КОНДЕНСАЦИЯЛАНҒАН КҮЙДІҢ ФИЗИКАСЫ
ФИЗИКА КОНДЕНСИРОВАННОГО СОСТОЯНИЯ
PHYSICS OF THE CONDENSED MATTER

UDC 538.95.405

S.A. Guchenko[1], E.N. Eremin[2], V.M. Yurov[1],
V.Ch. Laurinas[1], S.S. Kasimov[1], O.N. Zavatskaya[1]

[1]Ye.A. Buketov Karaganda State University, Kazakhstan;
[2]Omsk State Technical University, Russia
(E-mail: exciton@list.ru)

Autovole processes in deprivation plasma coatings

Autowave processes that arise when depositing plasma coatings are considered. Composite cathodes and stainless steel cathodes were used for the production of coatings. Microhardness measurements were made along and across the sample in an amount up to 50 pieces. Microhardness plots are periodic structures with a wavelength of the order of 10^{-4} m. The diffusion coefficient is of the order of 10^{-8} m^2/s, i.e. we have a system with small diffusion. The deposition of coatings in a plasma is a thermodynamically nonequilibrium process in an open system. The nonlinearity of the equations arises from the motion of the interface and the small diffusion of surface atoms. In this case, an autowave process arises. The experimental and theoretical results obtained by us fit into the model of macroscopic localization of plastic flow. In this model it is shown that the localization of plastic flow in metals and alloys has a pronounced wave character. The theory of crystallization of a cylinder of finite dimensions developed by us relates to problems with a moving interface and is called the Stefan problem. From a mathematical point of view, boundary-value problems of this type are fundamentally different from the classical problems of heat conduction or diffusion. Due to the dependence of the size of the flow transfer region on time, classical methods of separating variables and integral Fourier transforms are not applicable to this type of problems, since, remaining within the framework of classical methods of mathematical physics, it is not possible to coordinate the solution of the equation with the motion of the phase boundary boundary. The motion of the boundary of the phase difference leads to a nonlinearity of the system of equations, which leads to the appearance of autowaves.

Keywords: autowave, coating, plasma, diffusion, microhardness, crystallization.

Introduction

The terms «autowave process», «autowave» (AW) were proposed by R.V. Khokhlov, although the theory of autowaves was initiated by mathematicians — the work of R. Fisher (1937), A.N. Kolmogorov, G.I. Petrovsky and I.S. Piskunov (1937), N. Wiener and A. Rosenbluth (1946), A. Turing (1952) — long before their experimental discovery [1]. Subsequently, the AWP theory became an integral part of the theory of self-organization or synergetics [2–4].

A large class of AW-media can be conditionally described with the help of the following scheme. In an open distributed system, energy or a substance rich in energy comes from outside. These flows are controlled by the local properties of the regulating surface, or, more accurately, of the boundary layer of small thickness. In turn, the local properties of the surface depend both on the temperature waves, the concentration potential propagating along the thin boundary layer, and on the processes occurring in the substrate.

In the second class of AW media, surface effects are not so pronounced. Local feedback provides the presence of an N-shaped characteristic of the medium with a falling section of «negative» resistance in any elementary volume. Such media and spatio-temporal structures include the self-oscillating Belousov-Zhabotinsky reactions, domains in the electron-hole plasma of semiconductors, and a number of others [2].

The third class includes complex multiphase media in which nonequilibrium and AWP are supported by laser radiation energy, ion plasma energy, as in our experiments, thermochemical reactions and other sources. Such phenomena are determined not only by diffusion and heat transfer, but also by hydrodynamic flows, in particular convection, evaporation, boiling, surface tension. The formation of structures involving surface phenomena was considered in the monograph [5] and by us in [6].

Thus, an autowave is one of the results of self-organization in thermodynamically active nonequilibrium systems. This is a self-sustaining wave process that exists in nonlinear media containing distributed energy sources. The period, wavelength, propagation velocity, amplitude and other characteristics of the autowave are determined exclusively by local properties of the medium.

In addition to the motion of the combustion front, autowave processes include vibrational chemical reactions in active media, propagation of the excitation pulse along the nerve fiber, waves of chemical signaling in the colonies of certain microorganisms, autowaves in ferroelectric and semiconductor films, population autowaves, epidemics and many other phenomena [7–16].

Such a diversity of AWP leads to a variety of mechanisms of their origin, which are not always understood and not always described by simple mathematical models. This is the case with many AWP in condensed media and systems.

In this paper we consider the occurrence of AWP in the deposition of plasma coatings. This question has not been raised by anyone except our work [6], where some experimental results are partially reflected.

Autowaves in the active distributed kinetic system

The basis of models [1] describing the processes in the active distributed kinetic system are the equations of material balance:

$$\frac{\partial x_i}{\partial t} = F_i\left(x_1, x_2, ..., x_n\right) - \operatorname{div} I_i. \quad (1)$$

Here, x_i are interacting components, I_i is the flux of the *i*-th component:

$$I_i = Vx_i - \sum_{k=1}^{n} D_{ik} \operatorname{grad} x_k, \quad (2)$$

where V — is the directed velocity of the component and D_{ik} — is the matrix of the diffusion coefficients.

In the simplest case of a one-dimensional space, equations (1)–(2) are written as follows:

$$\frac{\partial x_i}{\partial t} = F_i\left(x_1, x_2, ..., x_n\right) + \frac{\partial}{\partial r}\left(\sum_{k=1}^{n} D_{ik}\left(x_1, x_2, ..., x_n\right)\frac{\partial x_i}{\partial r}\right). \quad (3)$$

The boundary conditions of systems (1)–(3) are determined by specific problems, but the conditions of «impenetrability» of the boundaries of a finite interval [0, L]:

$$\left.\frac{\partial x_i}{\partial t}\right|_{\substack{r=0\\ r=L}} = 0. \quad (4)$$

Under these conditions, the system is as autonomous as possible and the nature of the AWP is least affected by the influence of the borders.

