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On the application of mathematical methods for the research of vibration processes in mechanics

Article represents the study of applied problems of mathematics, whose mathematical modeling leads to boundary problems for equations in partial derivatives. Mathematical methods, applied to these models, enable to obtain exact analytical results. Detailed result is represented for boundary problem of oscillations of thin structures with boundary conditions in general terms. Application of spectral decomposition for sufficiently smooth function, characterizing the membrane deviation from equilibrium state, enables to define exact analytic representation of inflection function for studied problem. To calculate multilayer plates, method of finite elements is applied.

Keywords: oscillating motions, thin structures, orthonormal function system, inflection function, multilayer plates.

Integral instrument of mechanician, which is impossible to work without, is the wide range of mathematical methods, applied in mechanics.

Boundary problems for equations in partial derivatives, closely related to study if mechanical problems, became especially urgent due to developing extent of their application. Differential equations appear naturally at study of problems of classic mechanics, mechanics of continue, acoustics, optics, hydrodynamics, etc., at modeling of oscillatory processes, deformation processes, transportation processes, heat and weight exchange processes, dynamics, demolition, etc.

Thus, boundary problems for equations of mathematical physics are successfully applied, for example, in description of motion of continuum, where main equations of mechanics of continue are shown, which represent mass conservation law, energy conservation law, impulse law and law of conservation of angular momentum, peculiar for every physicist.

We consider the partial differential equations that describe mathematical models of mechanical and physical phenomena. We often use the second order partial differential equations of hyperbolic type in the problems of oscillation theory and we apply the parabolic equations in problems of mechanics, where the characteristics of the various elements of constructions are investigated under the influence of different temperatures [1].

We consider the partial differential equations that describe mathematical models of mechanical and physical phenomena. We often use the second order partial differential equations of hyperbolic type in the problems of oscillation theory and we apply the parabolic equations in problems of mechanics, where the characteristics of the various elements of constructions are investigated under the influence of different temperatures [1].

Method of Riemann functions, integral transformations, method of partition, finite elements method and other mathematical and numeral methods are effectively applied in different spheres of exact sciences, including problems of mechanics, such as dynamic problems for taut space, problems of fluid dynamics and its relations with elastic bodies, problems of oscillatory processes, etc.

Equation, defining the distribution of elastic waves in prismatic bar, whose longitudinal motions of points does not depend on coordinates in its cross section, and extension rigidity is the same that is in statics, and has the following type [1]

$$\rho F u_{tt} - EF u_{xx} = Q(t,x),$$

where u — moving; F — cross section area; Q — external axial load.

In the simplest cases, such as in problem of longitudinal oscillations of the bar, modeled by equations

$$u_{tt} - a^2 u_{xx} = Q(t,x),\ u_x(t,0) = 0,\ u(0,x) = \varphi(x),\ u_t(0,x) = \psi(x),\ 0 < t,\ x < \infty,$$

at application of integral cos-transformation [2], result may be obtained in evident analytic form

$$u(t,x)=\frac{1}{2}[\varphi(x+at)+\varphi(|x-at|)]+\frac{1}{2a}\left[\int\limits_0^{x+at}\psi(z)dz-sign(x-at)\int\limits_0^{|x-at|}\psi(z)dz\right]+$$

$$+\frac{1}{2a}\int\limits_0^t d\tau\left[\int\limits_0^{x+a(t-\tau)}Q(\tau,z)dz-sign(x-a(t-\tau))\int\limits_0^{x+a(t-\tau)}Q(\tau,z)dz\right].$$

Plane problem on longitudinal ambulatory plate distortions is defined with the following equations [1]

$$\nu_{tt}-\nu_{xx}-(1-2c^2)\omega_x=Q_0;$$

$$\omega_{tt}-c^2\omega_{zz}+3\omega+3(1-2c^2)\nu_x=0;$$

$$Q_0=\frac{1}{2}\int\limits_{-1}^{1}Q(t,x,y)dy,\ c=\sqrt{\frac{\mu}{\rho}},$$

where ν,ω — wave velocity in continuum; μ — Lame's constant.

