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ON COMPACTNESS OF COMMUTATORS FOR SINGULAR INTEGRAL OPERATOR ON GENERALIZED MORRY SPACES

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In this paper, we give the sufficient conditions for the compactness of the commutators of the singular integral operator Calderón-Zygmund in generalized Morrey spaces.

The compactness of the commutator for the singular integral operator Calderón-Zygmund on the Morrey spaces M_p^{λ} was considered in (4). The boundedness of the singular integral operator Calderón-Zygmund in generalized Morrey spaces was discussed in (1). The pre-compactness of sets on the generalized Morrey spaces $M_p^{w(\cdot)}$ was examined in (2).

Definition 1. Let $1 \le p \le \infty$ and let w be a measurable non-negative function on $(0,\infty)$ that is not equivalent to zero. The generalized Morrey space $M_p^{w(\cdot)} \equiv M_p^{w(\cdot)}(R^n)$ is defined as the set of all functions $f \in L_p^{loc}(R^n)$ with $\|f\|_{M_p^{w(\cdot)}} < \infty$, where

$$\|f\|_{M_p^{w(\cdot)}} = \sup_{x\in R^n, \quad r>0} \Big(w(r) \, \|f\|_{L_p(B(x,r))} \Big).$$

The space $M_p^{w(\cdot)}$ coincides with the Morrey space M_p^{λ} if $w(r) = r^{-\lambda}$, where $0 \le \lambda \le \frac{n}{p}$.

By $\Omega_{p\infty}$ we denote the set of all non-negative, measurable on $(0,\infty)$ functions, not equivalent to 0 and such that for some $t > 0$,

$$\|w(r) r^{\frac{n}{p}}\|_{L_\infty(0,t)} < \infty, \quad \|w(r)\|_{L_\infty(t,\infty)} < \infty.$$

The space $M_p^{w(\cdot)}$ is non-trivial if and only if $w \in \Omega_{p\infty}$ (3).

Next, we will provide the definition of the singular integral operator for the Calderón-Zygmund T, which plays a crucial role in harmonic analysis and potential theory.

$$Tf(x) = p.v. \int\limits_{R^n} \frac{f(y)K(x-y)}{|x-y|^n} dy.$$

Let $S^{n-1} = x \in R^n : |x| = 1$ be the unit sphere in R^n with surface measure $d\sigma$. Suppose that K satisfies the following conditions:

(i) K is a homogeneous function of degree zero on $R^n \backslash 0$, i.e.,

$$K(\mu x) = \mu K(x),\ \forall \mu > 0\ and\ x \in R^n \backslash \{0\} \tag{1}$$

(ii) K has zero on average on S^{n-1}, i.e.,

$$\int_{S^{n-1}} K(x') d\sigma(x') = 0 \tag{2}$$

(iii) $K \in\ Lip(S^{n-1})$, i.e.,

$$|K(x') - K(y')| \leq |x' - y'|,\ \forall x', y' \in S^{n-1}, \tag{3}$$

where $x' = \frac{x}{|x|}$ for any $x \neq 0$.

Definition 2. For a function $b \in L_{\text{loc}}(R^n)$, let M_b denote the multiplication operator $M_b f = bf$, where f is a measurable function. Then, the commutator for the singular integral operator T and the operator M_b is defined by the equation:

$$[b, T]\,(f)(x) = b(x)T(f)(x) - T(bf)(x) = \int\limits_{R^n} \frac{(b(x) - b(y))\,K(x-y)f(y)}{|x-y|^n} dy.$$

A function $b(x) \in L_\infty(R^n)$ belongs to the space $BMO(R^n)$ if $\|b\|_* = \sup_{Q \subset R^n} \frac{1}{|Q|} \int\limits_Q |b(x) - b_Q|\, dx < \infty$, where Q is ball in R^n and $b_Q = \frac{1}{|Q|} \int\limits_{R^n} f(y) dy$.

Definition 3. Let $VMO(R^n)$ denote the BMO-closure of the space $C_0^\infty(R^n)$, where $C_0^\infty(R^n)$ is the set of all functions from $C^\infty(R^n)$ with compact support.

