CALCULATING EXPANSION COEFFICIENTS OF THE DEFLECTION FUNCTION FOR PLATE BENDING

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Plates are rightly considered the most universal and widespread elements in virtually all sectors of science, technology and economy. During operation, the plate as a part of a mechanism or an independent structure is subjected to various influences (friction, deformations, the effects of various loads, temperature changes, vibrations, wear, etc.) which cause, first of all, the plate bending. Therefore, knowledge of the theory for the plate bending and of classical methods for calculating them is necessary for a modern engineer.

Analytical and numerical calculations are necessary in the production of material goods in all sectors of the national economy: from the production of household goods, cars, airplanes, ships and unique equipment to a wide variety of structures and space rockets, the details of which are plates. Analytical and numerical calculations are relevant at any time, since progress does not stand still, enterprises design new devices and equipment, new structures are being built, the creation of which is impossible without clear technical studies that specify the accuracy of calculations necessary for this design.

A variety of analytical and numerical calculation methods are used to study plate bending problems. One of these methods is the Levy method.

We consider the case of the platebending ($0 \le x \le a, 0 \le y \le b$), in which only two opposite edges have a hinge support (for example, $x = 0$ and $x = a$) and the other two edges have arbitrary boundary conditions. The mathematical model of the plate is completely determined by the function of deflection (vertical displacements) $W(x, y)$. When calculating by the Levy method, the desired deflection function $W(x, y)$ has the form [1]

$$W(x, y) = \sum_{n=1}^{\infty} [A_n \cdot ch\omega_n y + B_n \cdot sh\omega_n y + C_n \cdot y \cdot ch\omega_n y + D_n \cdot y \cdot sh\omega_n y + \varphi_n(y)] \sin \omega_n x, \qquad (1)$$

where A_n, B_n, C_n, D_n are arbitrary constants of integration, $\omega_n = \frac{n\pi}{a}$, φ_n is a particular integral that depends on the type of coefficients f_n and, consequently, on a given external load f [1].

To determine the four integration constants A_n, B_n, C_n, D_n, the boundary conditions defined at the edges of the plate $y = 0$ and $y = b$ are used. These boundary conditions, of course, can be different. In the general case, this leads to the solving a system of algebraic equations with respect to unknown constants A_n, B_n, C_n, D_n. However, it should be noted that the order of this system can increase, for example, if the load is given in the direction of the *y*-axis by a discontinuous law.

Obviously, various approximate methods can be used to find the constants A_n, B_n, C_n, D_n. It depends on what degree of accuracy is needed when solving a specific practical problem. In addition, it should be taken into account that the deflection function is defined as an infinite series, finding the sum of which is not always an easy task.Therefore, it is often necessary to limit ourselves to a finite number of the first terms of the series (1) for the deflection function, which naturally reduces the accuracy of the desired solution.

The calculation of the coefficients A_n, B_n, C_n, D_n in a general form in the case when one of the sides of the plate (for example, a side $y = 0$), parallel to the x axis, is supported by an elastic contour, and the other side is rigidly pinched, is given in [2].The elastic contour may be, for example, a beam, bending under the action of pressures applied to it. For this case the boundary conditions have the form

$$\left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} \right)\Bigg|_{y=0} = 0 , D\left[\frac{\partial^3 W}{\partial y^3} + (2-\nu) \frac{\partial^3 W}{\partial x^2 \partial y} \right]\Bigg|_{y=0} = \left(EJ \frac{\partial^4 W}{\partial x^4} \right)\Bigg|_{y=0} ; \; W\big|_{y=b} = 0 ,$$

$$\frac{\partial W}{\partial y}\bigg|_{y=b} = 0; .$$

where D is the cylindrical rigidity of the plate, EJ is the rigidity of the beam.

In the case when one of the sides of the plate parallel to the x axis is rigidly pinched, and the other side is free, under a uniformly distributed load $f = q$ of constant intensity, integration constants A_n, B_n, C_n, D_n are presented in [3].

This special case in a more general form, namely, for any type of external load f with full calculation of the integration constants A_n, B_n, C_n, D_n is studied in [4], where the side $y = 0$ is free, and the side is $y = b$ rigidly fixed.