If the mixing inside the «volume» [0, L] occurs rapidly enough, then in any part of it the processes are synchronous and the system is described by so-called «point» equations [1]:

$$\frac{\partial x_i}{\partial t} = F_i\left(x_1, x_2, ..., x_n\right). \quad (5)$$

Formally, from system (3) to (5) it is possible to go over with $D_{ik} \to \infty$. Physically, this means that the transition to (5) corresponds to the zero approximation with respect to the ratio of the characteristic diffusion times and chemical processes.

Dissipative structures for systems with small diffusion

In the monograph [14] an attempt is made to create a unified theory of dissipative Turing-Prigogine structures for systems of parabolic and hyperbolic equations with small diffusion. For this purpose, special asymptotic methods are developed for investigating the existence and stability problems of highly modal stationary regimes in singularly perturbed systems, which make it possible to obtain very subtle assertions about the unlimited growth of the number of stable dissipative structures (both stationary and periodic in time) with decreasing diffusion coefficients and with other fixed parameters.

As a model system in [14], an equation of the type (3):

$$\frac{\partial u}{\partial t} = \nu D \frac{\partial^2 u}{\partial t^2} + F(u). \quad (6)$$

Here the parameter ν>0 is responsible for the proportional change in the diffusion coefficients. The basic assumption about boundary value problem (6) is that $u = 0$ is its only spatially homogeneous equilibrium state, globally exponentially stable within the framework of the point model (5).

Macrolocalization of plastic flow

The monograph [12] shows that the macroscopic localization of the plastic flow has an autowave character and appears in all deformable materials, and the type of autowave localization is determined by the strain hardening law acting at the corresponding stage of the plastic flow process (Fig. 1).

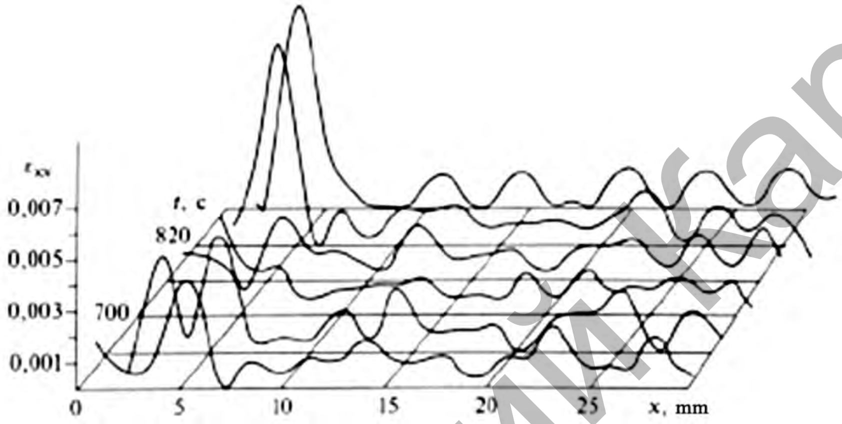

Figure 1. Formation of a high-amplitude fixed maximum of localization at the parabolic stage in a single crystal of Fe–Si [12]

The deformed state of plasma coatings is described in detail in [17].

The experimental results described in the monograph [12] point to several important circumstances related to plastic deformation:

– plastic deformation of metals and alloys throughout the process reveals a tendency to localization in different forms;
– the number of forms of localization of the plastic flow of all materials studied to date does not exceed four;
– the implementation of each form is determined by the deformation hardening law in force at this stage.

Perhaps the most interesting form of localization of plastic flow is that which is observed at the stage of linear strain hardening of mono- and polycrystalline materials. Schematically, this form of localization is shown in Figure 2. It can be seen that the localization picture has all the features of the wave process and can be characterized by the wavelength λ, the propagation velocity V_{aw}, and the frequency $\omega = 2\pi V_{aw}/\lambda$.

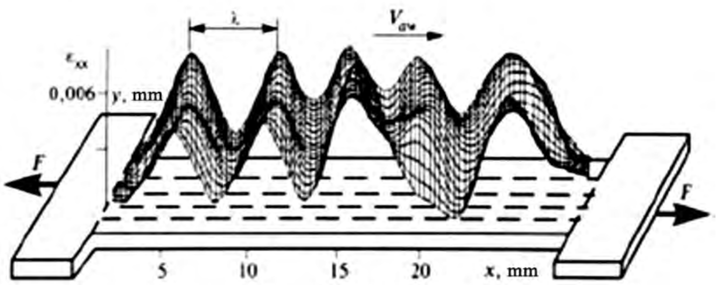

Figure 2. Coordinated motion with a constant velocity V_{aw} along the loading axis of the localization system ε_{xx} spaced apart at a distance λ, at the stage of linear hardening in a sample of a single-crystal Fe–Ni [12]

Results of the experiment

For the application of coatings, composite cathodes and cathodes made of 12X18H10T steel were used. With the help of these cathodes, coatings were applied on the NNB-6.6I1 unit on a steel substrate in argon and nitrogen gas for 40 minutes.

The microhardness of the coating was measured on a HVS-1000 A microhardness by the Vickers method along, across and along the diagonal of the samples. About 50 samples were studied. Some characteristic results are shown in Figures 3 and 4.

