+\frac{1}{2}\partial^{\mu}\varphi\partial_{\mu}\varphi-\frac{1}{2}m_b^2\varphi^2+\frac{i}{2}\left[\bar{\psi}\Gamma^{\mu}D_{\mu}\psi-(D_{\mu}\bar{\psi})\Gamma^{\mu}\psi\right]-$$
$$-V(\bar{\psi}\psi)-\lambda\bar{\psi}\varphi\psi+\frac{1}{2}m_v^2A\mu A^{\mu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\}.$$

Here sourses of gravity are the fermion field and its potential $V(\bar{\psi}\psi)$

$$L_f=\frac{i}{2}\left[\bar{\psi}\Gamma^{\mu}D_{\mu}\psi-(D_{\mu}\bar{\psi})\Gamma^{\mu}\psi\right]-V(\bar{\psi}\psi), \tag{1}$$

where ψ and $\bar{\psi}=\psi^{+}\gamma^{0}$ represent the spinor field and its adjoint, respectively. The covariant derivatives in (1)

$$D_{\mu}\psi=\partial_{\mu}\psi-\Omega_{\mu}\psi+iqA_{\mu}\psi,$$
$$D_{\mu}\bar{\psi}=\partial_{\mu}\bar{\psi}+\bar{\psi}\Omega_{\mu}-iq\bar{\psi}A_{\mu}.$$

Here q is a constant which couples the fermion field with the vector field A_{μ}. Moreover, Ω_{μ} is the spin connection

$$\Omega_{\mu}=-\frac{1}{4}g_{\rho}\sigma\left[\Gamma^{\rho}_{\mu\delta}-e^{b}(\partial_{\mu}e^{b}_{\delta})\right]\Gamma^{\delta}\Gamma^{\sigma},$$

where $\Gamma^{\nu}_{\sigma\lambda}$ is the Christoffel symbols.

The Lagrangian density of a massive scalar field ϕ without self-interaction potential

$$L_b=\frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi-\frac{1}{2}m_b^2\phi^2,$$

where m_b is mass of the scalar field.

The Lagrangian density of the massive vector field A_{μ}

$$L_v=\frac{1}{2}m_v^2A_{\mu}A^{\mu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where m_v is mass of the vector field and $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$.

The Lagrangian density corresponds to the Yukawa interaction between the fermionic and the scalar fields

$$L_Y=-\lambda\bar{\psi}\phi\psi,$$

where λ is the coupling constant of the Yukawa potential.

In order to study the evolution of a homogeneous and isotropic spatially flat universe, we use the Friedmann-Robertson-Walker metric

$$ds^2=-dt^2+a(t)^2(dx^2+dy^2+dz^2), \tag{2}$$

where $a(t)$ is scale factor of the universe, and the Ricci scalar is expressed as

$$R=6\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right),$$

Here we consider the case of time-like vector field, namely

$$A_{\mu}=(A_0(t),0,0,0).$$

This case is the only possible ansatz compatible with a homogeneous and isotropic space-time. We assume that the self-interaction potential of the fermion field is $V(u)=\xi u^{n}$, where ξ and n are constants and $u=\bar{\psi}\psi$ is biliner function. Then the corresponding point-like Lagrangian has the form of

$$L = 3a\dot{a}^2 - a^3 \frac{i}{2}(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi + 2iqA_0\bar{\psi}\gamma^0\psi) + \\ + a^3\{\xi u^n - \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m_b^2\varphi^2 + \lambda\bar{\psi}\varphi\psi - \frac{1}{2}m_v^2 A_0^2\}. \tag{3}$$

From the Euler-Lagrange equations and energy-momentum tensor, the complete system of equations of motion corresponding to the Lagrangian (3) take the form

$$3H^2 = \rho, \tag{4}$$

$$2\dot{H} + 3H^2 = -p, \tag{5}$$

$$\ddot{\phi} + 3H\dot{\phi} + \lambda u + m_b^2\phi = 0, \tag{6}$$

$$A_0 = q\frac{\bar{\psi}\gamma^0\psi}{m_v^2}, \tag{7}$$

$$\dot{\bar{\psi}} + \frac{3}{2}H\bar{\psi} - i\bar{\psi}\left[qA_0 + n\xi u^{n-1}\gamma^0 + \lambda\phi\gamma^0\right] = 0, \tag{8}$$

$$\dot{\psi} + \frac{3}{2}H\psi + i\left[n\xi u^{n-1}\gamma^0 + qA_0 + \lambda\phi\gamma^0\right]\psi = 0, \tag{9}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{10}$$

where

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_b^2\phi^2 + \xi u^n + \lambda\bar{\psi}\phi\psi + \frac{1}{2}q^2\frac{(\bar{\psi}\gamma^0\psi)^2}{m_v^2}. \tag{11}$$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_b^2\phi^2 + \xi(n-1)u^n + \frac{1}{2}q^2\frac{(\bar{\psi}\gamma^0\psi)^2}{m_v^2}. \tag{12}$$

The equations (4), (5) are the Friedmann equations; the equation (6) is the Klein-Gordon equation; the equation (7) is the vector field equation; (8) and (9) are Dirac equations; the equation (10) is the conservation law; (11), (12) are the energy density and pressure, respectively.