In a fairly widespread, but computationally simple case, when the edges of the plate parallel to the axis are rigidly pinched, the analytical expressions for the coefficients A_n, B_n, C_n, D_n have the form

$$A_n = -\varphi_n(0) , \quad B_n = \frac{\xi_1 (sh\omega_n b + b\omega_n ch\omega_n b) - \xi_2 b sh\omega_n b}{sh^2\omega_n b - b^2\omega_n^2} ,$$

$$C_n = \frac{\omega_n [\xi_1 (sh\omega_n b + b\omega_n ch\omega_n b) - \xi_2 b sh\omega_n b]}{b^2\omega_n^2 - sh^2\omega_n b} - \varphi_n'(0) , \quad D_n = \frac{\xi_1 b\omega_n^2 sh\omega_n b - \xi_2 (sh\omega_n b - b\omega_n ch\omega_n b)}{sh^2\omega_n b - b^2\omega_n^2}$$

$$;$$

$$\xi_1 = ch\omega_n b[\varphi_n(0) + b\varphi_n'(0)] - \varphi_n(b) , \quad \xi_2 = \varphi_n'(0) ch\omega_n b + \omega_n sh\omega_n b[\varphi_n(0) + b\varphi_n'(0)] - \varphi_n'(b) .$$

Due to the bulkiness of formulas for the determination of the coefficients A_n, B_n, C_n, D_n in the general case, and, consequently, due to the inconvenience and complexity of further use of these formulas, it is recommended that all calculations of the constants A_n, B_n, C_n, D_n be carried out for particular numerical values of a problem in each special case with given numerical parameters. This was also done when solving specific problems with the above boundary conditions. Analytical calculations are difficult, but improving the accuracy of the result obtained is an obvious fact.

Due to the clarity of the calculation algorithm, without any difficulty, Levy's solution can also be applied to the study of the bending of a plate whose sides parallel to the x axis have other boundary conditions. Levy's solution also easily applies to those cases when the sides of the plate contour parallel to the x axis are not completely rigid, but are relatively flexible beams that bend under the action of the pressures applied to them.

Finding analytical expressions for constants A_n, B_n, C_n, D_n makes it possible to obtain an analytical expression (formula) for the deflection function $W(x, y)$. And then the deflection function can be set with the accuracy that is necessary for solving a specific problem, only being limited by the number of members in the series (1) that will provide the required accuracy of the calculations of the object under study.

Setting the calculation accuracy necessary for the manufactured product undoubtedly entails an increase in the product quality.

In addition, this factor plays an important role in the design and calculation of titanium plates of body armor, solar panels, plates in an alkaline apparatus of water ionization, wall film heaters, Earth satellites, etc., as well as in such areas of production as instrumentation, mechanical engineering, aviation, space industry, etc.

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ЖАРТЫЛАЙ ШЕКСІЗ СЕРПІМДІЛІ ЖАЗЫҚТЫҚТЫҢ НЕГІЗГІ ТЕҢДЕУІНІҢ ЖАЛПЫ АНАЛИТИКАЛЫҚ ШЕШІМІ

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Жартылай серпімділі жазықтықтың стандарттық теңдеуі келесі түрде жазылады:

$$\nabla^2\nabla^2 F = \frac{\partial^4 F}{\partial x_1^4} + 2\frac{\partial^4 F}{\partial x_1^2 \partial x_3^2} + \frac{\partial^4 F}{\partial x_3^4} = 0 \qquad (1)$$

Бұл бигармоникалық теңдеудің (1) шешімін табу үшін $F(x_1, x_3)$ жылжу функциясы мына түрде қабылданып алынады [1]:

$$F(x_1, x_3) = \delta(x_3)\cdot W(x_1), \qquad (2)$$

мұнда $\delta(x_3)$ – таралу функциясы; $W(x_1)$ – майысу функциясы.

Енді осы жұмыстағы шешуші теңдеуді (1) өту теңдеуін $\frac{d^2W(x_1)}{dx_1^2} = -\bar{k}^2 W(x_1)$; $\frac{d^4W(x_1)}{dx_1^4} = \bar{k}_\omega^4 W(x_1)$ және (2) қолданып келесі жалпы түрде жазамыз:

$$\delta^{IV}(z_0) - 2\cdot k^2 \delta''(z_0) + k_\omega^4 \delta(z_0) = 0$$

Осытеңдеудің жалпы шешімін былайша анықтаймыз:

$$\lambda^2 = k^2\left(1 \pm \sqrt{1-\alpha}\right);\; k_\omega^4 = \alpha\cdot\left(k^2\right)^2$$

$$\delta(z_0) = \left[C_1\cos(\beta_1 z_0) + C_2\sin(\beta_1 z_0)\right]e^{-\alpha_1 z_0} + \left[C_3\cos(\beta_1 z_0) + C_4\sin(\beta_1 z_0)\right]e^{\alpha_1 z_0} \qquad (3)$$

мұнда

$$\alpha_1 = k\sqrt{\frac{\alpha+1}{2}},\; \beta_1 = k\sqrt{\frac{\alpha-1}{2}},$$

$$\alpha_2 = k\sqrt{1-\sqrt{1-\alpha}},\; \beta_2 = k\sqrt{1+\sqrt{1-\alpha}}$$

Жалпы шешімді (3) жартылай шексіз серпімділі жазықтыққа қолданайық $(z_0 \to \infty,\; \delta(z_0) = 0)$: