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## **Development of the algorithm for calculating the optimal molding modes of the BeO slurry using various rheological models**

This paper presents the results of calculating a mathematical model of the flow and heat transfer of thermoplastic beryllium oxide in a round channel of a molding installation. An algorithm for calculating the system of equations based on the Herschel-Bulkley rheological model has been developed. The finite-difference analogue of the equations system of motion, continuity, and energy is solved numerically using the Crank-Nicholson difference scheme. The three-parameter equation is used to test the consistency of experimental data, and how adequately the physical features of the non-isothermal flow of the slurry convey comparing to the Shvedov-Bingham model. The calculation results illustrate that the proposed model reflects the most important features of the thixotropic flow character of the slurry and is in better agreement with the experimental data of viscoplastic fluids. It provides the calculations of speed of viscous-plastic flow of the slurry based on Shvedov-Bingham and Herschel Bulkley's two rheological models considering the peculiarities of coagulation structure formation and flow mechanism with boundary conditions. As a result of calculations, the fields of velocity, temperature, and density were obtained, which describe the regularities of the flow and heat transfer of a thermoplastic slurry. The change in the Nusselt criterion along the length of the shaping cavity is shown, which coincides with the analytical solution of Nusselt under first kind boundary conditions. The optimal conditions for the process of ceramics molding by hot casting method have been found, allowing to obtain a hardened product with a homogeneous structure of beryllium ceramics at the outlet.

*Keywords:* thermoplastic slurry, beryllium oxide, ceramics, rheological model, Herschel Bulkley, thixotropic, viscous-plastic, non-isothermal.

### *Introduction*

High-density beryllium oxide ceramics are widely used in various fields of new technology due to a number of valuable properties and above all, unique thermal conductivity. The manufacture of high-tech beryllium ceramics by the slurry (extrusion) molding, similar to MIM (Metal Injection Molding) technology, includes the same physical processes and methods that reduce the cost of rejected products and significantly improve the quality of products.

During industrial tests, thermoplastic slurry has been used to obtain ceramic products — a highly viscous suspension with different organic binder contents: 9.5; 10.7 and 11.7% prepared from beryllium oxide powder (grade H1 specific surface area 1,57 m<sup>2</sup>/g). The organic binder includes three components: paraffin, beeswax, oleic acid in a ratio (82; 15 and 3%) [1].

The molding unit tank is filled with thermoplastic molten slurry and the mold is filled under pressure. In the molding process, there is a difficulty in the deformation behavior of the molding because of the high thermal conductivity of the dispersion medium. In the temperature range of 40–59 °C, the volume-phase characteristics of the slurry mass changes and the volume of the liquid phase increases. An increase in the volume of the liquid phase and the presence of an additional amount of binder lead to structural defects. Effective control of the molding process to reduce the content of the liquid phase while maintaining the high fluidity of the slurry was carried out under the influence of a pressure gradient resulting from ultrasonic (US) effects.

The present paper examines the viscous-plastic course of the beryllium ceramic molding process based on experimental data obtained from actual injection molding plants [2]. The reasonable choice and calculation of the optimal mode of the beryllium ceramic molding process are, from an energy point of view, a paramount technological task for the production of ceramic products.

Numerical calculations of the mathematical model of the process take into account the thixotropic-dilatant and complex rheological behavior of the slurry on rheological models, the dependence of thermo-physical characteristics on temperature, non-isothermal flow, and heat exchange when the aggregate state

changes. A detailed discussion of the conditions of heat exchange and phase conversion is not our task however, it was necessary to note the solidification temperature of the slurry.

*Calculation of non-isothermal flow based on the rheological model of a power-law fluid*

Non-isothermal movement of thermoplastic slurry in molding cavity of molding unit is considered (Fig. 1). The structure of the molding cavity is made in the form of coaxial pipes. Inner pipe with radius of  $r_1 = 0,045\text{ m}$  forms cavity and circular layer with radii of  $r_2 = 0,005$  and  $r_3 = 0,015$  — casing for circulation of cooling liquid. Liquid slurry with initial temperature of  $T_0 = 75^\circ\text{C}$  flows in and moves into forming cavity. According to experimental data, the maximum value of the molding speed does not exceed 2 mm/min.

As it moves, the slurry cools and solidifies acquiring a structural shape at the outlet of the pipe [3].