We can assume that the total energy density of the gravitational field as the sum $\rho = \rho_b + \rho_f + \rho_Y + \rho_A$ is associated with the bosonic, fermionic fields, as well as the Yukawa potential and the vector field, respectively. Their expressions are of the form

$$\rho_b = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_b^2\phi^2, \tag{13}$$

$$\rho_f = \xi u^n, \tag{14}$$

$$\rho_A = \frac{1}{2}q^2\frac{(\bar{\psi}\gamma^0\psi)^2}{m_v^2}. \tag{15}$$

$$\rho_Y = \lambda\bar{\psi}\phi\psi. \tag{16}$$

Total pressure of the gravitational field in the form as $p = p_b + p_f + p_A$ is associated with bosonic, fermionic and vecton fields, respectively. The pressure of a Yukawa type potential is zero. Their expressions are of the form

$$p_b = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m_b^2\phi^2, \tag{17}$$

$$p_f = \xi(n-1)u^n, \tag{18}$$

$$p_A = \frac{1}{2}q^2\frac{(\bar{\psi}\gamma^0\psi)^2}{m_v^2}, \tag{19}$$

$$p_Y = 0. \tag{20}$$

Results

The system of equations (4)–(12) has the following solution in the form of a hybrid function

$$a = a_0 e^{\alpha t} t^{\beta}, \tag{21}$$

where a_0, α and β are constants and $\alpha > 0$. By solving equations (11) and (12) together and taking into account the hybrid dependence on the scale factor, we can find the form of the scalar field function

$$\phi = \frac{-\dfrac{\lambda c}{a_0^3 e^{3\alpha t} t^{3\beta}} + \sqrt{\lambda^2 c^2 a_0^{-6} e^{-6\alpha t} t^{-6\beta} - 4m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \dfrac{12\alpha\beta}{t} - \dfrac{2\beta(3\beta-1)}{t^2})}}{2m_b^2}.$$

From equation (7), we obtain a solution for the vector field

$$A_0 = \frac{\grave{c}}{m_v^2 a_0^3 e^{3\alpha t} t^{3\beta}},$$

where we have previously derived the relation simplifying further calculations from the Dirac equations (8), (9) $\bar{\psi}\gamma^0\psi = \dfrac{\grave{c}}{a^3}$, $\grave{c}$ is constant.

For the fermionic field, we will search for a solution in the form

$$\psi_k = E_k(t) e^{iF_k(t)}, k = 0,1,2,3. \tag{22}$$

Expanding the Dirac equations (8), (9) in component terms and substituting in them the general form of the fermion field field (22), we find the exact value of the coefficients

$$E_k = E_{k0} a^{-\frac{3}{2}},$$

$$F_k = -\frac{q^2 \grave{c} 3^{\frac{3}{2}\beta-1} e^{-\frac{3}{2}\alpha t} t^{-3\beta} (\alpha t)^{\frac{3}{2}\beta} - WhittakerM(-\frac{3}{2}\beta, -\frac{3}{2}\beta + \frac{1}{2}, 3\alpha t)}{m_v^2 a_0^3 \alpha(-1+3\beta)} -$$

$$- n\xi\left(\frac{c^{n-1}(n-1)^{3\beta(n-1)} \alpha^{3\beta(n-1)-1} 3^{3\beta(n-1)-1} (WhittakerM(1-3\beta(n-1), 3\alpha(n-1)t))}{a_0^{3(n-1)}}\right) -$$

$$- \frac{\lambda c(1-3\beta) 3^{3\beta-1} \alpha^{3\beta-1}}{2a_0^3 m_b^2 (3\beta-1)} + F_{k0} + \int N dt,$$

where E_{k0} and F_{k0} are integration constants, $F_k = -F_m$, (k=0,1; m=2,3) and we introduced the notation

$$N = \frac{\sqrt{\lambda^2 c^2 a_0^{-6} e^{-6\alpha t} t^{-6\beta} - 4m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \dfrac{12\alpha\beta}{t} - \dfrac{2\beta(3\beta-1)}{t^2})}}{2m_b^2}.$$

From the form of the self-interaction potential of the fermion field chosen above, taking into account the form of the scale factor, we find its dependence on time t

$$V = \xi\left(\frac{c}{a_0^3 e^{3\alpha t} t^{3\beta}}\right)^n.$$

The total energy density and pressure of the model under study from the Friedman equations (4) and (5) are equal to

$$p = -3(\alpha + \frac{\beta}{t})^2 + \frac{2\beta}{t^2},$$

$$\rho = 3(\alpha + \frac{\beta}{t})^2.$$

The component-wise contributions of each of the fields to the total density (13)–(16), respectively, are

$$\rho_b = \frac{-\frac{\lambda c}{a_0^3 e^{3\alpha t} t^{3\beta}} \sqrt{\lambda^2 c^2 a_0^{-6} e^{-6\alpha t} t^{-6\alpha} - 4m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \frac{12\alpha t}{t} - \frac{2\beta(3\beta-1)}{t^2})}}{4m_v^2} +$$

$$+\frac{1}{2}\dot{\phi}^2 - \frac{\lambda^2 c^2 a_0^{-6} e^{-6\alpha t} t^{-6\alpha} - 2m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \frac{12\alpha\beta}{t} - \frac{2\beta(3\beta-1)}{t^2})}{4m_v^2},$$

$$\rho_f = \xi(\frac{c}{a_0^3 e^{3\alpha t} t^{3\beta}})^n,$$

$$\rho_Y = \frac{-\frac{\lambda^2 c^2}{a_0^6 e^{6\alpha t} t^{6\beta}} + \frac{\lambda c}{a_0^3 e^{3\alpha t} t^{3\beta}} \sqrt{\lambda^2 c^2 a_0^{-6} t^{-6\alpha} - 4m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \frac{12\alpha t}{t} - \frac{2\beta(3\beta-1)}{t^2})}}{2m_b^2},$$