Considering that axis stresses, averaged in cross-section $\sigma_x x$ are defined with equation

$$\sigma_{xx}=\nu_{xx}+(1-2c^2)\omega,$$

for infinite plate at

$$Q_0=2\delta(x),$$

which corresponds to compression of the right part of plate ($x>0$) with single force and strain of left part with the same force ($x<0$), and after application of Laplace transformation under t and Fourier transformation under x [3] and applying asymptomatic representations at reversion, we obtain

$$\sigma_x x\approx-\frac{1}{3}+\int\limits_0^{\eta}Ai(\tau)d\tau-J_0\left(\sqrt{6\frac{1-d^2}{1-c^2}t(t-x)}\right);$$

$$\eta=(x-ct)\left[\frac{1}{6}\left(1+c^2-d^2-\frac{c^2}{d^2}\right)\right]^{-\frac{1}{3}};$$

$$d^2=\frac{E}{(1-\nu^2)\rho},$$

where $Ai(\tau)$ — Airy stress function; $J_0(z)$ — Bessel function.

Consider the problem of vibrations of the infinite rod [1]

$$u_{xx}-u_{yy}+au_x+bu_y+cu=0,\ -\infty<x<\infty,\ 0<y<\infty;\ u(x,0)=\varphi(x),\ u_y(x,0)=g(x).$$

Using the method of the Riemann function [1], we find

$$u(x,y)=\left\{\frac{\varphi(x-y)\cdot e^{-\frac{a-b}{2}y}+\varphi(x+y)\cdot e^{-\frac{a+b}{2}y}}{2}\right\}-$$

$$-\frac{1}{2}e^{\frac{b}{2}y}\int\limits_{x-y}^{x+y}\left\{\frac{b}{2}J_0(\sqrt{c_1}\sqrt{(x-\varsigma)^2-y^2})-\sqrt{c_1}y\frac{J_1(\sqrt{c_1}\sqrt{(x-\varsigma)^2-y^2})}{\sqrt{(x-\varsigma)^2-y^2}}\right\}\cdot e^{-\frac{a}{2}(x-\varsigma)}\varphi(\varsigma)d\varsigma+$$

$$+\frac{1}{2}^{\frac{b}{2}y}\int\limits_{x-y}^{x+y}J_0\left(\sqrt{c_1}\sqrt{(x-\varsigma)^2-y^2}\right)\cdot e^{\frac{a}{2}(x-\varsigma)}g(\varsigma)d(\varsigma).$$

Applying the Laplace transformation [2] to the more general problem for the wave equation

$$u_{tt} = a^2 u_{xx} + c^2 u + f(x,y),\ -\infty < x < \infty,\ 0 < t < \infty;\ u(x,0) = \varphi(x), u_t(x,0) = g(x),$$

we receive the solution in the analytic form

$$u(x,t) = \frac{1}{2a} \cdot \int\limits_{x+at}^{x-at} g(\varsigma) I_0 \left(c\sqrt{t^2 - \frac{(x-\varsigma)^2}{a^2}} \right) d\varsigma +$$

$$+\frac{\varphi(x-at)+\varphi(x+at)}{2} + \frac{ct}{2a} \cdot \int\limits_{x-at}^{x+at} \varphi(\varsigma) \frac{I_1\left(c\sqrt{t^2 - \frac{(x-\varsigma)^2}{a^2}} \right)}{\sqrt{t^2 \frac{(x-\varsigma)^2}{a^2}}} d\varsigma +$$

$$+\frac{1}{2a} \cdot \int\limits_0^t dt \int\limits_{x-a(t-\tau)}^{x+a(t-\tau)} f(\varsigma,\tau) I_0 \left(c\sqrt{(t-\tau)^2 - \frac{(x-\varsigma)^2}{a^2}} \right) d\varsigma.$$

The boundary value problem for the heating of the infinite rod [1]

$$\frac{\partial u}{\partial t} = \frac{a^2}{x^\nu} \cdot \frac{\partial}{\partial x}(x^\nu \frac{\partial u}{\partial x}),\ 0 < x < \infty, 0 < t < \infty;\ \ u(0,t) = \varphi(t), u(\infty,t) = 0, u(x,0) = 0,$$

by applying the mathematical methods [3] has the following solution of this problem

$$u(x,t) \int\limits_0^t \frac{x^{2\eta} \cdot \varphi(\tau)}{\Gamma(\eta) \cdot (4a^2)^\eta \cdot (t-\tau)^{1+\eta}} \cdot \exp\left(-\frac{x^2}{4a^2 \cdot (t-\tau)} \right) d\tau.$$

Among thin structures, which combine lightness with high durability, we can emphasize film-type and membrane constructions. Thin structures (films, membranes, coatings, etc.) may be applied in all the spheres of manufacture and human life support.