Theorem Let $1 < p < \infty$, $w_1, w_2 \in \Omega_{p\infty}$. Let also, for $s \leq p$, the pair $w_1(r), w_2(r)$ satisfy the condition $\int\limits_r^\infty \left(1 + ln\frac{t}{r}\right) \frac{ess \inf\limits_{t<\tau<\infty} \frac{\tau^{\frac{n}{p}}}{w_1(\tau)}}{t^{\frac{n}{p}+1}} dt \leq C\frac{1}{w_2(r)}$, and, for $1 < p < s$, the pair $w_1(r), w_2(r)$ satisfy the condition $\int\limits_r^\infty \left(1 + ln\frac{t}{r}\right) \frac{ess \inf\limits_{t<\tau<\infty} \frac{\tau^{\frac{n}{p}}}{w_1(\tau)}}{t^{\frac{n}{p}-\frac{n}{s}+1}} dt \leq C\frac{r^{\frac{n}{s}}}{w_2(r)}$, where C does not depend on x and r. K satisfies the conditions (1), (2), (3), $b \in VMO(R^n)$.

Then the commutator $[b, T]$ is a compact operator from $M_p^{w_1(\cdot)}$ to $M_p^{w_2(\cdot)}$.

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THE INVERSE PROBLEM FOR A FOURTH-ORDER PARABOLIC EQUATION WITH A COMPLEX-VALUED COEFFICIENT

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This article presents the results of studies of inverse problems for a fourth-order parabolic equation with a variable complex-valued coefficient. The existence and uniqueness of the solution of inverse problems for a one-dimensional fourth-order equation is established

$$u_t\left(x,t\right)+\frac{\partial^4}{\partial x^4}u\left(x,t\right)+q\left(x\right)u\left(x,t\right)=f\left(x\right) \quad (1)$$

with Dirichlet boundary conditions

$$u\left(-1,t\right)=0,\ u\left(1,t\right)=0,\ u_{xx}\left(-1,t\right)=0,\ u_{xx}\left(1,t\right)=0,\ \ t\in\left[0,T\right]. \quad (2)$$

We have to find a pair of functions $u\left(x,t\right)$ and $f\left(x\right)$ satisfying equation (1) in the domain Ω and conditions

$$u\left(x,0\right)=\varphi\left(x\right),\ u\left(x,T\right)=\psi\left(x\right),\ x\in\left[-1,1\right], \quad (3)$$

where $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions and $q\left(x\right)=q_1\left(x\right)+iq_2\left(x\right)$. We will use $\Omega=\left\{-1<x<1,\ 0<t<T\right\}$ to denote an open domain, and $\bar{\Omega}=\left\{-1\le x\le 1,\ 0\le t\le T\right\}$ to denote a closed domain.

Let us introduce a non-self-conjugate fourth-order differential operator $L_q : D\left(L_q\right)\subset L_2\left(-1,1\right)\to L_2\left(-1,1\right)$ by the formula

$$L_q y=y^{IV}\left(x\right)+q\left(x\right)y\left(x\right),\ -1\le x\le 1,$$

with the domain of definition

$$D\left(L_q\right)=\left\{y\left(x\right)\in C^3\left[-1,1\right]:y^{IV}\left(x\right)\in L_2\left(-1,1\right)\right\}.$$

Theorem. Let $q\left(x\right)\in C^4\left[-1,1\right]$, and functions $\varphi,\ \psi$ are such that $\varphi,\ \ \psi,\ \ L_q\varphi,\ \ L_q\psi\in D\left(L_q\right)$. Then inverse problem (1), (2), (3) has a unique solution, which can be represented as Fourier series

$$u(x,t)=\varphi\left(x\right)+\sum_{k=0}^{\infty}\frac{\varphi_k-\psi_k}{1-e^{-\lambda_k T}}\left(e^{-\lambda_k t}-1\right)X_k\left(x\right),$$

and

$$f\left(x\right)=L_q\varphi\left(x\right)-\sum_{k=0}^{\infty}\frac{\varphi_k-\psi_k}{1-e^{-\lambda_k T}}\lambda_k\cdot X_k\left(x\right).$$

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