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Probability distribution of distances between local maximum of random number series

Кездейсоқ сандар тізбегінің локалді максимумдары арасындағы қашықтықтар ықтималдылығының таралуы

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Кездейсоқ тәуелсіз сандардың тізбегі қарастырылған. Егер $x_{i-1} < x_i >$ xi+1 болса, онда x_i — локальді максимум. Мақалада массалық ықтималдылық функциясы (PMF) f (d) d қашықтық пен локальді максимум арасында параметрлік емес екені көрсетілді. Бұл кез келген кездейсоқ сандардың тараулының ықтималдылығы үшін дұрыс. Орташа қашықтық 3-ке тең. PMF әдісінің санау әдісі келтірілді және оның деңгейі қашықтар үшін 2 және 29 арасында. Нәтиже кездейсоқ сандар тізбегінің қашықтықтарының таралуымен расталады. Олар кездейсоқ сандар псевдогенераторлар көмегімен жасалды немесе кездейсоқ сандардың табиғи көздерінен алынды.

Рассмотрена последовательность случайных независимых чисел. Если $x_{i-1} < x_i >$ xi+1, то x_i — локальный максимум. В работе показано, что функция массовой вероятности (PMF) f (d) от расстояний d между локальными максимумами является непараметрической. Это справедливо для любого распределения вероятности случайных чисел в последовательности. Среднее расстояние точно 3. Представлены методика вычисления этого PMF и его уровень для расстояний между 2 и 29. Результат подтверждается для распределения расстояний пробной последовательности случайных чисел, которые были созданы псевдогенераторами случайных чисел или получены из «истинных» источников случайного числа.

1. Average distance between local maximum

Let's take any number in the sequence and find out the probability that it's a local maximum.

Definition 1. *A number x_i is a local maximum, if the following condition is true $x_{i-1} < x_i > x_{i+1}$.*

First, we'll use a combinatorial approach. Consider the following sequence (fig.) of pseudo generators of random by MS Excel RAND () function:

1. 0.935536495
2. 0.191531578
3. 0.429049655
4. 0.308968021
5. 0.179540986
6. 0.401329789
7. 0.71581906
8. 0.604617962
9. 0.877254876
10. 0.973280207
11. 0.489033299
12. 0.912367351
13. 0.604552972
14. 0.039395302
15. 0.3780448
16. 0.55317569
17. 0.6308772
18. 0.373163479
19. 0.812434426
20. 0.560173882

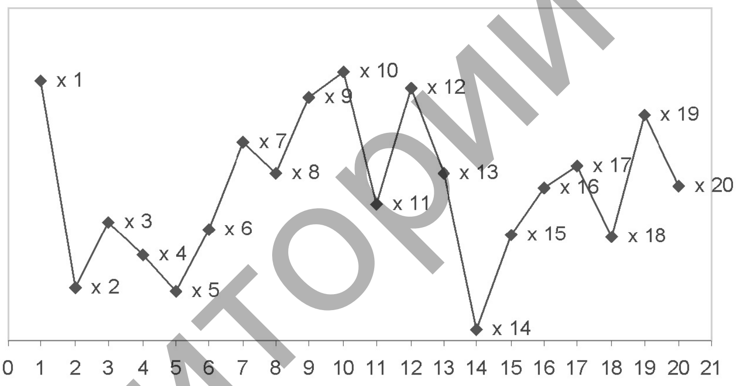

Fig. Sample random sequence

Let's take any three consequitive numbers x_{i-1}, x_i and x_{i+1}. For instance, for i=3 we have: x_2=0.191531578, x_3=0.429049655 and x_4=0.308968021. In this case, x_3 is a local maximum. If we denote the greatest value as 2, the least value as 0 and the value in the middle as 1, then we have a triplet (0,2,1). Any three consequitive numbers can be represented by such triplet. Out of six possible permutations

(0,1,2), (0,2,1), (1,0,2), (1,2,0), (2,0,1) and (2,1,0)

we are interested only in two combinations, which represent local maxima

(0,2,1) and (1,2,0).