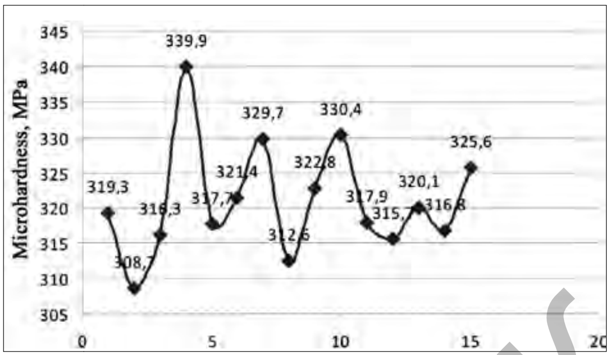

Figure 3. Results of measuring the microhardness of the sample Zn-Cu-Al + 12X18H10T

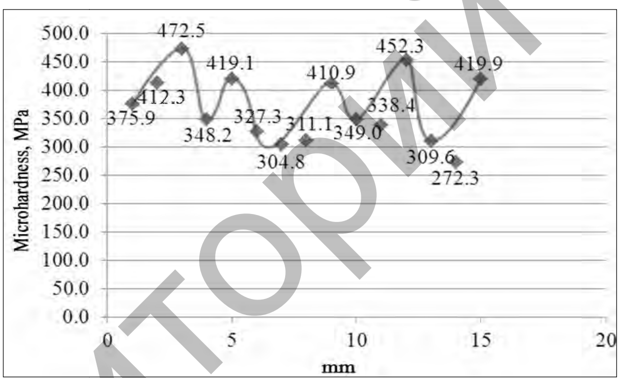

Figure 4. Results of measuring the microhardness of the Cu + 12X18H10T sample

Discussion of experimental results

We consider the problem of crystallization of a deposited coating in the form of a cylinder of finite dimensions with a moving interface. The nonstationary diffusion equation describing this process in a moving cylindrical coordinate system moving in accordance with the law β(t) has the form:

$$\frac{\partial C}{\partial t} = D\left[\frac{\partial^2 C}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right], \tag{7}$$

where D — is the diffusion coefficient.

The initial and boundary conditions are chosen in the general form:

$$C(r, z, t)|_{t=0} = \phi(r, z); \tag{8}$$

$$C(r, z, t)|_{r=R} = \gamma(z, t); \tag{9}$$

$$C(r, z, t)|_{z=0} = \gamma_1(r, t); \tag{10}$$

$$C(r, z, t)|_{z=\beta(t)} = \gamma_2(r, t). \tag{11}$$

The functions $\beta(t), \phi(r,z), \gamma(z,t), \gamma_1(r,t)$ and $\gamma_2(r,t)$ are assumed to be continuous. We seek the solution of the problem in the form:

$$C(r,z,t)=\sum_{k=o}^{\infty}\overline{C}_k(z,t)I_0(\lambda_{0k}r), \tag{12}$$

where λ_{ok} are the roots of equation

$$I_0(\lambda_{ok}R)=0 \tag{13}$$

and $I_0(\lambda_{ok}R)$ is a zero-order Bessel function satisfying equation:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dI(\lambda_{0k}r)}{dr}\right]+I_0(\lambda_{0k}r)=0\,; \tag{14}$$

$$\overline{C}_k(z,t)=\int_0^R C_k(r,z,t)I_0(\lambda_0 r)rdr\,. \tag{15}$$

Applying (15) and taking (12) and (13) into account, we reduce equation (7) to the form:

$$\frac{1}{D}\frac{\partial\overline{C}_k}{\partial t}=\frac{\partial^2\overline{C}_k}{\partial z^2}+\overline{\Phi}_k(z,t)-\overline{C}_k(z,t)\,. \tag{16}$$

Using the substitution $\overline{C}_k=\tilde{C}_k e^{-Dt}$ and transforming the boundary conditions similarly, we obtain the following problem:

$$\frac{1}{D}\frac{\partial\tilde{C}_k}{\partial t}=\frac{\partial^2\tilde{C}_k}{\partial z^2}+\tilde{\Phi}_k(z,t)\,; \tag{17}$$

$$\tilde{C}_k(z,t)\,|_{t=0}=\tilde{\phi}(z); \tag{18}$$

$$\tilde{C}_k(z,t)\,|_{z=0}=\tilde{\gamma}_1(t); \tag{19}$$

$$\tilde{C}_k(z,t)\,|_{z=\beta(t)}=\tilde{\gamma}_2(t), \tag{20}$$

in area $D:(t>0,\,0<z<\beta(t))$.

We seek the solution of the problem (17)–(20) in the form of a sum of potentials of the first and second kind, and also the two potentials of the double layer:

$$\tilde{C}_k(z,t)=\frac{1}{2\sqrt{D}}\int_0^{\ell}\frac{\tilde{\phi}(\xi)}{\sqrt{\pi t}}e^{-\frac{(z-\xi)^2}{4Dt}}d\xi+\int_6^t d\tau\int_0^{\ell}\frac{\tilde{\Phi}_k(\xi,\tau)}{2\sqrt{\pi D(t-\tau)}}e^{-\frac{(z-\xi)^2}{4D(t-\tau)}}d\xi+$$
$$+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{z}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{z^2}{4D(t-\tau)}}K_1(\tau)d\tau+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{z-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{[z-\beta(\tau)]^2}{4D(t-\tau)}}K_2(\tau)d\tau. \tag{21}$$