$$\rho_A = \frac{1}{2} q^2 \frac{`c^2}{a_0^3 e^{3\alpha t} t^{3\beta} m_v^2},$$

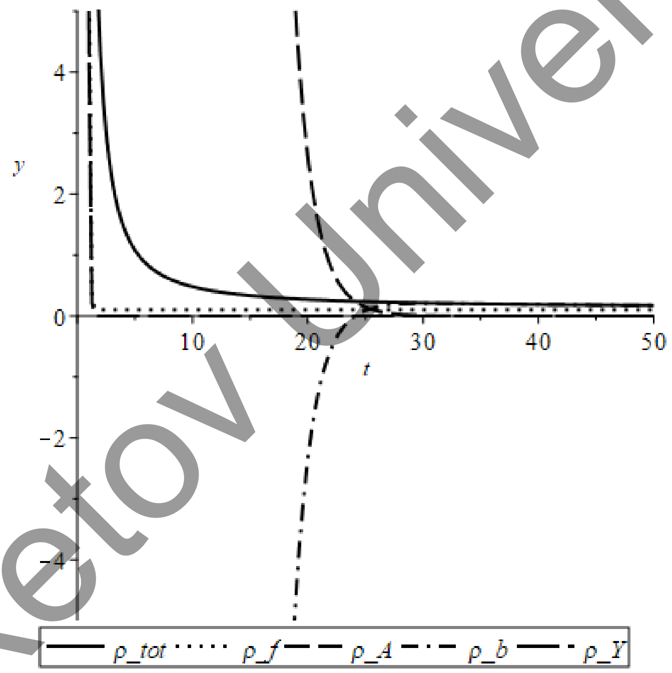


Figure 1. Energy density ρ depends on time t, at
$m = 10^{-6}, \xi = 5, c = 1, n = 2, a_0 = 1, \alpha = 0.2, q = 2, c` = 10, \phi = 2, \lambda = 2$

Figure 1 shows the total energy density ρ (solid line), fermionic field density ρ_f (dotted line), vector field density ρ_A (dash line), scalar field density ρ_b (dash-dotted line), potential density Yukawa type ρ_Y (open line). The component-wise contributions of each of the fields to the total pressure (17)–(20), respectively, are

$$p_b = -\frac{\frac{\lambda c}{a_0^3 e^{3\alpha t} t^{3\beta}} \sqrt{\lambda^2 c^2 a_0^{-6} e^{3\alpha t} t^{-6\alpha} - 4m_b^2(\xi(2-n)c^n (a_0 e^{\alpha t} t^{\beta})^{-3n} - 6\alpha^2 - \frac{12\alpha t}{t} - \frac{2\beta(3\beta-1)}{t^2})}}{4m_v^2} +$$

$$+\frac{1}{2}\dot{\phi}^2+\frac{\lambda^2c^2a_0^{-6}e^{6\alpha t}t^{-6\alpha}-2m_b^2(\xi(2-n)c^n(a_0e^{\alpha t}t^{\beta})^{-3n}-6\alpha^2-\frac{12\alpha\beta}{t}-\frac{2\beta(3\beta-1)}{t^2})}{4m_v^2},$$

$$p_f=\xi(n-1)(\frac{c}{a_0^3e^{3\alpha t}t^{3\beta}})^n,$$

$$p_A=\frac{1}{2}q^2\frac{`c^2}{a_0^6e^{6\alpha t}t^{6\beta}m_v^2},$$

$$p_Y=0.$$

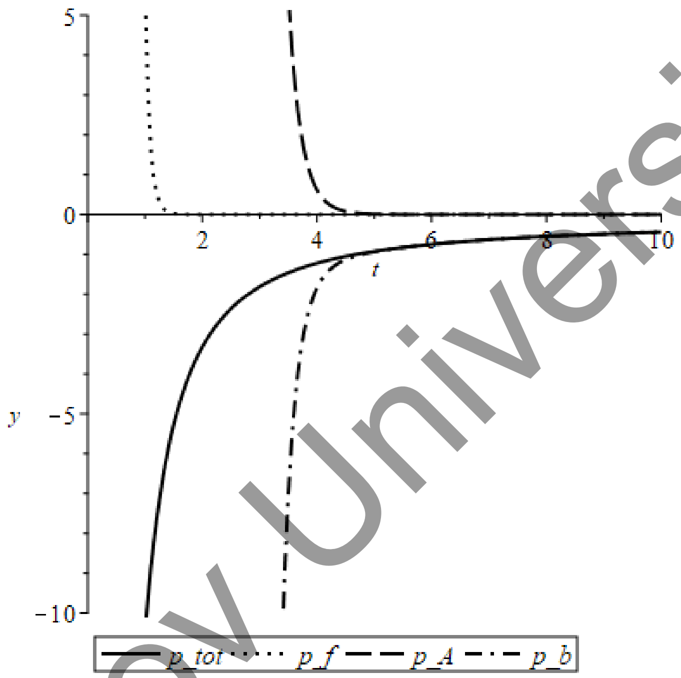


Figure 2. Pressure p and energy density ρ component contributions versus time t, at $m=10^{-6},\xi=5,c=1,n=2,a_0=1,\alpha=0.2,q=2,c`=10,\phi=2,\lambda=2$

Figure 2 shows the total pressure of the model p (solid line), pressure of the fermionic field p_f (dotted line), pressure of the vector field p_A (dash line) pressure of the scalar field p_b (dash-dotted line).

In the model under consideration, the scalar and fermion fields have negative pressure, while the massive vector field has a slight positive pressure. The Yukawa field does not contribute. In the early epoch, the bosonic field is responsible for the accelerated regime, but in later time, a transition to the slow regime occurs since the total pressure tends to zero.

