ON AN ABSOLUTE STABILITY OF CONTROL SYSTEMS WITH TACHOMETRIC FEEDBACK TAKING INTO ACCOUNT EXTERNAL LOAD

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Letov A.M.[1] proposed the following formula:

$$\dot{\xi} = \phi(\sigma) \cdot \psi(\nu), \tag{1}$$

where the function $\varphi(\sigma)$is continuous in σ and satisfies condition

$$\phi(0) = 0 \wedge \phi(\sigma)\sigma > 0 \quad \forall\sigma \neq 0. \tag{2}$$

He led to a convenient form for the study of the equation of the hydraulic actuator, taking into account the external load, obtained by V.A. Khokhlov[1]:

$$\dot{x} = \mu\sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F}\sqrt{p_0 - \Delta p sign\sigma} \cdot \sigma \tag{3}$$

Here $\mu\sqrt{\frac{g}{\gamma}} \cdot \frac{l}{F}$ is the constructive constant, μ is the flow coefficient, σ is the spool displacement, $p_0 = p_k - p_a$ is pressure in the supply line, p_k is pressure at the drain, Δp is pressure difference in the cameras of the actuator, determined by the load.

The function $\varphi(\sigma)$ replaces the expression $\sqrt{\frac{gp_0}{\gamma}} \cdot \frac{l}{F}\mu\sigma$, which determines the speed of an unloaded executor, and the multiplier $\psi(\nu) = \sqrt{1 - \frac{\Delta p}{p_0} sign\ \sigma}$ takes into account the influence of the load.

The multiplier $\psi(\nu)$, when ν depends on the deflection of the control element ξ, its speed $\dot{\xi}$and its acceleration $\ddot{\xi}$is determined as follows:

$$\psi(\nu) = \begin{cases} 1 & at \quad \nu \geq 1, \\ \sqrt{\nu} & at \quad 0 < \nu < 1, \\ 0 & at \quad \nu \leq 0, \end{cases} \tag{4}$$

where in the general case ν has the following form [1]:

$$\nu = 1 - (a\ddot{\xi} + b\dot{\xi} + c\xi) sign\sigma. \tag{5}$$

Here a, b, c are real numbers, and $sign\sigma$is the Kronecker function

We consider the problem of constructing on a given smooth program manifold Ω(t) the following indirect automatic control system with tachometric feedback taking into account external load [2]:

$$\dot{x} = f(t,\ x) - b_1\xi, \quad t \in I\Theta = [0,\ \infty),$$
$$\dot{\xi} = \phi(\sigma) \cdot \psi(\nu), \quad \sigma = p^T\omega - q\xi - N\dot{\xi}, \tag{6}$$

where the coefficients $b_1 \in R^n,\ p \in R^s$ are constant, $q,\ N$are constant coefficients of rigid and tachometric feedback,$\sigma$is a total impulse-signal and the differentiable function ξsatisfies the following conditions

$$\phi(0) = 0 \wedge \phi(\sigma)\sigma > 0 \quad \forall\sigma \neq 0,$$
$$\frac{d\phi}{d\sigma}|\sigma = 0 = \chi > 0, \tag{7}$$

a multiplier $\psi(\nu)$deforms the function $\phi(\sigma)$when the coordinates $\xi,\ \sigma$ change. Here,ν is a complex discontinuous function of the automatic control system. The program manifold Ω(t) is determined by the following equations

$$\Omega(t) \equiv \omega(t,x) = 0, \tag{8}$$

In the simplest case, it looks like this:

$$\nu = 1 - c\xi sign\sigma. \quad (9)$$

Definition 1. The program manifold of an indirect control system with rigid and tachometric feedback, taking into account the external load, is called absolutely stable if it is globally stable on solutions of system (6) for any $\omega(t_0, x_0)$and $\phi(\sigma)$, $\psi(\nu)$satisfying conditions (7), (9).

Statement of the problem. Find a condition for the absolute stability of the program manifold of the indirect control system with rigid and tachometric feedback, taking into account the external load

Duetothefact that the manifold (8) is an integral manifold also for the system (6) - (9) and taking the Erugin function to be linear with respect to the vector function ω:

$$F(t,\ x,\ \omega) = -A\omega, \quad (10)$$

we arrive at the following system with respect to ω:

$$\dot{\omega} = -A\omega - b\xi, \quad t \in I\Theta = [0,\ \infty),$$
$$\dot{\xi} = \phi(\sigma)\cdot\psi(\nu), \quad \sigma = p^T\omega - q\xi - N\dot{\xi}, \quad (11)$$

where $b = Hb_1$, $H = \frac{\partial\omega}{\partial x}$ and $-A(s \times s)$ is a constant Hurwitz matrix, the nonlinearity $\phi(\sigma)$satisfies conditions (7), and the multiple $\psi(\nu)$ is determined by formula (9).

System (11) is reduced to the canonical form [1]

$$\dot{\eta} = -\rho\eta + \sigma,$$
$$\dot{\sigma} = \beta^T\eta - M\xi - N\varphi(\sigma,\eta)sign\sigma, \quad (12)$$

where$\rho = diag(\rho_1, \dots, \rho_s)$, β, M, Nh are constants.

For system (12), we construct the Lyapunov function as follows

$$V = \sum_{i=1}^{s}\sum_{k=1}^{s}\frac{l_i \cdot l_k}{\rho_i + \rho_k}\eta_i\eta_k + \frac{1}{2}\sum_{k=1}^{s} L_k\eta_k^2 + \sum_{i=1}^{s-m} C_i\eta_{m+i}\eta_{m+i+1} + \frac{1}{2}l_{s+2}\,\sigma^2.$$

Here $l_{s+2} > 0$, $l_1, \dots, l_m$are real, $l_{m+1}, \dots, l_{s+1}$are complex pairwise conjugate numbers and L_k, $C_i$are positive real numbers.

The following theorem is valid.

Theorem. If the Erugin function is linear with respect to $\omega$and there areL_k, C_i positive real numbers , $l_{s+2} > 0$, in addition, the nonlinearity $\phi(\sigma)$ satisfies condition (6), and the function $\psi(\nu)$ satisfies conditions (7) and (4) - (5), then in order for the program manifold of the automatic system of indirect control with rigid and tachometric feedback, taking into account the external load, was absolutely stable with respect to the vector function ω , it suffices to satisfy equalities

$$L_k + l_{s+2}\beta_k + 2l_k\sum_{i=1}^{s+1}\frac{l_i}{\rho_i + \rho_k} = 0 \quad (k = 1, \dots, m),$$

$$C_j + l_{s+2}\beta_{m+j} + 2l_{m+j}\sum_{i=1}^{s}\frac{l_i}{\rho_i + \rho_k} = 0 \quad (j = 1, \dots, s-m+1),$$

where $l_1, \dots, l_m$are real and $l_{m+1}, \dots, l_{s+1}$are complex pairwise conjugate numbers.

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