In all the spheres of human life technical and economical problems are tried to be solved on the base of films, membranes and coatings. These are problems of friction and wearing, problems of corrosion and erosion, problems of absorption of waves of specified range, problems of protection from high temperatures and fire, problems of protection from viruses and bacteria, problems of mechanisms, products and water conservation, disinfection, etc.

Creation of new films, membranes and coatings with specified operating characteristics is one of perspective directions of mechanics development.

To create new films, membranes and coatings with specified operating characteristics and durability, it is necessary to study temperature-time, mechanic (including oscillatory), chemical and other impacts may cause the demolition processes in material structure, that is why necessary characteristics of films, membranes and coatings are provided mainly by calculation of influence of such impacts on durability and material characteristics necessary for exploitation [4].

Oscillatory process of plane thin structure (membrane) is described with the following equation in second-order partial derivatives [5]:

$$u_{tt} = a^2(u_{xx} + u_{yy}), \tag{1}$$

where $a^2 = \frac{T}{\rho}$; T — tension on a membrane; ρ — membrane density.

Let us study plane uniform rectangular membrane, edge-fixed, with legs b and c in plane XOY, $0 \leq x \leq b$, $0 < y < c$.

Membrane inflection function, i.e its derivations from equilibrium state in point x, y at the moment of time t, shall be denoted with $u(x,y,t)$.

Let us study the process when membrane oscillation is caused by specified primary deviation and specified primary velocity.

To find the function $u(x, y, t)$ we have the following boundary value problem: find the solution of the oscillation equation (1) in the region $0 < x < b,\ 0 < y < c,\ t > 0$ under the initial conditions

$$u(x, y, 0) = \varphi(x, y); \tag{2}$$

$$u_t(x, y, 0) = \psi(x, y), \tag{3}$$

and boundary conditions

$$\alpha_1 u(0, y, t) + \beta_1 u_x(0, y, t) = 0,\ \alpha_2 u(b, y, t) + \beta_2 u_x(b, y, t) = 0; \tag{4}$$

$$\gamma_1 u(x, 0, t) + \theta_1 u_y(x, 0, t) = 0,\ \gamma_2 u(x, c, t) + \theta_2 u_y(x, c, t) = 0, \tag{5}$$

where φ and ψ are given functions; $\alpha_1, \beta_i, \gamma_i, \theta_i$ are given numbers and

$$\alpha_i^2 + \beta_i^2 \neq 0, \gamma_i^2 + \theta_i^2 \neq 0, i = 1, 2.$$

The solution of problem (1)–(5) is founded by Fourier method in the form function, not identically zero

$$u(x, y, t) = \upsilon(x, y) \cdot T(t). \tag{6}$$

Substituting (6) into (1) and dividing the variables, we obtain

$$\frac{T"}{a^2 T} = \frac{\upsilon_{xx} + \upsilon_{yy}}{\upsilon} = -\sigma, \tag{7}$$

where σ is constant of separation of variables, which for convenience of calculations we take with a minus sign, without assuming anything about her sign.

From (7) we obtain the differential equation for the

$$T" + a^2 \sigma T = 0 \tag{8}$$

and the following boundary-value problem for the function $\upsilon(x, y)$

$$\upsilon_{xx} + \upsilon_{yy} + \sigma \upsilon = 0; \tag{9}$$

$$\alpha_1 \upsilon(0, y) + \beta_1 \upsilon_x(0, y) = 0, \alpha_2 \upsilon(b, y) + \beta_2 \upsilon_x(b, y) = 0; \tag{10}$$

$$\gamma_1 \upsilon(x, 0) + \theta_1 \upsilon_y(x, 0) = 0, \gamma_2 \upsilon(x, c) + \theta_2 \upsilon_y(x, c) = 0, \tag{11}$$

where the boundary conditions (10), (11) are obtained by direct substitution (6) into (4), (5).