Cosmography

Cosmography ensures to test cosmological models that do not contradict the cosmological principle [34]. The components of the dark energy introduced by us into the model change the equations of motion, but do not affect the relationship between the kinematic characteristics. The expansion of the scale factor in a Taylor series in the vicinity of the current time instant t_0 leads to an expression that depends only on the metric (2) and is completely independent of the model [35, 36].

$$a(t)=a_0+\dot{a}(t_0)+\frac{1}{2!}\ddot{a}(t_0)(t-t_0)^2+\frac{1}{3!}\dddot{a}(t_0)(t-t_0)^3+\frac{1}{4!}\ddddot{a}(t_0)(t-t_0)^4, \quad (23)$$

where 0 means the current value of the quantity and terms above the fifth order have been omitted. Functions in terms of derivatives of the scale factor and their values at the hybrid law scale factor (21) are Hubble parameter

$$H(t)=\frac{1}{a}\frac{da}{dt}=\alpha+\frac{\beta}{t}.$$

Deceleration parameter

$$q(t)=-\frac{1}{a}\frac{d^2a}{dt^2}\left(\frac{1}{a}\frac{da}{dt}\right)^{-2}=-1+\frac{2\alpha\beta t}{\alpha^2t^2+\beta^2}.$$

Jerk parameter

$$j(t)=\frac{1}{a}\frac{d^3a}{dt^3}\left(\frac{1}{a}\frac{da}{dt}\right)^{-3}=1+\frac{3\alpha\beta t\left(\alpha t+\beta\right)}{\alpha^3t^3+\beta^3}.$$

Snap parameter

$$s(t)=\frac{1}{a}\frac{d^3a}{dt^3}\left(\frac{1}{a}\frac{da}{dt}\right)^{-3}=1+\frac{\alpha\beta t\left(4\alpha^2t^2+6\alpha\beta t^2+3\alpha\beta t+4\beta^2\right)}{\alpha^4t^4+\beta^4}.$$

The parameters of deceleration, jerk, and snap are dimensionless. Using them, one can rewrite the equation (23) as

$$a(t)=a_0\left[1+H_0(t-t_0)-\frac{1}{2!}q_0H_0^2(t-t_0)^2+\frac{1}{3!}j_0H_0^3(t-t_0)^3+\frac{1}{4!}s_0H_0^4(t-t_0)^4\right].$$

An accelerated increase in the scale factor occurs at $q<0$. An accelerated increase in the expansion rate, $H>0$, corresponds to $q<-1$.

Energy conditions

In general, relativity and modified theories of gravity, the distribution of mass, momentum and angular momentum must have values for any field and are described by the energy momentum tensor or matter tensor. However, Einstein's field equation does not impose restrictions on the types of state of matter or non-gravitational regions admissible in the space-time model. This is a strong point, since general relativity should be as independent as possible from any assumptions of non-gravitational physics. The weak point is that the Einstein equation admits solutions by properties that most cosmologists regard as non-physical, i.e., too unusual to fit in the real [34] universe.

The energy conditions are such criteria. They describe the properties characteristic of all states of matter and all non-gravitational areas that are studied in physics. Energy conditions can eliminate many non-physical solutions of the Einstein equations. In cosmology, these four energy conditions are of great importance.

Null Energy Condition (NEC)

$$\rho+p\geq 0.$$

Weak Energy Condition (WEC)

$$\rho\geq 0,\quad \rho+p\geq 0.$$

Strong Energy Condition (SEC)

$$\rho+3p\geq 0,\quad \rho+p\geq 0.$$

Dominant Energy Condition (DEC)

$$\rho\geq 0,\quad -\rho\leq p\leq\rho.$$

For hybrid solution (21), the energy-momentum components are read as follows

NEC

$$2\beta t^{-2}\geq 0 \tag{24}$$

WEC

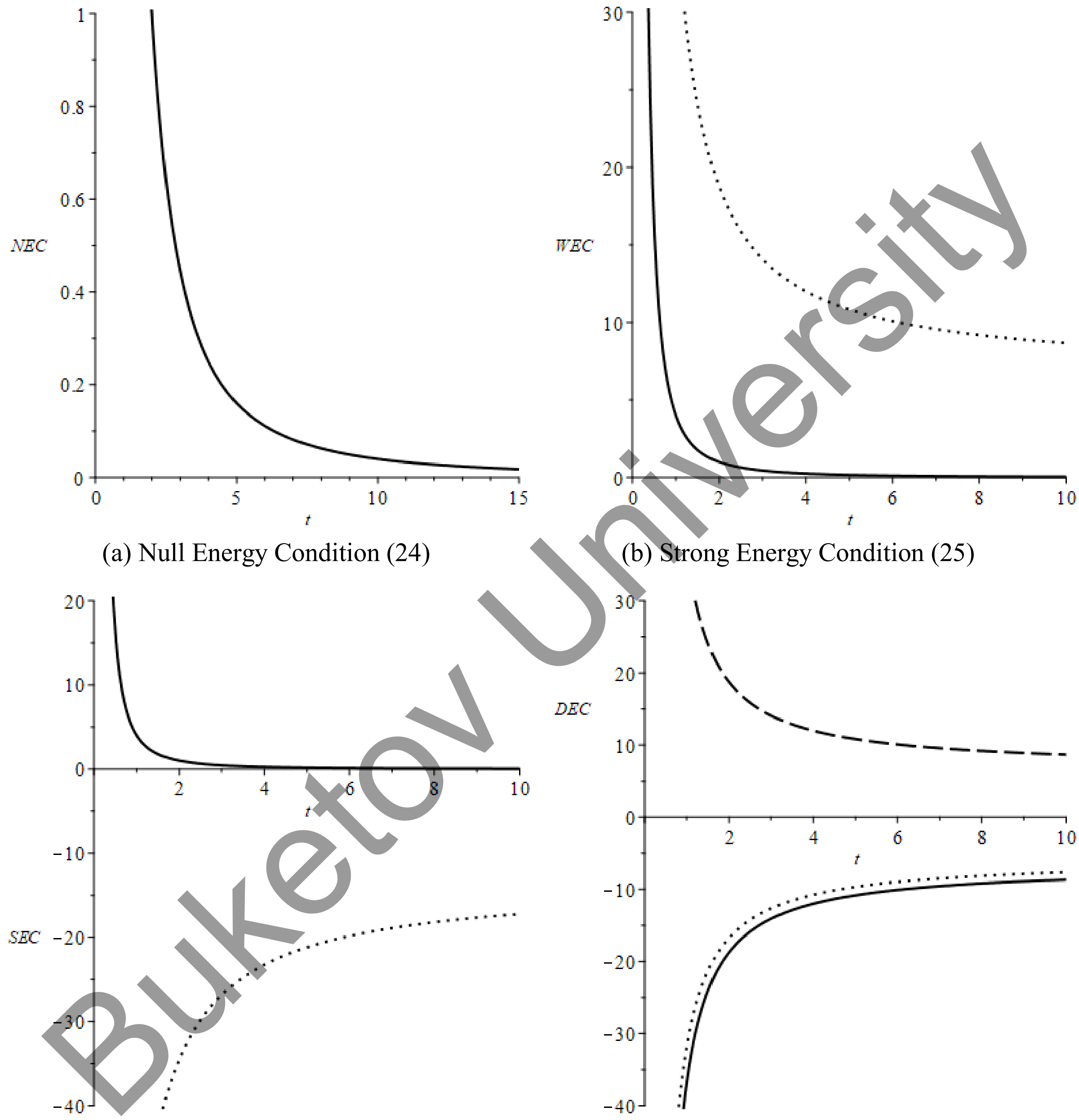
$$3(\alpha+\frac{\beta}{t})^2\geq 0,\quad 2\beta t^{-2}\geq 0. \tag{25}$$

