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PERFECT JONSSON VARIETIES AND QUASIVARIETIES

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Definition 1. A is an algebraically prime model of theory T, if A is a model of $T$and A may be isomorphically embedded in each model of the theory$T$.

Definition 2. The inductive theory T is called the existentially prime if:

1) it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T;

1) classE_T non trivial intersects with class AP_T, i.e. $AP_T \cap E_T \neq \emptyset$.

Definition 3.The theory T is called convex if for any its model A and for anyfamily $\{B_i | i \in I\}$ of substructures of A, which are models of thetheory T, the intersection$\bigcap_{i \in I} B_i$is a model of T , provided it isnon-empty. If in addition such an intersection is never empty, then$T$ is called strongly convex.

Definition 4.Model C of Jonsson theory T is called semantic model, if it is ω^+-homogeneous-universal.

Definition 5.The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^*, i.e. $T^* = Th(C)$.

Definition 6.Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^*.

Definition 7.Let T be the Jonsson theory. A companion of a Jonsson theory T is a theory $T^\#$ of the same signature that satisfies the following conditions:

1) $(T^\#)_\forall = T_\forall$;
2) for any Jonsson theory T', if $T_\forall = T'_\forall$, then $T^\# = (T')^\#$;
3) $T_{\forall\exists} \subseteq T^\#$.

The natural interpretations of the companion$T^\#$are T^*, T^0, T^f, T^M, T^e.

Definition 8.A set X is called a Jonsson set, if hold the following conditions:

1) X is a definable set by some existential formula $\varphi(\bar{x}, \bar{y}) = \exists \bar{y} \psi(\bar{x}, \bar{y})$;
2) $cl(X) = M, M \in E_T$.

Let K be the class of structures of countable signature σ.

Definition 9. We call a K - Jonsson variety if

1) K is a variety in the usual sense [2; 269];
2) $Th_{\forall\exists}(K)$is a Jonsson theory.

Definition 10. We call a K - Jonsson quasivariety if

1) K - is a quasivariety in the usual sense [2; 269];
2) $Th_{\forall\exists}(K)$is a Jonsson theory.

Consider the $JSpV\ (K)$ - Jonsson spectrum of the Jonsson varieties of class K, where K is the Jonsson variety:

$JSpV\ (K) = \{T / T$is a Jonsson theory, $T = Th_{\forall\exists}(N)$; $N \subseteq K$; N is a subvariety of $K\}$.

Consider the $JSpQV\ (K)$ - Jonsson spectrum of the Jonsson quasivarieties of the class K, where$K$ is the Jonsson quasivariety:

$JSpQV\ (K) = \{T / T$is a Jonsson theory, $T = Th_{\forall\exists}(N)$; $N \subseteq K$; N is a subquasivariety of $K\}$.

Then $JSpQV\ (K) /_{\bowtie}$ is denoting the factor set of the Jonsson spectrum of Jonsson quasivariety of the class K by the relation ⋈.

It is clear that $JSpQV\ (K) \subseteq JSp(A)$ for any model $A \in K$, where $JSp(A)$ is the Jonsson spectrumof the model A .

We say that class K_1 $JSpQV$ -cosemantic to class $K_2(K_1 \overset{\bowtie}{}_{JSpQV} K_2)$ if $JSpQV\ (K_1)\ /_{\bowtie} = JSpQV\ (K_2)\ /_{\bowtie}$.

In the framework of studying the properties of perfect Jonsson varieties of algebras, the following result is obtained.

Theorem. Let K be a class of algebras of some signature σ, and let Tbe a complete inductivetheory,$E_T \neq \emptyset$. Then, the class Kfromexistentially closed models of the theory T,$K \subseteq E_T$. Let $[T] \in PJSp(K)$. $E_{[T]}$ coincide with $Mod(Th_{\forall\exists}(E_{[T]}))$.

Also for all who have an interest in the particular case of the above-considered materials, one canfind out in the following resources [1-3].

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THE PROPERTIES OF EXISTENTIALLY PRIME SUBCLASSES OF PERFECT JONSSON THEORIES

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Definition 1 [1, p. 80]. A theory T is called a Jonsson theory if the following conditions hold:

1. T has at least one infinite model,
2. T is $\forall\exists$-axiomatizable,
3. T possesses AP and JEP.

Definition 2 [2]. A theory T is called existentially prime, if $AP_T \cap E_T \neq \emptyset$, where AP_T is the class of algebraically prime models of T.

Let T be a Jonsson theory. For any model $\mathfrak{M} \in E_T$ there are the inner world $IW(\mathfrak{M})$ and the outer world $OW(\mathfrak{M})$ defined in [3].

Definition 3 [3]. Let T be a Jonsson theory and let $\mathfrak{M}$ be an existentially closed model of T. $IW_T(\mathfrak{M}) = \{\mathfrak{N} \in E_T \mid f: \mathfrak{N} \to \mathfrak{M}$ is said to be the inner world of $\mathfrak{M}$ for the theory T, where f is an isomorphism.

Definition 4 [3]. Let T be a Jonsson theory. $\left(OW_T(\mathfrak{M})\right) = \{\mathfrak{N}' \in E_T \middle|$ there exists$\mathfrak{N} \cong \mathfrak{M}, \mathfrak{M} \subseteq \mathfrak{N}\}$ is called the outer world of A for the theory T.

The result of the paper is the following Theorem, where some properties of the inner world of a fixed theory $T$are formulated.

Theorem. Let T be a perfect Jonsson existentially prime theory and let $\mathfrak{A}, \mathfrak{B} \in E_T$. Then

1. $OW(\mathfrak{A}) \subseteq OW(\mathfrak{B})$ whenever $\mathfrak{A} \subseteq \mathfrak{B}$;
2. $OW(\mathfrak{A}) = OW(\mathfrak{B})$ whenever $\mathfrak{A} \prec \mathfrak{B}$;
3. $OW(\mathfrak{A}) = \cap \{B \mid \mathfrak{B} \subseteq \mathfrak{A}$ and $\mathfrak{B} \vDash T$ whenever $\mathfrak{A}$ is a universal model of T.

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