To solve problem (9)–(11), we again apply Fourier method. The solution of the boundary value problem (9)–(11) will be sought in the form

$$\upsilon(x, y) = X(x) \cdot Y(y), \tag{12}$$

where the function $\upsilon(x, y) \neq 0$. We substitute (12) into (9) and divide the variables

$$\frac{X"}{X} = -\frac{Y"}{Y} - \sigma = -\eta, \tag{13}$$

where η is constant of separation of variables. From the relation (13) and from the boundary conditions (10), (11) we obtain for the definition of the functions $X(x)$ and $Y(y)$ the following one-dimensional spectral eigenvalue problems, where $\tau = \sigma - \eta$

$$\begin{cases} X" + \eta X = 0; \\ \alpha_1 X(0) + \beta_1 X'(0) = 0; \\ \alpha_2 X(b) + \beta_2 X'(b) = 0; \end{cases}$$

$$\begin{cases} Y" + \tau \cdot Y = 0; \\ \gamma_1 Y(0) + \theta_1 Y'(0) = 0; \\ \gamma_2 Y(c) + \theta_2 Y'(c) = 0. \end{cases} \tag{14}$$

Note. We consider the problem for the differential equation with a parameter ν

$$\begin{cases} Z" + \nu \cdot X = 0; \\ h_1 Z(0) + g_1 Z'(0) = 0; \\ h_2 Z(l) + g_2 Z'(l) = 0; \end{cases} \tag{15}$$

where $Z = Z(z); 0 < z < l; g_i (i = 1, 2)$ — given number, and define under which parameter values of v problem has a nontrivial solution.

Under direct calculations [5] obtained that the problem (15) have nontrivial solution in following cases:

1) $\nu = 0$ under the condition

$$g_1 h_2 - h_1 (h_2 l + g_2) = 0; \tag{16}$$

2) $\nu < 0$.

In remark spectral problems (14) have eigenvalues and eigen functions, $\eta = \tau = 0$ if under performance of condition (16) for matching parameters and $\eta < 0,\ \tau < 0$.

We introduce the notation $\eta = \lambda^2, \tau = \mu^2$, then problem (14) will take of the form

$$\begin{cases} X" + \lambda^2 X = 0; \\ \alpha_1 X(0) + \beta_1 X'(0) = 0; \\ \alpha_2 X(b) + \beta_2 X'(b) = 0; \end{cases}$$

$$\begin{cases} Y" + \mu^2 X = 0; \\ \gamma_1 Y(0) + \theta_1 Y'(0) = 0; \\ \gamma_2 Y(c) + \theta_2 Y'(c) = 0. \end{cases} \tag{17}$$

If spectral problems solved (17) we obtained that eigenvalues of problem (17) is $\lambda_1, ...\lambda_n, ...$ and $\mu_1, ...\mu_m, ...$ equation

$$tg\lambda b = \frac{(\alpha_2\beta_1 - \alpha_1\beta_2)\lambda}{\alpha_1\alpha_2 + \beta_1\beta_2\lambda^2}, \quad tg\mu c = \frac{(\gamma_2\theta_1 - \gamma_1\theta_2)\mu}{\gamma\gamma - \theta_1\theta_2\mu^2},$$

root respectively, and eigen functions is function with view as

$$X_n(x) = A_n(\beta_1\lambda_n \cos\lambda_n x - \alpha_1 \sin\lambda_n x), Y_m(y) = B_m(\theta_1\mu_m cos\mu_m y - \gamma_1 sin\mu_m y). \tag{18}$$