Using conditions (19), (20), we obtain a system of integral equations:

$$\tilde{\gamma}_1'(t)=\frac{K_1(t)}{2D}-\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta^2(\tau)}{4D(t-\tau)}}K_2(\tau)d\tau;$$
$$\tilde{\gamma}_2'(t)=\frac{K_2(t)}{2D}+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(t)-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{[\beta(t)-\beta(\tau)]^2}{4D(t-\tau)}}K_2(\tau)d\tau+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta^2(t)}{4D(t-\tau)}}K_1(\tau)d\tau\,, \tag{22}$$

where

$$\tilde{\gamma}_1'(t)=\tilde{\gamma}_1(t)-\frac{1}{2}\int_0^{\ell}\frac{\tilde{\phi}(\xi)}{\sqrt{\pi Dt}}e^{-\frac{\xi^2}{4Dt}}d\xi-\int_0^t d\tau\int_0^{\ell}\frac{\tilde{\Phi}_k(\xi,\tau)}{2\sqrt{\pi D(t-\tau)}}\cdot e^{-\frac{\xi^2}{4D(t-\tau)}}d\xi\,;$$

$$\tilde{\gamma}_2'(t)=\tilde{\gamma}_2(t)-\frac{1}{2}\int_0^{\ell}\frac{\tilde{\phi}(\xi)}{\sqrt{\pi Dt}}e^{-\frac{[\beta(t)-\xi]^2}{4Dt}}d\xi-\int_0^t d\tau\int_0^{\ell}\frac{\tilde{\Phi}_k(\xi,\tau)}{2\sqrt{\pi D(t-\tau)}}\cdot e^{-\frac{[\beta(t)-\xi]^2}{4D(t-\tau)}}d\xi.$$

Eliminating from the first equation of system (22) and substituting in the equation $K_1(t)$, we have:

$$\tilde{\gamma}_2'(t)=\frac{K_2(t)}{2D}+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(t)-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{[\beta(t)-\beta(\tau)]^2}{4D(t-\tau)}}K_2(\tau)d\tau+\frac{2D}{\sqrt{\pi}}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta^2(t)}{4D(t-\tau)}}\tilde{\gamma}_1'(\tau)d\tau+$$
$$+\frac{D}{8\pi}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta^2(t)}{4D(t-\tau)}}\left(\int_0^\tau\frac{\beta(\tau_1)}{\left[D(t-\tau_1)\right]^{3/2}}e^{-\frac{\beta^2(t)}{4D(t-\tau)}}K_2(\tau_1)d\tau_1\right)d\tau. \qquad (23)$$

Introducing the notation

$$q(t)=\tilde{\gamma}_2(t)-\frac{D}{2\sqrt{\pi}}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}\tilde{\gamma}_1'(\tau)e^{-\frac{\beta^2(t)}{4D(t-\tau)}}d\tau \qquad (24)$$

and calculating the integral in (23), we obtain

$$-\frac{K_2(t)}{2D}+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(t)-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{[\beta(t)-\beta(\tau)]^2}{4D(t-\tau)}}K_2(\tau)d\tau+\frac{1}{4\sqrt{\pi}}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta(t)^2}{4D(t-\tau)}}K_2(\tau)d\tau=q(t). \qquad (25)$$

Denoting,

$$\lambda=\frac{1}{2\sqrt{D}},\quad f(t)=2Dq(t),\ K(t,\tau)=\frac{\lambda}{\sqrt{\pi}}\frac{\beta(t)-\beta(\tau)}{(t-\tau)^{3/2}}\ e^{-\lambda^2\frac{[\beta(t)-\beta(\tau)]^2}{(t-\tau)}}+\frac{\lambda}{\sqrt{\pi}}\frac{\beta(t)}{(t-\tau)^{3/2}}e^{-\lambda^2\frac{\beta(t)^2}{(t-\tau)}}, \qquad (26)$$

we obtain the integral equation

$$K_2(t)-\int_0^t K(t,\tau)\,K_2(t,\tau)\,d\tau=f(t). \qquad (27)$$

The integral equation (27) is Volterra in $C(0,\ell)$, if and only if:

$$\lim_{t\to 0}\int_0^t K(t,\tau)d\tau=0.$$

Indeed, taking into account that $e^{-z}<1$ for $z>0$, it is easy to show that the above equality holds. Then for equation (27) there exists a unique solution that has the form:

$$K_2(t)=\sum_{n=0}^{\infty}K_{2,n}(t); \qquad (28)$$
$$K_{2,0}(t)=f(t);$$
$$K_{2,1}(t)=\int_0^t K(t,\tau)\,K_{2,0}(\tau)d\tau;$$
$$K_{2,2}(t)=\int_0^t K(t,\tau)\,K_{2,1}(\tau)d\tau;$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$K_{2,n}(t)=\int_0^t K(t,\tau)\,K_{2,n-1}(\tau)d\tau$$

and (28) converges absolutely and uniformly in the topology $C(0,\ell)$. Then

$$K_1(t)=2D\tilde{\gamma}_1(t)+\frac{D}{2\sqrt{\pi}}\int_0^t\frac{\beta(t)}{\left[D(t-\tau)\right]^{3/2}}e^{-\frac{\beta^2(\tau)}{4D(t-\tau)}}\sum_{n=0}^{\infty}K_{2,n}(\tau)d\tau. \qquad (29)$$

Performing the inverse transformation, we finally have:

$$C(r,z,t)=\sum_{\kappa=0}^{\infty}J_0(\lambda_{o\kappa}r)\left\{e^{-Dt}\ \left[\frac{1}{2D\sqrt{\pi}}\int_0^t e^{\frac{(z-\xi)^2}{4Dt}}dt\times\right.\right.$$
$$\times\left(\int_0^\ell\phi(r,\xi)I_0(\lambda_{ok}r)rdr\right)d\xi+\frac{RI_1(\lambda_{ok}R)}{2\sqrt{\pi}D}\int_0^t d\tau\int_0^\ell\frac{\gamma(\xi,\tau)}{\sqrt{t-\tau}}e^{-Dt}e^{-\frac{(z-\xi)^2}{4D(t-\tau)}}d\xi+$$