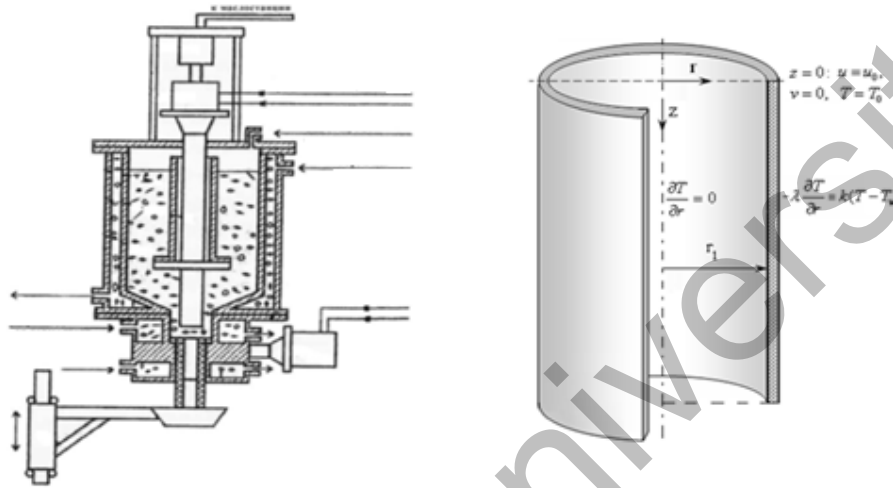


Figure 1. Diagram of industrial ultrasonic (UZ) molding plant

The wall of the molding cavity is cooled by water circulating in the annular casing. The cooling zone is divided into three parts, in the first compartment the water temperature  $T_{w1} = 73^\circ\text{C}$ , in the second  $T_{w2} = 59^\circ\text{C}$ , in the third  $T_{w3} = 45^\circ\text{C}$ . The total length of the cavity  $L = 0,108\text{ m}$ : the length of the hot, warm, and cold part of the pipe is equal to  $L_1 = 0,022\text{ m}$ ,  $L_2 = 0,045\text{ m}$ ,  $L_3 = 0,041\text{ m}$ , respectively.

For numerical calculations of non-isothermal flow and heat exchange of thermoplastic slurry, a mathematical model is presented by the following system of equations of motion, continuity and heat exchange [4].

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial r} = -\frac{dP}{dz} + \frac{\partial \tau_0}{\partial r} + \frac{\partial}{\partial r} \left( B \frac{\partial u}{\partial r} \right) + \rho g; \quad (1)$$

$$\frac{\partial(\rho u)}{\partial z} + \frac{1}{r} \frac{\partial(\rho v r)}{\partial r} = 0; \quad (2)$$

$$\rho u c_p \frac{\partial T}{\partial z} + \rho v c_p \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda r \frac{\partial T}{\partial r} \right) + L_k \frac{d\rho}{dt} + B \left( \frac{\partial u}{\partial r} \right)^2. \quad (3)$$

The system of equations (1) – (3) is closed by the Herschel-Bulkley rheological equation, in which the quasi-Newtonian viscosity is determined by the relation [5; 6]:

$$B = k(T) \left| \frac{\partial u}{\partial r} \right|^{n-1} \quad (4)$$

In (1) – (3),  $z, r$  – cylindrical coordinates;  $u, v$  – speed components;  $p, \rho, T, \tau_0, B, \lambda, c_p, L_k$  – pressure, density, temperature, yield strength, quasi-Newtonian viscosity coefficient, thermal conductivity, the slurry heat capacity and crystallization heat, respectively. Density, coefficients of heat capacity, viscosity and thermal conductivity of the slurry depend on temperature and their dependencies are determined by empirical formulas obtained on the basis of experimental data [7].

The effect of mechanical energy dissipation is not significant, however, taken into account in the calculations. The condition of maintaining the mass flow rate of the current medium in the mass-forming cavity makes it possible to find the pressure drop necessary to control the molding speed:

$$2\pi \int_0^{r_1} \rho u r dr = \pi r_1^2 \rho_0 u_0 \quad (5)$$

The speed and temperature distribution at the inlet is assumed to be constant along the channel section, accordingly, all thermo-physical characteristics of the slurry are constant. Boundary conditions are recorded at channel input:

$$\text{at } z = 0: u = u_0, v = 0, T = T_0; \quad (6)$$

on the walls of the channel in the area of the liquid state of the slurry for speed, adhesion conditions are set:

$$\text{at } z > 0, r = 0: \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0, v = 0; \quad (7)$$

and in the field of solid plastic state — conditions of non-flowing and sliding:

$$\text{at } z > 0, r = r_i, v = 0, -\frac{dp}{dz} = \frac{2}{r_i} \left( \tau_{0i} + \left( k \frac{\partial u}{\partial r} \right)^n \right), i = 1, 2. \quad (8)$$