$\sigma = \tau + \eta = \lambda^2 + \mu^2$, considering $\tau = \sigma - \eta$ from (14). So, we obtained that eigenvalues $\sigma_{n,m} = \lambda_n^2 + \mu_m^2$ correspond eigen functions according to (12), (18):

$$\upsilon_{nm}(x, y) = X_n(x)Y_m(y) = A_{nm}(\beta_1\lambda_n cos\lambda_{nx} - \alpha_1 \cos\lambda_n x)(\theta_1\mu_m cos\mu_m y - \gamma_1 sin\mu_m y), \tag{19}$$

where $A_{nm} = A_n \cdot B_m$ — constant. Choose it in such a way as to norm of function υ_{nm} with weight at 1 was total 1 that is to say function orthonormaling

$$\int_0^b\int_0^c \upsilon_{nm}^2 dxdy = A_{nm}^2 \int_0^b (\beta_1\lambda_n \cos\lambda_n x - a_1 \sin\lambda_n x)^2 dx \cdot \int_0^c (\theta_1\mu_m \cos_m y - \gamma_1 \sin\mu_m y) dy = 1;$$

$$A_{nm} = \frac{1}{\sqrt{\int\limits_0^b (\beta_1\lambda_n \cos\lambda_n x - a_1 \sin\lambda_n x)^2 dx \cdot \int\limits_0^c (\theta_q\mu_m \cos_m y - \gamma_1 \sin\mu_m y) dy}}. \tag{20}$$

Calculation of coefficient A_{nm} is time-consuming and impractical in general cases according to the formula (20). Notably easy and rational way of calculation of coefficient A_{nm} is calculate in each concrete cases of border-line conditions spectral problem, in contrast with using awkward-to-handle and difficulty-memorizing formula, which was gotten under integral calculation in (20).

Return to original problem (1)–(5). From (19) we have

$$\upsilon_{nm}(x, y) = A_{nm}(\beta_1\lambda_n cos\lambda_{nx} - \alpha_1 \cos\lambda_{nx})(\theta_1\mu_m cos\mu_m y - \gamma_1 sin\mu_m y),$$

where coefficient A_{nm} is ccalculated in each concrete cases of border-line conditions. Find the general solution of equation (8) with $\sigma_{Tn} = \lambda_n^2 + \mu_n^2$

$$T_{nm}(t) = C_{nm} \cos a\sqrt{\sigma_{nm}t} - D_{nm} \sin a\sqrt{\sigma_{nm}t},$$

where C_{nm}, D_{nm} — the arbitrary constant.

Returning to the original problem (1)–(5), we obtain that the particular solutions according to (6) will have view

$$u_{nm}(x, y, t) = \upsilon_{nm}(x, y) \cdot T_{nm}(t) = \upsilon_{nm}(x, y)(C_{nm} \cos a\sqrt{\sigma_{nm}t} + D_{nm} \sin a\sqrt{\sigma_{nm}t}).$$

According to the superposition solution of the equation (1) with the boundary conditions (2), (3) have view

$$u(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} u_{nm}(x, y)T_{nm}(t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(C_{nm} \cos a\sqrt{\sigma_{nm}t} + D_{nm} \sin a\sqrt{\sigma_{nm}t}) \cdot u_{nm}(x, y). \qquad (21)$$

Using initial conditions (4), (5), correlation between (21) and property of u_{nm} function orthonormaling, we find value of constant C_{nm} and D_{nm}.

$$u(x, y, 0) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}u_{nm}(x, y) = \varphi(x, y), \ \Rightarrow C_{nm} \int_0^b\int_0^c \varphi(x, y)u_{nm}dxdy;$$

$$u(x, y, 0) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{nm}a\sqrt{\sigma_{nm}}u_{nm}(x, y) = \psi(x, y), \ \Rightarrow$$

$$\Rightarrow D_{nm} = \frac{1}{a\sqrt{\delta_{nm}}} \int_0^b\int_0^c \psi(x, y)u_{nm}(x, y)dxdy.$$

So then we get analytical form of problem solving (1)–(5):

$$u(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(C - nmcosa\sqrt{\sigma_{nm}}t + D_{nm}sina\sqrt{\sigma_n m}t) \cdot u_{nm}(x, y),$$

where

$$u_{nm}(x, y,) = A_{nm}(\beta_1\lambda_n cos\lambda_n x - \alpha_1 \cos\lambda_n x)(\theta_1\mu_m cos\mu_m y - \gamma_1 sin\mu_m y);$$

$\lambda_1, ..., \lambda_n, ...$ - is equation root $tg\lambda b = \frac{(a_2\beta_1 - a_1\beta_2)\lambda}{a_1a_2 + \beta_1\beta_2\lambda^2}$;