SEC

$$-6(\alpha+\frac{\beta}{t})^2+\frac{6\beta}{t^2}\geq 0, \quad 2\beta t^{-2}\geq 0. \tag{26}$$

DEC

$$3(\alpha+\frac{\beta}{t})^2\geq 0, \quad -3(\alpha+\frac{\beta}{t})^2\leq -3(\alpha+\frac{\beta}{t})^2+\frac{2\beta}{t^2}\leq 3(\alpha+\frac{\beta}{t})^2. \tag{27}$$



(a) Null Energy Condition (24) (b) Strong Energy Condition (25)

(c) Weak Energy Condition (26) (d) Dominant Energy Condition (27)

Figure 3. Energy Condition

Figure 3a shows NEC ($\rho+p$ is a solid line); the curve $\rho+p$ does not cross the abscissa axis, that is, over the entire time interval $\rho+p\geq 0$. Figure 3b shows WEC (ρ is a solid line, $\rho+p$ is a dotted line); curves $\rho+p$ and ρ are above the abscissa axis throughout the entire time interval, that is, conditions $\rho\geq 0$ and $\rho+p\geq 0$ are satisfied throughout the entire time interval. Figure 3c shows the SEC ($\rho+3p$ is a dotted line, $\rho+p$ is a solid line); the $\rho+3p$ curve is located below the abscissa axis, that is, the $\rho+3p\geq 0$ condition is violated and the SEC is not fulfilled. Figure 3d shows DEC (ρ is a dash line, $-\rho$

is a solid line, p is a dotted line); the curve ρ is located above the abscissa axis and curve p lies between curves $-\rho$ and ρ, that is, the condition $\rho \geq 0$ and $-\rho \leq p \leq \rho$ is satisfied throughout the entire time interval. These conditions impose simple and model-independent constraints on behavior of the energy density and pressure. For our model, the zero energy condition, the strong energy condition, the dominant energy condition are satisfied, and the weak energy condition, which is not mandatory, is not satisfied.

Conclusions

We have investigated the scalar, fermion, and massive vector fields' influence and their interactions on the dynamics and evolution of cosmological regimes under conditions of a homogeneous and isotropic spatially flat universe. Observing the behavior of pressure of the scalar field with respect to the fermion and vector fields, we can infer that it is this field, to a greater extent, that is responsible for the accelerated regime in the early universe. An extended set of parameters was found to describe the kinematics of cosmological expansion: the deceleration parameter q, the jerk parameter j, and the snap parameter s. All the resulting parameters satisfy the latest observational data for $\alpha > 1$. Pressure in the model under consideration is negative and tends to zero at a later time. Therefore, at a later time, there is a transition to a slow mode.

Acknowledgments

This study was funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan AP09261147.

References

1 Perlmutter S. Measurements of Ω and γ from 42 High-Redshift Supernovae / S. Perlmutter, G. Aldering, G. Goldhaber, et al. // Astrophysical Journal, 1999. — Vol. 517, No. 2. — P. 564.

2 Riess A.G. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant / A.G. Riess, A.V. Filippenko, P. Challis, et al. // Astrophysical Journal, 1998. — Vol. 116, No. 3. — P. 1009.

3 Riess A.G. Type Ia supernova discoveries at z>1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution / A.G. Riess et al. // Astrophysical Journal, 2004. — Vol. 607. — P. 665–687.

4 Spergel D.N. First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters / D.N. Spergel, L. Verde, et al. // Astrophysical Journal Supplements Series, 2003. — Vol. 148, No. 1. — P. 175.

5 Nojiri S. The future evolution and finite-time singularities in F (R)-gravity unifying the inflation and cosmic acceleration / S. Nojiri, S.D. Odintsov // Physical Review D, 2008. — Vol. 78. — P. 046006.

6 Nojiri S. Introduction to Modified Gravity and Gravitational Alternative for Dark Energy / S. Nojiri, S.D. Odintsov // International Journal of Geometric Methods in Modern Physics, 2007. — Vol. 04, No.01. — P. 115–145.

7 Bamba K. Future of the universe in modified gravitational theories: Approaching to the finite-time future singularity / K. Bamba, S. Nojiri, S.D. Odintsov // Journal of Cosmology and Astroparticle Physics, 2008. — Vol. 0810. — P. 045.

8 Bamba K. Finite-time future singularities in modified Gauss-Bonnet and F (R, G) gravity and singularity avoidance / K. Bamba, S.D. Odintsov, L. Sebastiani, S. Zerbini // The European Physical Journal C, 2010. — Vol. 67. — P. 295.