Heat exchange on the outer wall is determined in accordance with the temperature value in the cooling contour of the cavity. We denote the temperature of water in hot, warm, cold contours  $T_1, T_2, T_3$ , respectively, and then we have boundary conditions for the temperature on the outer wall:

$$\begin{aligned} \text{at } 0 \leq z < l_1, r = r_1, -\lambda \frac{\partial T}{\partial r} &= k(T - T_1); \\ \text{at } l_1 \leq z < l_2, r = r_1, -\lambda \frac{\partial T}{\partial r} &= k(T - T_2); \\ \text{at } l_2 \leq z < l_3, r = r_1, -\lambda \frac{\partial T}{\partial r} &= k(T - T_3). \end{aligned} \quad (9)$$

The heat transfer coefficient  $k$  is in the standard manner [8].

The system of equations (1) – (3) is given to dimensionless variables. As a result of transition to dimensionless variables of the Reynolds equation, Bingham, Nusselt, Prandtl and Bio:

$$\begin{aligned} \tilde{z} = \frac{z}{r_1}, \tilde{r} = \frac{r}{r_1}, \tilde{u} = \frac{u}{u_0}, \tilde{v} = \frac{v}{u_0}, \tilde{\rho} = \frac{\rho}{\rho_0}, \tilde{\tau} = \frac{\tau}{\tau_0}, \tilde{T} = \frac{T}{T_0}, \tilde{P} = \frac{P}{\rho_0 u_0^2}, \\ \tilde{L}_k = \frac{Lk}{c_{p0} T_0}, Re = \frac{\rho U^{2-n} r_1^n}{\mu}, Bin = \frac{\tau_0 r_1^{n-1}}{\mu U^n}, Nu = \frac{\alpha_1 r_1}{\lambda}, Pr = \frac{c_p \mu}{\lambda}, Bi = \frac{K r_1}{\lambda}. \end{aligned}$$

grid points in each coordinate direction  $Oz$  and  $Or$  designates  $\Delta z, \Delta r$  and are calculated by the formulas:

$$\Delta z = z_{n+1} - z_n, \quad \Delta r = r_{j+1} - r_{j-1}, \quad \Delta r_+ = r_{j+1} - r_j, \quad \Delta r_- = r_j - r_{j-1}, \quad r_{j+1/2} - r_{j-1/2} = \Delta \tilde{r}$$

where  $\Delta z, \Delta r$  vary in the range  $0 \leq \Delta \leq 1$ .

The difference analogues of the equations of motion (1) and energy (3) are obtained according to the Crank-Nicolson scheme. Then the equations of motion can be represented as:

$$\begin{aligned} & \frac{[\theta(\rho_j^{n+1} u_j^{n+1}) + (1 - \theta)\rho_j^n u_j^n](u_j^{n+1} - u_j^n)}{\Delta z} + \\ & \frac{\theta(\rho_j^{n+1} v_j^{n+1})(u_{j+1}^{n+1} - u_{j-1}^{n+1}) + (1 - \theta)(\rho_j^n v_j^n)(u_{j+1}^n - u_{j-1}^n)}{r_{j+1} - r_{j-1}} + \\ & = - \left( \frac{dp}{dz} \right)^{n+1} + \frac{\theta}{r_{j+1/2} - r_{j-1/2}} \left[ B_{j+1/2}^{n+1} \frac{u_{j+1}^{n+1} - u_j^{n+1}}{r_{j+1} - r_j} - B_{j-1/2}^{n+1} \frac{u_j^{n+1} - u_{j-1}^{n+1}}{r_j - r_{j-1}} \right] \\ & + \frac{1 - \theta}{r_{j+1/2} - r_{j-1/2}} \left[ B_{j+1/2}^n \frac{u_{j+1}^n - u_j^n}{r_{j+1} - r_j} - B_{j-1/2}^n \frac{u_j^n - u_{j-1}^n}{r_j - r_{j-1}} \right] + \frac{\tau_{0,j+1}^{n+1} - \tau_{0,j-1}^{n+1}}{r_{j+1} - r_{j-1}}. \end{aligned}$$

The resulting difference equation is nonlinear with respect to the desired quantities, therefore, to find them, it is necessary to iterate along the nonlinearity or apply the Newton linearization method [9].