$\mu_1, ..., \mu_m, ...$ - is equation root $tg\mu c = \frac{(\gamma_2\theta_1 - \gamma_1\theta_2)\mu}{\gamma_1\gamma_2 + \theta_1\theta_2\mu^2}$.

$$A_{nm} = \frac{1}{\sqrt{\int_0^b(\beta_1\lambda_n \cos\lambda_n x - a_1 \sin\lambda_n x)^2dx \cdot \int_0^c(\theta_1\mu_m \cos_m y - \gamma_1 \sin\mu_m y)dy}};$$

$$C_{nm} = \int_0^b\int_0^c \varphi(x, y)u_{nm}(x, y)dxdy, \ D_{nm} = \frac{1}{a\sqrt{\delta_{nm}}}\int_0^b\int_0^c \psi(x, y)u_{nm}(x, y)dxdy.$$

So then we get analytical solution of boundary-value problem, which describe fluctuation of beamless plate structure for general cases of border-line conditions. Boundary-value problems and equation in partial derivative is ubiquitous in mechanics of continua, liquid mixture, beamless plate structure, introduction to fracture mechanics, shape memory alloy mechanics, differential modeltheory of viscoelasticity, theory of hereditary elasticity, flow theory and in other mechanic fields [1].

The remarkable thing is that the similar boundary-value problem as investigating problem above, can describe different processes in varied fields of knowledge such as mechanic, physics, chemistry, biology, economics etc, insomuch as mathematical modeling various processes lead to the same tasks under using basic conservation law (energy, mass, particle number). Analytical solution could be used in applied problem of variety science, which have result as represented boundary-value problem under mathematical modeling, in adding number with border-line conditions in general view.

Finite elements method be able used for calculating multilayer plates under inability getting accurate analytical solution [6]. Following calculation formula as a result of using basic classical correlation for displacement, voltage, balance equation for multilayer plates, is [7]:

− for contact conditions in stratum boundary, for voluntary constant

$$u_1^{i-1} = u_1^i, \;\; H\varphi_{i-1}(a_{i-1}) = H\varphi_i(a_{i-1});$$

$$C_{i-1}^0 - a_{i-1} = C_1^0 - a_{i-1}, \;\; C_{i-1}^0 = C_i^0 = C^0;$$

$$\tau_{13}^{i-1} = \tau_{13}^i, \;\; E_0H^2\psi_{i-1}(a_{i-1}) = E_0H^2\psi_i(a_{i-1});$$

$$A_{i-1}^0 - \beta_{i-1}(C^0a_{i-1} - \frac{a_{i-1}^2}{2}) = A_{i-1}^0 - \beta_i(C^0a_{i-1} - \frac{a_{i-1}^2}{2});$$

$$\sigma_3^{i-1} = \sigma_3^i, \;\; E_0H^2\sigma_{i-1}(a_{i-1}) = E_0H^3\sigma_i(a_{i-1});$$

$$B_{i-1}^0 - A_{i-1}^0 + \beta_{i-1}(C_{i-1}^0\frac{a_{i-1}^2}{2}\frac{a_{i-1}^3}{6}) = B_i^0 - A_i^0 + \beta_i(C_i^0\frac{a_{i-1}^2}{2}\frac{a_{i-1}^3}{6});$$

− for voluntary constant

$$C^0 = \frac{1}{2}\frac{\beta_n - \sum\limits_{k=2}^{n}(\beta_k - \beta_{k-1}a_{k-1}^2)}{\beta_n - \sum\limits_{k=2}^{n}(\beta_k - \beta_{k-1}a_{k-1}}, \;\; A_i^0 = C^0\sum_{k=2}^{i}(\beta_k - \beta_{k-1})a_{k-1} - \frac{1}{2}\sum_{k=2}^{i}(\beta_k - \beta_{k-1})a_{k-1}^2;$$

$$B_i^0 = \sum_{k=2}^{i}(A_k^0 - A_{k-1}^0) - \frac{C^0}{2}\sum_{k=2}^{i}(\beta_k - \beta_{k-1})a_{k-1}^2 + \frac{1}{6}\sum_{k=2}^{i}(\beta_k - \beta_{k-1})a_{k-1}^3;$$