9 Bamba K. Cosmological evolution of equation of state for dark energy in G-essence models / K. Bamba, O. Razina, K. Yerzhanov, R. Myrzakulov // International Journal of Modern Physics, 2013. — Vol. 67. — P. 310.

10 Padmanabhan T. Accelerated expansion of the universe driven by tachyonic matter / T. Padmanabhan // Physical Review D, 2002. — Vol. 66. — P. 021301.

11 Feinstein A. Power-law inflation from the rolling tachyon / A. Feinstein // Physical Review D, 2002. — Vol. 66. — P. 063511.

12 Abramo L.R.W. Cosmological dynamics of the tachyon with an inverse power-law potential / L.R.W. Abramo, F. Finelli // Physics Letters B, 2003. — Vol. 575, No. 3. — P. 165.

13 Aguirregabiria J.M. Tracking solutions in tachyon cosmology / J.M. Aguirregabiria, R. Lazkoz // Physical Review D, 2004. — Vol. 69. — P. 123502.

14 Aguirregabiria J.M. Phantom k-essence cosmologies / J.M. Aguirregabiria, L.P. Chimento, R. Lazkoz // Physical Review D, 2004. — Vol. 70. — P. 023509.

15 Calcagni G. Tachyon dark energy models: Dynamics and constraints / G. Calcagni, A.R. Liddle // Physical Review D, 2006. — Vol. 74. — P. 043528.

16 Myrzakulov R. Cosmological solutions in F (T) gravity with the presence of spinor fields / R. Myrzakulov, D. Saez-Gomez, P. Tsyba // International Journal of Geometric Methods in Modern Physics, 2015. — Vol. 12. — P. 1550023.

17 Yerzhanov K. g-Essence as the cosmic speed-up / K. Yerzhanov, K. Yesmakhanova, P. Tsyba, N. Myrzakulov, G. Nugmanova, R. Myrzakulov // Astrophysics and Space Science, 2012. — Vol. 341, No. 2. — P. 681–688.

18 Avelino P. The politics of sustainability transitions / P. Avelino et al. // Journal of Environmental Policy and Planning, 2016. — Vol. 8, No. 2. — P.70.

19 Joyce A. Trodden M. Beyond the cosmological standard model / A. Joyce, B. Jain, J. Khoury // Physics Reports, 2015. — Vol. 568, No. 2. — P. 1–98.

20 Bamba K. Dark Energy Cosmology: The Equivalent Description via Different Theoretical Models and Cosmography Tests / K. Bamba, S. Capozziello, S. Nojiri, S. Odintsov // Astrophysics and Space Science, 2012. — Vol. 342. — P. 155–228.

21 Copeland E.J. Dynamics of dark energy / E.J. Copeland, M. Sami, S. Tsujikawa // International Journal of Modern Physics D, 2006. — Vol. 15, No. 11. — P. 1753–1935.

22 Peebles J.E. The cosmological constant and dark energy / J.E. Peebles, B. Ratra // Reviews of Modern Physics, 2003. — Vol. 75. — P. 559.

23 Capozziello S. Classifying and avoiding singularities in the alternative gravity dark energy models / S. Capozziello, M. De Laurentis, S. Nojiri, S.D. Odintsov // Physical Review D, 2009. — Vol. 79. — P. 559.

24 Ribas M.O. Fermions as sources of accelerated regimes in cosmology / M.O. Ribas, F.P. Devecchi, G.M. Kremer // Physical Review D, 2005. — Vol. 72. — P. 3502.

25 Samojeden L.L. Fermions in Brans-Dicke cosmology / L.L. Samojeden, F.P. Devecchi, G.M. Kremer // Physical Review D, 2010. — Vol. 81. — P. 7301.

26 Samojeden L.L. Accelerated expansion in bosonic and fermionic 2D cosmologies with quantum effects / L.L. Samojeden, G.M. Kremer, F P. Devecchi // Europhys. Letters, 2009. — Vol. 87. — P.1

27 Razina O. Cosmological Einstein-Maxwell model with g-essence / O. Razina, P. Tsyba, B. Meirbekov, R. Myrzakulov // International Journal of Modern Physics D, 2019. — Vol. 28, No. 10. — P. 1950126.

28 Razina O. Power solution of the f (R)-gravity with Maxwell term and g-essence / O. Razina, P. Tsyba, Z.M. Sagidullayeva // Bulletin of the university of Karaganda-Physics, 2019. — Vol. 1, No. 93. — P. 94–102.

29 Ribas M.O. Fermions in a Walecka-type cosmology / M.O. Ribas, P. Zambianchi, F.P. Devecchi, G.M. Kremer // Europhysics Letters, 2012. — Vol. 97. — P. 5.

30 Kulnazarov I. G-essence with Yukawa Interactions / I. Kulnazarov, K. Yerzhanov, O. Razina, Sh. Myrzakul, P. Tsyba, R. Myrzakulov // The European Physical Journal C, 2011. — Vol. 71, No. 7. — P. 1698.

31 Tsyba P. Scenario of the evolution of the universe with equation of state of the Weierstrass type gas / P. Tsyba, O. Razina, Z. Barkova, S. Bekov, R. Myrzakulov // Journal of Physics: Conference Series, 2019. — Vol. 1391. — P. 012162.

32 Razina O.V. Cosmological Yang-Mills model with k-essence / O.V. Razina, P.Yu. Tsyba, R. Myrzakulov, B. Meirbekov, Z. Shanina // Journal of Physics Conference Series, 2019. — Vol. 1391. — P. 012164.

33 Razina O. G-essence cosmologies with scalar-fermion interactions / O. Razina, Y. Myrzakulov, N. Serikbayev, S. Myrzakul, G. Nugmanova, R. Myrzakulov // European Physical Journal Plus, 2011. — Vol. 126, No. 9. — P. 85.