.

$$+\frac{1}{4\sqrt{\pi}}\int_0^t \frac{z}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{z^2}{4D(t-\tau)}} K_1(\tau)d\tau + \frac{1}{4\sqrt{\pi}}\int_0^t \frac{z-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{\left[z-\beta(\tau)\right]^2}{4D(t-\tau)}} K_2(\tau)d\tau \Bigg]\Bigg\}. \quad (30)$$

Thus, an analytical solution is obtained for the problem of crystallization of a cylinder of finite dimensions. We shall consider the isothermal case and homogeneous boundary conditions. In this case, the problem takes the form:

$$\frac{\partial C}{\partial t} = D\left[\frac{\partial^2 C}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)\right]; \quad (31)$$

$$\left.\begin{array}{l} C(r,z,t)|_{t=0} = 0 \\ C(r,z,t)|_{r=R} = C_0 = const \\ C(r,z,t)|_{z=0} = C_0 = const \\ C(r,z,t)|_{z=\beta(t)} = C_0 = const \end{array}\right\}, \quad (32)$$

where C_0 is the value of the concentration of adatoms on the sample surface.

The integral transforms found by us made it possible to reduce the problem (31)–(32) to Voltaire's equations of the second kind in a Banach space. Then the general solution of the problem takes the form:

$$C(r,z,t) = \sum_{\kappa=0}^{\infty} I_0(\lambda_{o\kappa} r)\Bigg\{ e^{-Dt}\Bigg[\frac{RI_1(\lambda_{o\kappa}R)}{2\sqrt{\pi D}}\int_0^t d\tau \int_0^H \frac{C_1}{\sqrt{t-\tau}} e^{-Dt} e^{-\frac{(z-\xi)^2}{4Д(t-\tau)}} d\xi +$$

$$+\frac{1}{4\sqrt{\pi}}\int_0^t \frac{z}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{z^2}{4Д(t-\tau)}} K_1(\tau)d\tau + \frac{1}{4\sqrt{\pi}}\int_0^t \frac{z-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{\left[z-\beta(\tau)\right]^2}{4D(t-\tau)}} K_2(\tau)d\tau \Bigg]\Bigg\}. \quad (33)$$

We need to calculate the integrals:

$$I_1 = \int_0^t d\tau \int_0^H \frac{C_0}{\sqrt{t-\tau}} e^{-D\tau} e^{-\frac{(z-\xi)^2}{4D(t-\tau)}} d\xi; \quad (34)$$

$$I_2 = \int_0^t \frac{z}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{z^2}{4D(t-\tau)}} K_1(\tau)d\tau; \quad (35)$$

$$I_3 = \int_0^t \frac{z-\beta(\tau)}{\left[D(t-\tau)\right]^{3/2}} e^{-\frac{\left[z-\beta(\tau)\right]^2}{4D(t-\tau)}} K_2(\tau)d\tau. \quad (36)$$

For large measurement times t, the integrals I_2 are I_3 negligibly small and $e^{-Дt} \to 1$. Then the problem reduces to calculating the integral I_1:

$$I_1 = \int_0^t d\tau \int_0^H \frac{\gamma(\xi,\tau)}{\sqrt{t-\tau}} e^{-D\tau} e^{-\frac{(z-\xi)^2}{4D(t-\tau)}} d\xi = C_0 \int_0^t \frac{e^{-D\tau}}{\sqrt{t-\tau}} I_1'(\tau)d\tau. \quad (37)$$

To calculate $I_1'(\tau)$ in (37) we make a change of variables $y = \frac{z-\xi}{\sqrt{4D(t-\tau)}}$, then we get:

$$I_1'(\tau) = \sqrt{4D(t-\tau)}\left[\int_0^{z_2} e^{-y^2}dy - \int_0^{z_1} e^{-y^2}dy\right], \quad (38)$$

where

$$z_1 = \frac{z}{\sqrt{4D(t-\tau)}}, \quad z_2 = \frac{z-H}{\sqrt{4D(t-\tau)}}.$$

The integrals in square brackets represent a function:

$$erfz = \frac{2}{\sqrt{\pi}}\int_0^z e^{-y^2}dy. \quad (39)$$

Using formula (39) and its expansion in a series, after simple calculations we obtain:

$$I_1'(\tau) = \left[ze^{-\frac{z^2}{\sqrt{4D(t-\tau)}}} - (z-H)e^{-\frac{(z-H)^2}{\sqrt{4D(t-\tau)}}} \right]. \tag{40}$$

Substituting (40) into (38), and calculating, we obtain:

$$I_1 = \frac{2\sqrt{D}}{z} \cdot U_1. \tag{41}$$

Restricting ourselves to the first term in the sum (33), for the stationary concentration we have the following expression:

$$C(r,z) = \frac{C_0 R}{\sqrt{\pi} z} I_0\left(\frac{2r}{R}\right). \tag{42}$$

When obtaining (42), we took into account that the equation $I_0(\lambda_{o\kappa} r) = 0$ implies $\lambda_0 = 2r/R$ and $I_1(2) = 1$. The radial and axial components of the concentration gradient, taking into account (42), will be equal to:

$$\frac{\partial C}{\partial r} = \frac{2}{z}\frac{C_0}{\sqrt{\pi}} I_1\left(\frac{2r}{R}\right); \tag{43}$$

$$\frac{\partial C}{\partial z} = \frac{RC_0}{\sqrt{\pi} z^2} I_0\left(\frac{2r}{R}\right). \tag{44}$$

Equations (43) and (44) can be easily integrated from 0 to *R*, but it is better to analyze the wave process using gradients. Both equations show the wave nature of the solidification of the coating.