$$\frac{\theta \rho_j^{n+1} u_j^{n+1} u_j^n + (1 - \theta) \rho_j^n u_j^n u_j^{n+1} - \theta \rho_j^{n+1} u_j^{n+1} u_i^n}{\Delta z} + \frac{\theta \rho_j^{n+1} v_j^{n+1} (u_{j+1}^{n+1} - u_{j-1}^{n+1})}{r_{j+1} - r_{j-1}} -$$

$$\begin{aligned}
 & -\frac{\theta}{r_{j+1/2} - r_{j-1/2}} \left[ \frac{B_{j+1/2}^{n+1}}{r_{j+1} - r_j} (u_{j+1}^{n+1} - u_j^{n+1}) - \frac{B_{j-1/2}^{n+1}}{r_j - r_{j-1}} (u_j^{n+1} - u_{j-1}^{n+1}) \right] - \\
 & -\theta \frac{\tau_{0,i+1}^{n+1} - \tau_{0,i-1}^{n+1}}{r_{j+1} - r_{j-1}} = -\left(\frac{dp}{dz}\right)^{n+1} + \frac{(1-\theta)\rho_i^n u_i^n u_i^n}{\Delta z} - \frac{(1-\theta)\rho_i^n v_i^n (u_{i+1}^n - u_{i-1}^n)}{r_{j+1} - r_{j-1}} + \\
 & + \frac{1-\theta}{r_{j+1/2} - r_{j-1/2}} \left[ \frac{B_{j+1/2}^n}{r_{j+1} - r_j} (u_{j+1}^n - u_j^n) - \frac{B_{j-1/2}^n}{r_j - r_{j-1}} (u_j^n - u_{j-1}^n) \right] \equiv -\left(\frac{dp}{dz}\right)^{n+1} + \\
 & + RHS_j^n + (1-\theta) \frac{\tau_{0,j+1}^n - \tau_{0,j-1}^n}{r_{j+1} - r_{j-1}}.
 \end{aligned}$$

Accuracy of calculating the flow strongly depends on the way of calculation  $B_j^{n+1}$ , then it can be written as:

$$\begin{aligned}
 B_{j+1/2}^{n+1} &= \frac{1}{2} (B_{j+1}^{n+1} - B_j^{n+1}), \\
 B_{j-1/2}^{n+1} &= \frac{1}{2} (B_{j-1}^{n+1} - B_j^{n+1}). \\
 \frac{\theta}{r_{j+1/2} - r_{j-1/2}} \left[ \frac{B_{j+1/2}^{n+1}}{r_{j+1} - r_j} (u_{j+1}^{n+1} - u_j^{n+1}) - \frac{B_{j-1/2}^{n+1}}{r_j - r_{j-1}} (u_j^{n+1} - u_{j-1}^{n+1}) \right] &= \frac{\theta}{2\Delta r_j} * \\
 * \left[ \frac{(B_{j+1}^{n+1} + B_j^{n+1})(u_{j+1}^{n+1} - u_j^{n+1})}{\Delta r_j^+} - \frac{(B_j^{n+1} + B_{j-1}^{n+1})(u_j^{n+1} - u_{j-1}^{n+1})}{\Delta r_j^-} \right].
 \end{aligned}$$

After linearizing nonlinear elements in the equation of motion to find  $u_j^{n+1}$ , the difference analogue of the equation is reduced to the three-point form:

$$-a_j^n u_{j+1}^{n+1} + b_j^n u_j^{n+1} - c_j^n u_{j-1}^{n+1} = \delta_j^n - \left(\frac{dp}{dz}\right)^{n+1}.$$

When calculating internal flows, the pressure gradient, which is an unknown quantity, in the process of solving it, will be determined by splitting method from condition of preservation of mass flow rate:

$$\int_0^1 \rho_j^{n+1} u_j^{n+1} r dr = \frac{1}{2} \tag{10}$$

For determining  $\left(\frac{dp}{dz}\right)^{n+1}$  the splitting method is used [9]:

$$u_j^{n+1} = \varphi_j^{n+1} + \left(-\frac{dp}{dz}\right)^{n+1} \cdot S_j^{n+1} \tag{11}$$

Thus, the solution algorithm  $u_j^{n+1}$  has the form: 1) from difference analogs  $\varphi_j^{n+1}, S_j^{n+1}$  the desired variables are calculated by the sweep method; 2) by found values  $\varphi_j^{n+1}, S_j^{n+1}$  definite integrals are found by the Simpson method and the pressure gradient  $\left(\frac{dp}{dz}\right)^{n+1}$ ; 3) calculated  $u_j^{n+1}$ .