34 Болотин Ю.Л. Расширяющаяся Вселенная: замедление или ускорение? / Ю.Л. Болотин, Д.А. Ерохин, О.А. Лемец // Успехи физических наук. — 2012. — Т. 55. — № 9. — С. 876–918.

35 Arjona R. Novel null tests for the spatial curvature and homogeneity of the Universe and their machine learning reconstructions / R. Arjona, S. Nesseris // Physical Review D, 2021. — Vol. 103. — P. 063537.

36 Capozziello S. Cosmography of f (R) gravity / S. Capozziello, V.F. Cardone, V. Salzano // Physical Review D, 2008. — Vol. 78. — P. 063504.

Г.С. Алтаева, О.В. Разина, П.Ю. Цыба

Мультиөрісті космологиялық модельдегі космография

Авторлар фермиондық өрісті, скалярлық өрісті және Юкава әсерлесуі бар векторлық өрісті қамтитын космологиялық модельді талдаған. Бұл модель материяның әртүрлі типті түрлері Әлемнің динамикасына қосқан үлесін зерттеуге мүмкіндік береді. Жазықтықта, біртекті және изотропты кеңістік-уақытта бұл байланыс Әлемнің үдемелі ұлғаюын қамтамасыз ете алады. Динамикалық теңдеулерді космологиялық реконструкциялауда гибридті теңдеулер шешімнің көмегімен жорамалдап алынады. Бұл шешім космография және энергетикалық күй арқылы зерттеліп қарастырылды. Зерттелетін бұл модельде нөлдік энергия шарты, күшті энергия шарты, басым энергетикалық шарты қанағаттандырылады, ал міндетті емес әлсіз энергия шарты қанағаттандырылмайды. Космографиялық параметрлердің – q баяулату параметрі, j серпілу параметрі және s басу параметрлерінің масштаб коэффициентінің гибридті мәніне қалай байланысты болуы анықталып көрсетілген. Алынған талдауда космографиядан және алынған модельден тәуелсіз нәтижелерді гравитацияның теориялық негізделген болжамдарымен байланыстыруға мүмкіндік береді. Гравитациялық өріс энергиясының жалпы тығыздығы мен жалпы қысымы бозондық өріс, фермиондық өріс, векторлық өрістермен, сондай-ақ Юкава типінің потенциалымен байланысты үлестер қосындысы түрінде болып табылады. Зерттелетін модельде ерте дәуірде бозондық өріс жеделдетілген режимге жауапты болып табылады. Фермиондық өріс және векторлық өрістер оң қысым мәніне ие, сондықтан Әлемнің үдемелі ұлғаюын баяулатады. Кейінірек баяу режимге көшу жүреді, өйткені жалпы қысым нөлге ұмтылады.

Кілт сөздер: скалярлы өріс, фермиондық өріс, Юкава типті әсерлесу, векторлық өріс, космография, тежеу параметрі.

Г.С. Алтаева, О.В. Разина, П.Ю. Цыба

Космография в мультиполевой космологической модели

Авторами проанализирована космологическая модель, содержащая фермионное поле, скалярное поле и векторное поле с взаимодействием Юкавы. Такая модель позволяет исследовать вклад различных типов материи в динамику Вселенной. В плоском, однородном и изотропном пространстве времени эта связь может обеспечивать ускоренное расширение Вселенной. Космологическая реконструкция динамических уравнений получена с помощью гибридного решения. Это решение исследуется космографией и энергетическим состоянием. В исследуемой модели выполняется нулевое энергетическое условие, сильное энергетическое условие, доминирующее энергетическое условие и не выполняется слабое энергетическое условие, которое не является обязательным. Показано, как можно связать космографические параметры — параметры замедления q, рывка j и щелчка s с гибридным значением масштабного фактора. Полученный анализ дает возможность связать независимые от модели результаты, полученные из космографии, с теоретически обоснованными предположениями гравитации. Найдены полная плотность и давление энергии гравитационного поля в виде суммы вкладов, которые связаны с бозонным, фермионным, векторным полями, а также потенциалом типа Юкавы. В исследуемой модели в раннюю эпоху бозонное поле является ответственным за ускоренный режим. Фермионное и векторное поля имеют положительное значение давления и, следовательно, замедляют ускоренное расширение Вселенной. В позднее время происходит переход в замедленный режим, так как общее давление стремится к нулю.

Ключевые слова: скалярное поле, фермионное поле, взаимодействие типа Юкавы, векторное поле, космография, параметр замедления.

References

1 Perlmutter, S., Aldering, G., & Goldhaber, G., et al. (1999). Measurements of Ω and ʌ from 42 high-redshift supernovae. *Astrophysical Journal*, 517(2), 565–586.

2 Riess, A.G., Filippenko, A.V., & Challis, P. et al. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astrophysical Journal,* 116(3), 1009–1038.

3 Riess, A G., Strolger, L.G., & Tonry, J., et al. (2004). Type Ia supernova discoveries at z > 1 from the hubble space telescope: evidence for past deceleration and constraints on dark energy evolution. *Astrophysical Journal*, 607, 665–687.

4 Spergel, D.N., Verde, L., & Peiris, H.V., et al. (2003). First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters. *Astrophysical Journal, Supplement Series*, 148(1), 175.

5 Nojiri, S., & Odintsov, S.D. (2008). Future evolution and finite-time singularities in F (R) gravity unifying inflation and cosmic acceleration. *Physical Review D,* 78, 046006.

6 Nojiri, S., & Odintsov, S.D. (2007). Introduction to modified gravity and gravitational alternative for dark energy. *International Journal of Geometric Methods in Modern Physics,* 4(1), 115–145.

7 Bamba, K., Nojiri, S., & Odintsov, S.D. (2008). Future of the universe in modified gravitational theories: Approaching to the finite-time future singularity. *Journal of Cosmology and Astroparticle Physics*, 0810, 045.