In our experiments, approximation (5) can not be applied. This is clearly seen in Figures 3 and 4, where the wavelength is of the order of 10–4 m, i.e. the mass transfer rate is $\sim 10^{-4}$ m/s. Since the mass transfer rate $V \approx \sqrt{D/t}$, for the diffusion coefficient we get the estimate $D \sim 10^{-8}$ m^2/s. This corresponds to the regime of small diffusion.

Comparison of Figures 1 and 2 with Figures 1 and 4 shows their similarity. This suggests that the mechanism of formation of autowaves in both cases has similar features and is described by Bessel functions (Fig. 5).

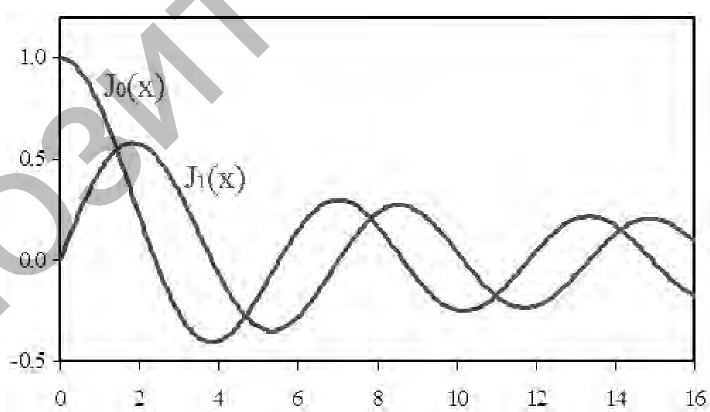

Figure 5. Graphs of Bessel functions

The experimental and theoretical results obtained by us fit into the model of macroscopic localization of the plastic flow developed in [12]. In this paper it is shown that the localization of plastic flow in metals and alloys has a pronounced wave character. In the light-slip, linear and parabolic strain hardening stages, as well as in the pre-fracture stage, the localization patterns observed are different types of wave processes. Analysis of the wave characteristics of such processes made it possible to measure the propagation velocity ($\sim 10^{-4}$ m/s), wavelength ($\sim 10^{-2}$ m), and establish that the dispersion relation for such waves is quadratic.

The theory of crystallization of a cylinder of finite dimensions developed by us relates to problems with a moving interface and is called the Stefan problem [18]. From a mathematical point of view, boundary-value problems of this type are fundamentally different from the classical problems of heat conduction or diffusion. Due to the dependence of the size of the flow transfer region on time, classical methods of separating variables and integral Fourier transforms are not applicable to this type of problems, since, remaining within the framework of classical methods of mathematical physics, it is not possible to coordinate the solution of the equation with the motion of the phase boundary boundary. The motion of the boundary of the phase difference leads to a nonlinearity of the system of equations, which leads to the appearance of autowaves.

Conclusion

The deposition of coatings in a plasma is a thermodynamically nonequilibrium process in an open system. The nonlinearity of the equations arises from the motion of the interface and the small diffusion of surface atoms. In this case, an autowave process arises. The solution we obtained about the crystallization of a cylinder of finite dimensions using the theory of thermal potentials is valid for boundary value problems of the first type. The solution of the second and third boundary value problems in the finite domain encounters considerable difficulties. Studies in this area continue to this day.

The work was carried out under the program of the Ministry of Education and Science of the Republic of Kazakhstan. Grants No. 0118RK000063 and No. F.0780.

References

1 Васильев В.А. Автоволновые процессы / В.А. Васильев, Ю.М. Романовский, В.Г. Яхно. — М.: Наука, 1987. — 240 с.

2 Хакен Г. Информация и самоорганизация: Макроскопический подход к сложным системам / Г. Хакен — М.: Мир, 1991. — 240 с.

3 Заславский Г.М. Физика хаоса в гамильтоновых системах / Г.М. Заславский. — М.; Ижевск: Ин-т компьютерных исследований, 2004. — 288 с.

4 Анищенко В.С. Динамическое и статистическое описание колебательных систем / В.С. Анищенко, Т.Е. Вадивасова, Л. Шиманский-Гайер. — М.; Ижевск: Ин-т компьютерных исследований, 2005. — 156 с.

5 Kahrig E. Dissipative Structuren / E. Kahrig, H. Beberdich. — Leipzig: VEB Georg Thieme, 1977. — 342 p.

6 Лауринас В.Ч. Автоволновые процессы при формировании ионно-плазменных покрытий / В.Ч. Лауринас, О.Н. Завацкая, В.М. Юров, С.А. Гученко // Вестн. Караганд. ун-та. Сер. Физика. — 2013. — № 1(69). — С. 57–68.

7 Davydov V.A. Autowaves in the moving excitable media / V.A. Davydov, N.V. Davydov, V.G. Morozov, M.N. Stolyarov, T. Yamaguchi // Condensed Matter Physics. — 2004. — Vol. 7, No. 3(39). — P. 565–578.

8 Kolobov A.V. Autowaves in the Model of Infiltrative Tumour Growth with Migration-Proliferation Dichotomy / A.V. Kolobov, V.V. Gubernov, A.A. Polezhaev // Math. Model. Nat. Phenom. — 2011. — Vol. 6, No. 7. — P. 1–12.

9 Герасев А.П. Неравновесная термодинамика автоволновых процессов в слое катализатора / А.П. Герасев // УФН. — 2004. — Т. 174, № 10. — С. 1061–1087.