Similarly, the heat transfer equation is represented in finite-difference form by the Crank-Nicolson scheme.

### Results and Discussion

Calculation of non-isothermal flow and heat transfer of thermoplastic slurry which based on the Bulckley-Herschel model gives the following results. The calculations provided for two identical operating parameters similar to the Bingham model [10]. Figure 2 (b) shows that the velocity profiles have a more filled form, and decreasing of the velocity on the axis is associated with a high consistency of the slurry mass. The developed profile of the shear rate increases the cooling effect from the side of the wall and this, in turn, leads to intensive heat transfer between the slurry and the cooling liquid, hence, to an increase in viscosity. As can be seen from Figure 2 (a), in hot and warm contours with the slurry which located in the central region of the pipe occurs heat transfer due to convection. When transition to warm contour  $z = 23 \text{ mm}$ , large changes of temperature along the channel radius are demonstrated which are related to the temperature gradient. Further, the temperature distribution in the core of the flow over the cross sections is reduced and remains almost uniform in the cold contour (Fig. 2, a). Beginning with  $z = 63 \text{ mm}$ , the speed distribution

becomes more uniform because of the balance between the cooling effect and the internal phenomenon of convection. The filled profile of the shear rate of the considering model does not always ensure the uniformity of the structure of the slurry mass, however, with the correct choice of thermal parameters, positive results can be obtained.

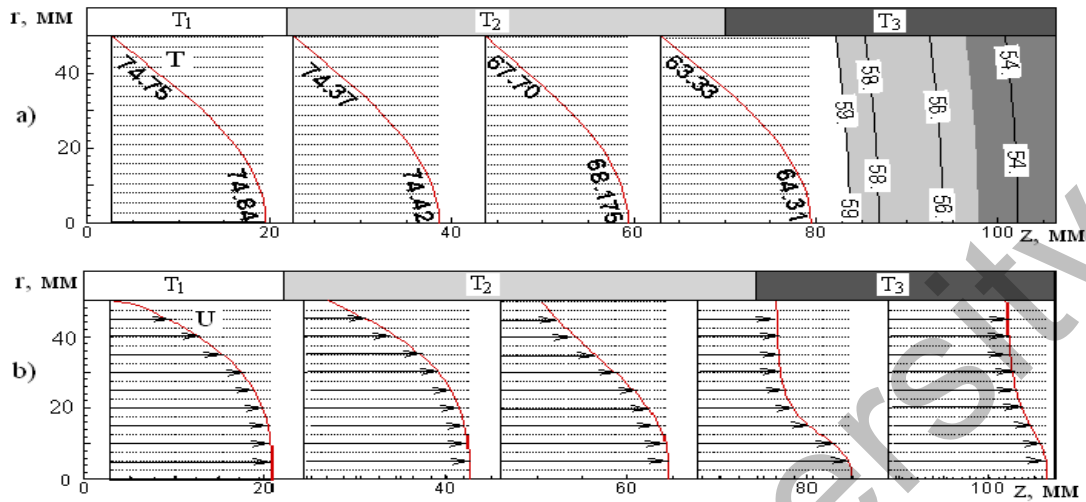


Figure 2. Temperature (a) and speed distribution (b) along the pipe length:

$$u = 1 \text{ mm/min}, \quad r_1 = 45 \text{ mm}, \quad Re = 3,52 \cdot 10^{-4}$$

As the thickness of the round pipe decreases, the temperature of crystallization is reached near the wall at a distance of  $z = 59 \text{ mm}$ , and the crystallization front at a sharp rate covers the entire crosscut layer of the round channel (Fig. 3). The developed speed profile facilitates rapid heat removal from the inside of the slurry to the wall. At transition to cold contour the slurry is in solid-plastic state and the temperature of the slurry at all points shall be equal to temperature of cooling liquid of this contour [11]. The solidification begins with a cold contour, followed by a sharp increase in the density of the thermoplastic slurry. The calculated data of the speed profile, obtained using the Bulkley-Herschel model, have a more filled form with a constant core in the central part of the flow (Fig. 3), which is in qualitative agreement with the data of the analytical solution of the motion of a non-Newtonian fluid in a round channel.

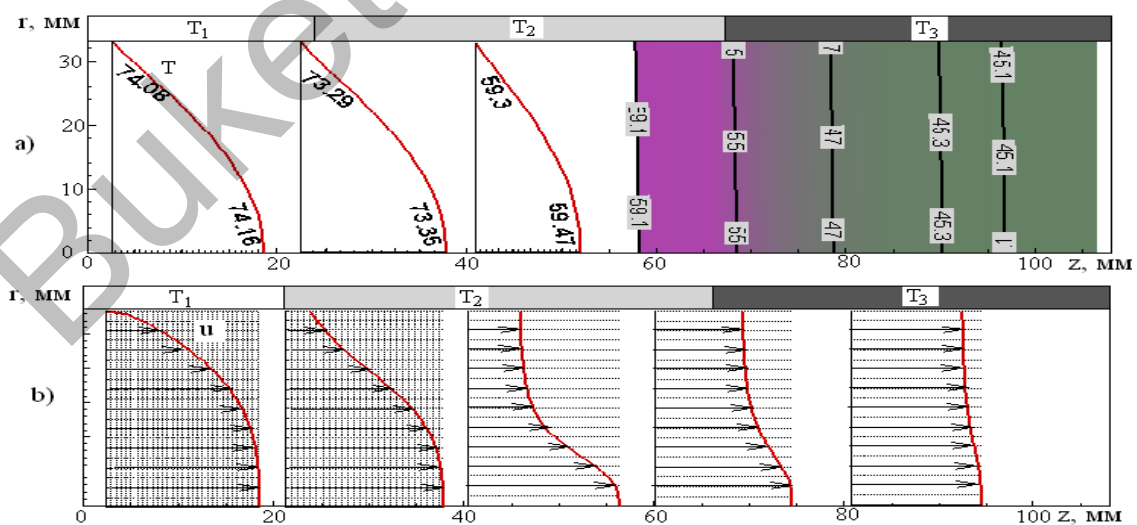


Figure 3. Temperature (a) and speed distribution (b) along the pipe length:

$$u = 1 \text{ mm/min}, \quad r_1 = 33 \text{ mm}, \quad Re = 1,95 \cdot 10^{-4}$$

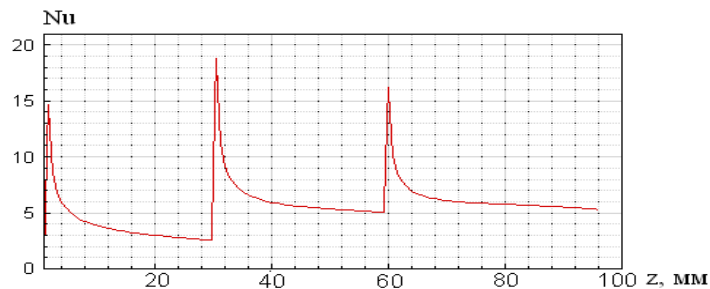


Figure 4. Nusselt criterion change along the pipe length

The regularities of the change of temperature and longitudinal speed profiles cause the change in the Nusselt number (Fig. 4). The presented dependence of the  $Nu$  number on  $Z$  testifies that in the warm and cold contours, the change in slurry viscosity with temperature has a stronger effect on heat transfer than in the hot contour, where the temperature gradient is insignificant. When falling temperature is different, Nusselt number decreases much faster (warm contour) than in a cold contour, where the rising temperature head slows down the fall  $Nu$ . The problem of heat transfer for a laminar flow of a non-Newtonian fluid, in a round pipe at a constant wall temperature, was solved to verify the numerical method. The change in the Nusselt criterion (dimensionless heat transfer coefficient) along the pipe length decreases monotonically in each contour and tends to a constant value that coincides with the analytical Nusselt solution under *first-kind boundary condition* [12].

As can be seen from Figure 5, the calculation data, obtained using the Herschel-Bulkley model, indicate good agreement with the results of experiments and industrial tests. In the calculated temperature range, the change in density in the liquid state is  $(2.355\text{--}2.38)\text{ g/cm}^3$ , in solid plastic —  $(2.38\text{--}2.43)\text{ g/cm}^3$ .