Therefore, if we take any three consequitive numbers, then the probability that the number in the middle is a local maximum is

$$P_{\max} = \frac{2}{6} = \frac{1}{3}.$$

If we have $3 \cdot N$ numbers in the sequence, then N of them are local maxima. It also means that the average distance between maxima should be exactly 3.

Now, let's introduce some additional notation and use an operator approach. When $x_{i-1} < x_i$ we'll put an operator U, i.e. the sequence goes üp». Alternatively, when $x_{i-1} > x_i$ we'll put an operator D, i.e. the sequence goes «down». For our sample sequence, the corresponding operator sequence is:

1. 0.935536495
2. 0.191531578 D

3. 0.429049655 U
4. 0.308968021 D
5. 0.179540986 D
6. 0.401329789 U
7. 0.71581906 U
8. 0.604617962 D
9. 0.877254876 U
10. 0.973280207 U
11. 0.489033299 D
12. 0.912367351 U
13. 0.604552972 D
14. 0.039395302 D
15. 0.3780448 U
16. 0.55317569 U
17. 0.6308772 U
18. 0.373163479 D
19. 0.812434426 U
20. 0.560173882 D

We can apply our new notation to triplets and see that

$$(0,1,2) \text{ becomes } \langle UU \rangle$$

$$(0,2,1) \text{ and } (1,2,0) \text{ become } \langle UD \rangle$$

$$(1,0,2) \text{ and } (2,0,1) \text{ become } \langle DU \rangle$$

$$(2,1,0) \text{ becomes } \langle DD \rangle.$$

Having a new notation, we can use it to compute the probability of appearance of a local maximum in the middle of any triplet. We are interested in triplets represented by $\langle UD \rangle$, because it's the only expression, which represents a local maximum in the middle of a triplet. We shall use a standard cumulative distribution function (CDF) F (x), defined as a probability of $x \le x_i$:

$$F(x_i) = \Pr(x \le x_i) = \int_{-\infty}^{x_i} f(x) \cdot dx,$$

where f (x) is PDF (probability density function) of x. This also could be written as

$$F(x_i) = \int_0^{F(x_i)} dF(x).$$

On the other hand, probability of $x > x_i$ is

$$\int_{F(x_i)}^{1} dF(x) = 1 - F(x_i).$$

Now, we can write the following formula

$$\langle UD \rangle = \int_0^1 dF(x_{i-1}) \cdot \int_{F(x_{i-1})}^{1} dF(x_i) \cdot \int_0^{F(x_i)} dF(x_{i+1}).$$

The first integral declares that the first number in a triplet $\langle UD \rangle$ can have any value. The next integral says that the number in the middle of a triplet should be greater than the first number. Finally, the third integral is for the trailing number of a triplet, which should be less than the previous number. It's easy to compute the probability as follows

$$\int_0^1 dF(x_{i-1}) \cdot \int_{F(x_{i-1})}^{1} dF(x_i) \cdot \int_0^{F(x_i)} dF(x_{i+1}) = \int_0^1 dF(x_{i-1}) \cdot \int_{F(x_{i-1})}^{1} dF(x_i) \cdot F(x_i);$$

$$\int_0^1 \mathrm{dF}\,(\mathrm{x}_{i-1})\cdot \int_{\mathrm{F}\,(\mathrm{x}_{i-1})}^1 \mathrm{dF}\,(\mathrm{x}_i)\cdot \mathrm{F}\,(\mathrm{x}_i) = \int_0^1 dF(x_{i-1})\cdot\left(\frac{1}{2}-\frac{F(x_{i-1})^2}{2}\right);$$

$$\int_0^1 dF(x_i-1)\cdot\left(\frac{1}{2}-\frac{F(x_i-1)^2}{2}\right)=\frac{1}{2}-\frac{1}{6}=\frac{1}{3}.$$

We got the same number in both combinatorial and operator approaches.

2. PMF of distances between local maxima

Now, it's time to advance our notation. Let's break up $\langle UD\rangle$ into pieces.