− for internal effort

$$M = \sum_{i-1}^{n}\int\limits_{a_{i-1}}^{a_i}\sigma_1^i z dz = -DC_\beta\frac{d^2W}{dx_1^2}, \;\; D = \frac{E_0H^3}{12};$$

$$C_\beta = 12\sum_{i=1}^{n}\left\{\frac{C_0}{2}\beta_i - (a_i^2 - a_{i-1}^2) - \frac{1}{3}\beta_i(a_i^3 - a_{i-1}^3)\right\};$$

$$Q = \sum_{i=1}^{n}\int\limits_{a_{i-1}}^{a_i}\tau_{13}^i dz = DA_\beta\frac{d^3W}{dx_1^3}; \tag{22}$$

$$A_\beta = 12\sum_{i=1}^{n}\left\{A_i^0(a_i - a_{i-1} - \beta_i\left[\frac{C_0}{2}(a_i^2 - a_{i-1}^2) - \frac{1}{6}(a_i^3 - a_{i-1}^3)\right]\right\};$$

$$\sigma_3^n = q = E_0H^3\delta_n(1)\frac{d^4W}{dx_1^4} = DB_\beta\frac{D^4W}{dx_1^4};$$

$$B_\beta = 12\delta_n(1), \;\; C_\beta = A_\beta.$$

Calculation of multilayer plates under given algorithm[7], where resolving equation have view:

$$DB_\beta \frac{d^4W}{dx_1^4} = q,$$

introduce proportion $\eta_i = \frac{D_i}{D_0}$; were D_0 — basic plate modulus one of layer, which is chosen the first from below; D_i — plate modulus of other layers.

Then integral parameters C_η and A_η adding as multiplierin basic formula of finite elements, which calculated by formula (22), but with an allowance instead.

$$F = C_\eta K \cdot V, \;\; M = -C_\eta B \cdot V, \;\; Q = -A_\eta C \cdot V_\eta \text{ [3]}.$$

Cooperative using analytical (for example,integral transformation) and numerical methods is quite often effective for solution multidimensional problems of mechanic, under this possible take results where any one of them in separateway are practically powerless.

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Механикадағы тербелмелі процестерді зерттеу кезінде математикалық әдістерді қолдану қосымшасы туралы

Мақалада қолданбалы механика есептері мен тапсырмаларын зерттеу жұмыстары көрсетілген, оларды математикалық модельдеу дербес туындылар теңдеулеріне шектік есептерге әкеледі. Арнайы модельдерге қоса берілген математикалық тәсілдер өз кезегінде аналитикалық нақты шешімдерді алуға мүмкіндік береді. Жалпы түрде шектес шарттармен қоса жұқабүйірлі құрылымдар тербелісі шеткі есебі үшін нақты шешім алуға болады. Тегіс функция үшін спектралды ыдырауды қолдану тепе-теңдік жағдайларында мембраналардың ауытқуы сипатталды, ол зерттелетін есеп үшін функциялардың иілу кезінде нақты күйін анықтауға мүмкіндік береді. Көпқабатты пластиналарды есептеу үшін шектік элементтер тәсіл пайданылған.

Кілт сөздер: тербелістер, жұқабүйірлі құрылымдар, ортонормаланған функция жүйесі, иілу функциясы, көпқабатты пластиналар.

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О приложении математических методов к исследованию колебательных процессов в механике

В статье представлено исследование прикладных задач механики, математическое моделирование которых приводит к краевым задачам для уравнений в частных производных. Математические методы, приложенные к данным моделям, позволяют получить точные аналитические решения. Подробное решение представлено для краевой задачи колебаний тонкостенных конструкций с граничными условиями в общем виде. Использование спектрального разложения для достаточно гладкой функции, характеризующей отклонение мембраны от положения равновесия, позволяет определить точное аналитическое представление функции прогиба для исследуемой задачи. Для расчета многослойных пластин использован метод конечных элементов.

Ключевые слова: колебания, тонкостенные конструкции, ортонормированная система функций, функция прогиба, многослойные пластины.

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