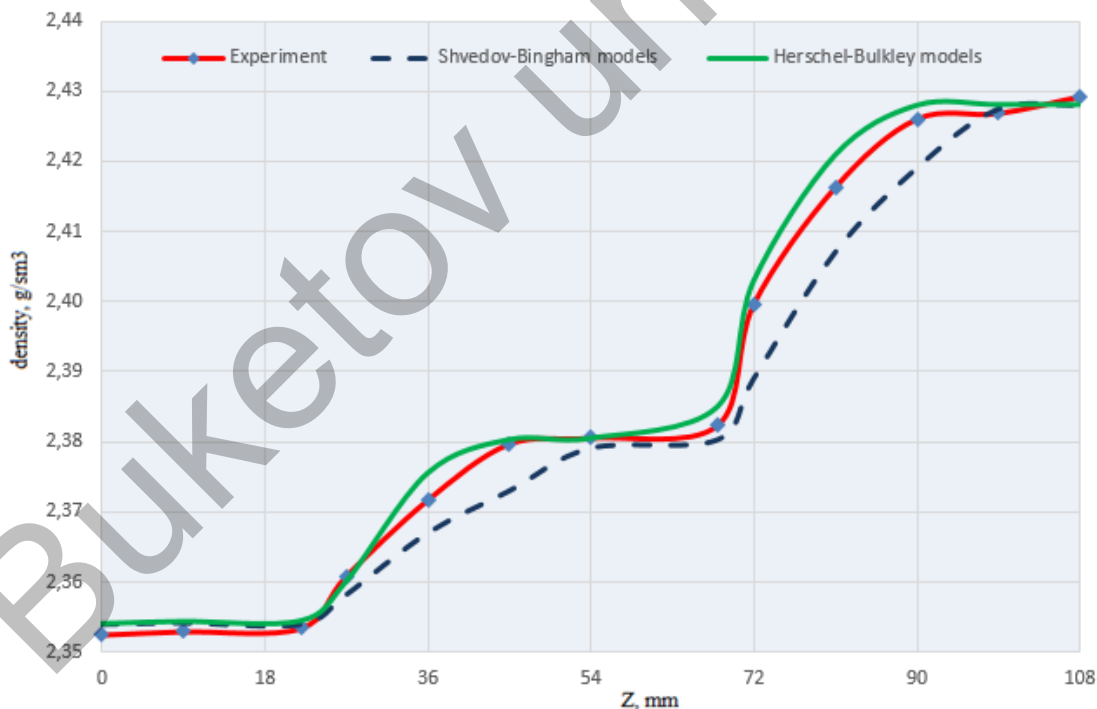


Figure 5. Density change of the slurry along the pipe length

The obtained results of calculations of the process of thermoplastic slurry solidification under the specific conditions of the round pipe of the molding installation make it possible to clearly represent the kinetics of solidification depending on the molding modes, the structure of the molding mass and the peculiarities of the configuration of the articles.

### Conclusions

Numerical calculations of non-isothermal flow and heat exchange of thermoplastic slurry based on Shvedov-Bingham and Herschel-Bulkley two rheological models are compared with experimental data. The application of the Herschel-Bulkley rheological model describing the non-thermal flow and the gradual transition from the flowing state after the destruction of the slurry mass structure to the solid state is justified by the thixotropic-dilatant properties of the slurry. The rheological behavior of the viscoplastic slurry is complex when different flow modes exist at different shear rate intervals depending on the duration of treatment of the slurry with exposure. Rheological parameters of viscoplastic slurry are determined experimentally in the studied area, where with existing measurement methods reliable results.

The given results of mathematical model calculations help to find optimal parameters of thermal regime of the slurry molding in forming cavity to reduce production costs of molding system. The calculated data show the applicability of the Herschel-Bulkley rheological model in a wider range of molding systems compared to the Shvedov-Bingham model. The optimum conditions of the process of molding of BeO thermoplastic slurry were found, which make it possible to obtain a solidified product with a unified structure of beryllium ceramics at the outlet.

The results of the study, obtained using the Herschel-Bulkley rheological model, lead to the following conclusion:

- speed profiles have a more filled appearance with a constant core in the central portion of the flow;
- unified speed distribution contributes to unified fields of temperature, density and improvement of other thermo-physical properties of the slurry;
- speed profiles with a constant core in the central part of the stream satisfy the theoretical data.