8 Bamba, K., Odintsov, S.D., & Sebastiani, L., et al. (2010). Finite-time future singularities in modified Gauss-Bonnet and F (R, G) gravity and singularity avoidance. *The European Physical Journal C,* 67, 295–310.

9 Bamba, K., Razina, O., Yerzhanov, K., & Myrzakulov, R. (2013). Cosmological evolution of equation of state for dark energy in G-essence models. *International Journal of Modern Physics,* 67, 310.

10 Padmanabhan, T. (2002). Accelerated expansion of the universe driven by tachyonic matter. *Physical Review D,* 66, 021301.

11 Feinstein, A. (2002). Power-law inflation from the rolling tachyon. *Physical Review D,* 66, 063511.

12 Abramo, L.R.W., & Finelli, F. (2003). Cosmological dynamics of the tachyon with an inverse power-law potential. *Physics Letters B*, 575(3), 165.

13 Aguirregabiria, J.M., & Lazkoz, R. (2004). Tracking solutions in tachyon cosmology. *Physical Review D*, 69, 123502.

14 Aguirregabiria, J.M., Chimento, L.P., & Lazkoz, R. (2004). Phantom k-essence cosmologies. *Physical Review D,* 70, 023509.

15 Calcagni, G., & Liddle, A.R. (2006). Tachyon dark energy models: Dynamics and constraints. *Physical Review D,* 74, 043528.

16 Myrzakulov, R., Saez-Gomez, D., & Tsyba, P. (2015). Cosmological solutions in F (T) gravity with the presence of spinor fields. *International Journal of Geometric Methods in Modern Physics*, 12, 1550023.

17 Yerzhanov, K., Yesmakhanova, K., Tsyba, P., Myrzakulov, N., Nugmanova, G., & Myrzakulov, R. (2012). g-Essence as the cosmic speed-up. *Astrophysics and Space Science*, 341(2), 681–688.

18 Avelino, P. et al. (2016). The politics of sustainability transitions. *Journal of Environmental Policy and Planning,* 8(2), 70.

19 Joyce, A., Jain, B., Khoury, J., & Trodden, M. (2015). Beyond the cosmological standard model. *Physics Reports*, 568(2), 1–98.

20 Bamba, K., Capozziello, S., Nojiri, S., & Odintsov, S. (2012). Dark Energy Cosmology: The Equivalent Description via Different Theoretical Models and Cosmography Tests. *Astrophysics and Space Science,* 342, 155–228.

21 Copeland, E.J., Sami, M., & Tsujikawa, S. (2006). Dynamics of dark energy. *International Journal of Modern Physics D,* 15(11), 1753–1935.

22 Peebles, J.E., & Ratra, B. (2003). The cosmological constant and dark energy. *Reviews of Modern Physics,* 75, 559.

23 Capozziello, S., De Laurentis, M., Nojiri, S., & Odintsov, S.D. (2009). Classifying and avoiding singularities in the alternative gravity dark energy models. *Physical Review D,* 79, 559.

24 Ribas, M.O., Devecchi, F.P., & Kremer, G.M. (2005). Fermions as sources of accelerated regimes in cosmology. *Physical Review D,* 72, 3502.

25 Samojeden, L.L., Devecchi, F.P., & Kremer, G.M. (2010). Fermions in Brans-Dicke cosmology. *Physical Review D,* 81, 7301.

26 Samojeden, L.L., Kremer, G.M., & Devecchi, F.P. (2009). Accelerated expansion in bosonic and fermionic 2D cosmologies with quantum effects. *Europhys. Letters*, 87, 1.

27 Razina, O., Tsyba, P., Meirbekov, B., & Myrzakulov, R. (2019). Cosmological Einstein-Maxwell model with g-essence. *International Journal of Modern Physics D*, 28(10), 1950126.

28 Razina, O., Tsyba, P., & Sagidullayeva, Z.M. (2019). Power solution of the f (R)-gravity with Maxwell term and g-essence. *Bulletin of the university of Karaganda-Physics,* 1(93), 94–102.

29 Ribas, M.O., Zambianchi, P., Devecchi, F.P., & Kremer, G.M. (2012). Fermions in a Walecka-type cosmology. *Europhysics Letters,* 97, 5.

30 Kulnazarov, I., Yerzhanov, K., Razina, O., Myrzakul, Sh., Tsyba, P., & Myrzakulov, R. (2011). G-essence with Yukawa Interactions. *European Physical Journal C*, 71(7), 1698.

31 Tsyba, P., Razina, O., Barkova, Z., Bekov, S., & Myrzakulov, R. (2019). Scenario of the evolution of the universe with equation of state of the Weierstrass type gas. *Journal of Physics: Conference Series*, 1391, 012162.

32 Razina, O.V., Tsyba, P.Yu., Myrzakulov, R., Meirbekov, B., & Shanina, Z. (2019). Cosmological Yang-Mills model with k-essence. *Journal of Physics Conference Series*, 1391, 012164.

33 Razina, O., Myrzakulov, Y., Serikbayev, N., Myrzakul, S., Nugmanova, G., & Myrzakulov, R. (2011). G-essence cosmologies with scalar-fermion interactions. *European Physical Journal Plus,* 126(9), 85.

34 Bolotin, Yu.L., Erokhin, D.A., & Lemets, O.A. (2012). Rasshiriaiushchaiasia Vselennaia: zamedlenie ili uskorenie? [Expanding Universe: slowdown or speedup?]. Uspekhi fizicheskikh nauk*, 55(9),* 876–918 [in Russian].

35 Arjona, R., & Nesseris, S. (2021). Novel null tests for the spatial curvature and homogeneity of the Universe and their machine learning reconstructions. *Physical Review D, 103,* 063537.

36 Capozziello, S., Cardone, V.F., & Salzano, V. (2008). Cosmography of f (R) gravity. *Physical Review D*, 78, 063504.