10 Елькин Ю.Е. Автоволновые процессы / Ю.Е. Елькин // Математическая биология и биоинформатика. — 2006. — Т. 1, № 1. — С. 27–40.

11 Хищенко К.В. О волне плавления в металле при быстром нагреве мощным импульсом тока / К.В. Хищенко, С.И. Ткаченко, П.Р. Левашов // Письма в ЖТФ. — 2006. — Т. 32, Вып. 3. — С. 67–74.

12 Зуев Л.Б. Физика макролокализации пластического течения / Л.Б. Зуев, В.И. Данилов, С.А. Баранникова. — Новосибирск: Наука, 2008. — 328 с.

13 Майер Р.В. Компьютерное моделирование автоволновых процессов / Р.В. Майер // Потенциал. — 2009. — № 7(55). — С. 42–49.

14 Мищенко Е.Ф. Автоволновые процессы в нелинейных средах с диффузией / Е.Ф. Мищенко, В.А. Садовничий, А.Ю. Колесов, Н.Х. Розов. — М.: Физматлит, 2010. — 395 с.

15 Хабибуллин И.Л. Автоволновой режим нагрева диэлектрических сред электромагнитным излучением / И.Л. Хабибуллин, Ф.Ф. Назмутдинов, А.Ф. Габзалилов // Теплофизика и аэромеханика. — 2010. — Т. 17, № 2. — С. 229–236.

16 Глызин С.Д. Автоволновые процессы в кольцевой нейронной цепи с однонаправленной связью / С.Д. Глызин, А.Ю. Колесов, Н.Х. Розов // Моделирование и анализ информационных систем. — 2015. — Т. 22, № 3. — С. 404–419.

17 Барвинок В.А. Управление напряженным состоянием и свойства плазменных покрытий / В.А. Барвинок. — М.: Машиностроение, 1990. — 384 с.

18 Gupta S.C. The Classical Stefan Problem: Basic Concepts, Modelling and Analysis / S.C. Gupta. — Amsterdam: Elsevier, 2003. — 385 p.

С.А. Гученко, Е.Н. Еремин, В.М. Юров, В.Ч. Лауринас, С.С. Касымов, О.Н. Завацкая

Плазмалық қаптамаларды тұндыру барысындағы автотолқындық үрдістер

Плазмалық қаптамаларды тұндыру кезінде автотолқын үрдістері қарастырылды. Жабынды алу үшін композициялық катод пен болаттан жасалған катодтың тот баспайтын түрлері қолданылды. Микроқаттылығын өлшеу үлгінің бойы және көлденең түрде 50 рет өлшенді. Микроқаттылық кестесі толқын ұзындығы шамамен 10^{-4} м кезеңдік құрылымын көрсетеді. Диффузия коэффициенті шамамен 10^{-8} м^2/с, яғни аз диффузиялық жүйемен жұмыс жасаймыз. Плазмада қаптаманы тұндыру ашық жүйеде термодинамикалық теңдеспеген үдерісті көрсетеді. Фазалар шекара бөлігінің қозғалысы мен жер бетіндегі атомдарының кіші диффузиясынан теңдеулердің қисықтығы туындайды. Бұл жағдайда автотолқын үрдісі пайда болады. Біз алған эксперименттік және теориялық нәтижелер пластикалық ағымдағы макроскопиялық оқшаулау моделіне төселеді. Бұл модельде пластикалық ағымдағы оқшаулауда металл мен қоспада толқын сипатының айқындығы көрсетілген. Біз дамытып отырған теория, яғни, цилиндрдің түпкілікті мөлшерін кристалдандыру фазалардың шекарасын жылжымалы бөлу міндеттеріне жатады және Стефан міндеті деп аталады. Математикалық тұрғысынан мұндай түрдегі шеттік есептер классикалық жылуөткізгіштік есептерінен немесе диффузиядан түбегейлі жақсы. Мөлшердің тәуелділік салдарынан көшіру облысы уақыт ағынының ауысуына байланысты осы міндеттердің түріне айнымалыларды бөлу және интегралдық өзгерістердің Фурье классикалық әдістерін қолдануға болмайды, өйткені классикалық математикалық физика әдістердің аясында отырып, фаза бөлімінің шекараларын қозғалысы мен теңдеуінің шешімін келістіру мүмкін емес. Фазалар бөлігінің шекаралар қозғалысы сызықтық емес теңдеулер жүйесіне әкеледі, бұдан автотолқын пайда болады.

Кілт сөздер: автотолқын, қаптама, плазма, диффузия, микроқаттылық, кристаллизация, аз диффузиялық жүйе.

С.А. Гученко, Е.Н. Еремин, В.М. Юров, В.Ч. Лауринас, С.С. Касымов, О.Н. Завацкая

Автоволновые процессы при осаждении плазменных покрытий

Рассматриваются автоволновые процессы, возникающие при осаждении плазменных покрытий. Для получения покрытий использовались композиционные катоды и катоды из нержавеющей стали. Проводились измерения микротвердости вдоль и поперек образца в количестве до 50 штук. Графики микротвердости представляют собой периодические структуры с длиной волны порядка 10^{-4} м. Коэффициент диффузии имеет порядок 10^{-8} м^2/с, т.е. мы имеем систему с малой диффузией. Осаждение покрытий в плазме представляет собой термодинамически неравновесный процесс в открытой системе. Нелинейность уравнений возникает из-за движения границы раздела фаз и малой диффузии поверхностных атомов. В этом случае возникает автоволновой процесс. Полученные нами экспериментальные и теоретические результаты укладываются в модель макроскопической локализации пластического течения. В этой модели показано, что локализация пластического течения в металлах и сплавах имеет ярко выраженный волновой характер. Развитая нами теория кристаллизации цилиндра конечных размеров относится к задачам с подвижной границей раздела фаз и носит название «задача Стефана». С математической точки зрения краевые задачи такого типа принципиально отличны от классических задач теплопроводности или диффузии. Вследствие зависимости размера области переноса потока от времени к этому типу задач неприменимы классические методы разделения переменных и интегральных преобразований Фурье, так как, оставаясь в рамках классических методов математической физики, не удаётся согласовать решение уравнения с движением границы раздела фаз. Движение границы раздела фаз приводит к нелинейности системы уравнений, что и приводит к возникновению автоволн.