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З.К. Саттинова, Т.Н. Бекенов, Б.К. Ассилбеков, Г.И. Рамазанова, К.М. Дюсенов  
**Әртүрлі реологиялық модельдерді қолдана отырып, ВеО термопласт шликерін құюдың оптималды шарттарын есептеу алгоритмін жасау**

Мақалада бериллий тотығы термопласт шликерінің құю қондырғысының дөңгелек каналындағы ағысы мен жылуалмасуының математикалық моделі есебінің қорытындысы келтірілген. Ағысты сипаттайтын Гершель–Балкли реологиялық модель негізіндегі теңдеулер жүйесінің сандық шешу алгоритмі құрастырылған. Қозғалыс, үзіліссіздік және энергия теңдеулер жүйесінің соңғы-айырымды аналогы Кранк–Никольсон схемасы арқылы алынған. Үшпараметрлі теңдеуді қолданудағы мақсат изотермиялық емес, ағысты қаншалықты эксперименттік деректерге сәйкес адекватты сипаттай алатындығын тексеру және Шведов–Бингам моделімен алынған есептеу нәтижелерімен салыстыру. Гершель–Балкли моделін қолданудың ерекшелігі жылжу жылдамдығының кең интервалында қисық сызықты шликер ағысын ескерудің күрделілігі болып саналады. Шликердің тиксотроп-дилатантты қасиетіне сәйкес ультрадыбысты өңдеуден кейінгі шликердің құрылымы бұзылып, қисық сызықты аққыштығы жылжу жылдамдығының үлкен интервалын қамтиды. Есептеулер нәтижесі ұсынылған модель шликердің тиксотропты ағысының ең басты ерекшеліктерін көрсететіндігі және эксперимент деректерімен сәйкестігі шликер ағысы жылдамдығы мен тығыздығының өзгеруі арқылы салыстырылды. Шликердің тұтқыр-пластикалық ағысының жылдамдығын, шликердің коагуляциялық құрылым тузу ерекшелігі мен ағыстың шекаралық шарттарын ескеріп, Шведов Бингам және Гершель–Балкли реологиялық модельдері негізінде сандық есептеулермен жүргізілді. Есептеулер нәтижесінде ағыс және жылуалмасу заңдылықтарын сипаттайтын жылдамдық, температура өрістерінің таралуы және тығыздықтың өзгеруі алынды. Бірінші текті шекаралық шартта Нуссельт критерийінің өзгеруі аналитикалық шешімімен сәйкес келетін жылуалмасу критерийінің канал бойымен өзгеруі көрсетілген. Құю процесі соңында біртекті құрылымды керамика өнімін алу үшін шликерді формалау процесінің тиімді шарттары анықталған.

*Кілт сөздер:* термопласт шликері, бериллий тотығы, керамика, реологиялық модель, Шведов-Бингам, Гершель–Балкли, тиксотропия, тұтқыр-пластикалық, изотермиялық емес.

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**Разработка алгоритма расчета оптимальных режимов литья термопластичного шликера ВеО с использованием различных реологических моделей**

В статье представлены результаты расчета математической модели течения и теплообмена термопластичного оксида бериллия в круглом канале литейной установки. Разработан алгоритм численного расчета системы уравнений на основе реологической модели Гершель–Балкли. Конечно-разностный аналог системы уравнений движения, непрерывности и энергии решается численным методом с использованием разностной схемы Кранка–Никольсона. Приведенные экспериментальные данные позволяют оценить деформационное поведение формовочной массы, а также установить зависимость реологических и теплофизических свойств термопластичного шликера от температуры. Учитывая особенности коагуляционного структурообразования и механизма течения с граничными условиями, в статье проводились расчеты скоростей вязкопластичного течения шликера на основе двух реологических моделей Шведова–Бингама и Балкли–Гершеля. Трехпараметрическое уравнение применено с целью проверки согласованности адекватности экспериментальным данным неизотермического течения шликера по сравнению с моделью Шведова–Бингама. Данная модель для описания реологического поведения шликера связана со сложностью учета нелинейности кривой течения в широких пределах изменения скорости сдвига. Тиксотропно-дилатантное свойство шликера такому ограничению не удовлетворяет, и нелинейность кривой течения после разрушения структуры проявляется в широких пределах изменения скорости сдвига. В нашем случае применение модели Балкли–Гершеля позволило адекватно отразить реологическое поведение шликера, включая нелинейность кривой течения и вязкие эффекты среды. В результате расчетов были получены поля скорости, температуры и плотности, описывающие закономерности течения и теплообмена термопластичного шликера. Показано изменение критерия Нуссельта по длине формообразующей полости, совпадающей с аналитическим решением Нуссельта при граничных условиях первого рода. Найдены оптимальные условия процесса формования керамики способом горячего литья, которые позволяют получить на выходе отвердевшее изделие с однородной структурой.

*Ключевые слова:* термопластичный шликер, оксид бериллия, керамика, реологическая модель, Шведов-Бингам, Гершель-Балкли, тиксотропия, вязко-пластичный, неизотермический.

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