Ключевые слова: автоволна, покрытие, плазма, диффузия, микротвердость, кристаллизация, система с малой диффузией.

References

1 Vasilev, V.A., Romanovskii, Yu.M., & Yakhno, V.G. (1987). *Avtovolnovye protsessy [Autowave processes]*. Moscow: Nauka [in Russian].

2 Haken, G. (1991). *Informatsiia i samoorhanızatsiia: Makroskopicheskii podkhod k slozhnym sistemam [Information and self-organization: A macroscopic approach to complex systems]*. Moscow: Mir [in Russian].

3 Zaslavskii, G.M. (2004). *Fizika khaosa v hamiltonovykh sıstemakh [The physics of chaos in Hamiltonian systems]*. — Moscow-Izhevsk: Institut kompiuternykh issledovanii [in Russian].

4 Anishchenko, V.S., Vadivasova, T.E., & Shimanskii-Gaier, L. (2005). *Dinamicheskoe i statisticheskoe opisanie kolebatelnykh sistem [Dynamic and statistical description of oscillatory systems]*. Moscow-Izhevsk: Institut kompiuternykh issledovanii [in Russian].

5 Kahrig, E. & Beberdich, H. (1977). *Dissipative Structuren*. Leipzig: VEB Georg Thieme.

6 Laurinas, V.Ch., Zavatskaia, O.N., Yurov, V.M., & Guchenko, S.A. (2013). Avtovolnovye protsessy pri formirovanii ionno-plazmennykh pokrytii [Autowave processes in the formation of ion-plasma coatings]. *Vestnik Karahandinskoho universiteta. Seriia Fizika — Bulletin of the Karaganda University. Ser. Physics, 1(69)*, 57–68 [in Russian].

7 Davydov, V.A., Davydov, N.V., Morozov, V.G., Stolyarov, M.N., & Yamaguchi, T. (2004). Autowaves in the moving excitable media. *Condensed Matter Physics, 7, 3(39)*, 565–578.

8 Kolobov, A.V., Gubernov, V.V., & Polezhaev, A.A. (2011). Autowaves in the Model of Infiltrative Tumour Growth with Migration-Proliferation Dichotomy. *Math. Model. Nat. Phenom., 6, 7*, 1–12.

9 Gerasev, A.P. (2004). Neravnovesnaia termodinamika avtovolnovykh protsessov v sloe katalizatora [Nonequilibrium thermodynamics of autowave processes in the catalyst layer]. *UFN — UFN, 174, 10*, 1061–1087 [in Russian].

10 Elkin, Yu.E. (2006). Avtovolnovye protsessy [Autowave processes]. *Matematicheskaia biolohiia i bioinformatika — Mathematic biology and bioinformatics, 1, 1*, 27–40 [in Russian].

11 *Khishchenko, K.V., Tkachenko, S.I., & Levashov, P.R.* (2006). O volne plavleniia v metalle pri bystrom nahreve moshchnym impulsom toka [On the melting wave in a metal with rapid heating by a powerful current pulse]. *Pisma v JTF — Letters to JTPh, 32*, 3, 67–74 [in Russian].

12 Zuev, L.B., Danilov, V.I., & Barannikova, S.A. (2008). *Fizika makrolokalizatsii plasticheskoho techeniia [The physics of macrolocalization of plastic flow].* Novosibirsk: Nauka [in Russian].

13 Maier, R.V. (2009). Kompiuternoe modelirovanie avtovolnovykh protsessov [Computer simulation of autowave processes] // *Potentsial — Potential*, *7(55)*, 42–49 [in Russian].

14 Mishchenko, E.F., Sadovnichii, V.A., & Kolesov A.Yu., Rozov N.Kh. (2010). *Avtovolnovye protsessy v nelineinykh sredakh s diffuziei [Autowave processes in nonlinear media with diffusion]*. Moscow: Fizmatlit [in Russian].

15 Habibullin, I.L., Nazmutdinov, F.F. & Gabzalilov, A.F. (2010). Avtovolnovoi rezhim nahreva dielektricheskikh sred elektromahnitnym izlucheniem [Autowave mode of heating of dielectric media by electromagnetic radiation]. *Teplofizika i aeromehanika — Thermophysics and aeromechanics*, 17, 2, 229–236 [in Russian].

16 Glyzin, S.D., Kolesov, A.Yu., & Rozov, N.Kh. (2015). Avtovolnovye protsessy v koltsevoi neironnoi tsepi s odnonapravlennoi sviaziu [Autowave processes in a ring neural network with unidirectional coupling]. *Modelirovanie i analiz informatsionnykh sistem — Modeling and analysis of information systems,* 22, 3, 404–419 [in Russian].

17 Barvinok, V.A. (1990). *Upravlenie napriazhennym sostoianiem i svoistva plazmennykh pokrytii [Stress state control and properties of plasma coatings]*. Moscow: Mashinostroenie [in Russian].

18 Gupta, S.C. (2003). *The Classical Stefan Problem: Basic Concepts, Modelling and Analysis.* Amsterdam: Elsevier.