Definition 2. *Operators* $\langle ,U,Dand\rangle$ *are defined as*

$$\langle\ \ \psi(\mathrm{x}) = \int_0^1 \mathrm{d}\psi(\mathrm{x}),$$

$$\mathrm{U}\cdot\psi(\mathrm{x}) = \int_{\mathrm{z}\,(\mathrm{x})}^1 \mathrm{d}\psi(\mathrm{x})\cdot\psi(\mathrm{x}),$$

$$\mathrm{D}\cdot\psi(\mathrm{x}) = \int_0^{\mathrm{z}\,(\mathrm{x})} \mathrm{d}\psi(\mathrm{x})\cdot\psi(\mathrm{x}),$$

$$\rangle = 1.$$

Armed with this notation let's look at any quintet of numbers from i−1 to i+3. If it happens so that the numbers come like (0,2,1,4,3), then we got two local maxima $x_i=2$ and $x_{i+2}=4$. The distance between these maxima is (i+2) − i=2. This quintet can be represented by an expression)UDUD∗. Such quintet in our sample sequence can be found at i=10: (x_9=0.877254876, x_{10}=0.973280207, x_{11}=0.489033299, x_{12}=0.912367351, x_{13}=0.604552972).

Definition 3. *If* x_i *is a local maximum, and the next nearest maximum is the number* x_j*, the the distance between maxima is* $j-i$*.*

Now, we can compute the probability of the distance between local maxima equal to 2

$$\Pr(d=2)=f_m(2)=\frac{\langle UDUD\rangle}{P_{\max}}=\frac{\langle UDUD\rangle}{\langle UD\rangle}, \tag{1}$$

where d is a distance between local maxima and f_m (d) is a probability mass function (PMF) of the distribution of these distances.

Notice the denominator. It is necessary to divide the probability of the quintet by the probability of the maximum in its first three numbers (triplet). Consider the quintet (x_{10}=0.973280207, x_{11}=0.489033299, x_{12}=0.912367351, x_{13}=0.604552972, x_{14}=0.039395302) from the sample sequence above. Its operator expression is $\langle DUDD\rangle$, which doesn't seem to represent two maxima on distance 2. Let's add the number x_9=0.877254876 and look at the resulting sextet. This sextet's operator expression is $\langle UDUDD\rangle$. It starts with $\langle UDUD\rangle$. Clearly, our original quintet is a part of a sextet with two maxima on distance 2. Therefore, we have to take into account those quintets, which are not represented by $\langle UDUD]\rangle$, but these quintets could be parts of sought quintets. Probability $P_{\max}$ in denominator includes those quintets, which would be left unaccounted otherwise.

This formula can also be interpreted in terms of conditional probabilities as follows

$$\Pr\,(\mathrm{A}|\mathrm{B})=\frac{\Pr(\mathrm{A}\cap\mathrm{B})}{\Pr(\mathrm{B})}, \tag{2}$$

where event A∩B is a quintet with one maximum in its head and one maximum in its tail, event B is the maximum in first three numbers of a quintet, and event A|B is two maxima on a given distance from each other.

Using the same methodology it's easy to show that the probability of the distance 3 is

$$\Pr(d=3)=f_m(3)=\frac{\langle UDUUD\rangle+\langle UDDUD\rangle}{P_{\max}}=\frac{\langle UDU^2D\rangle+\langle UD^2UD\rangle}{P_{\max}}. \tag{3}$$

In order to see why there are two terms in the numerator, consider these two sextets (0,2,1,3,5,4) and (0,3,2,1,5,4). The following is the table with formulae for the next 3 distances

$$\Pr(d=4)=f_m(4)=\frac{\langle UDU^3D\rangle+\langle UD^2U^2D\rangle+\langle UD^3D\rangle}{P_{\max}}; \tag{4}$$

$$\Pr(d=5)=f_m(5)=\frac{\langle UDU^4D\rangle+\langle UD^2U^3D\rangle+\langle UD^3U^2D\rangle+\langle UD^4UD\rangle}{P_{\max}}; \tag{5}$$

$$\Pr(d=6)=\frac{\langle UDU^5D\rangle+\langle UD^2U^4D\rangle+\langle UD^3U^3D\rangle+\langle UD^4U^2D\rangle+\langle UD^5UD\rangle}{P_{\max}}. \tag{6}$$

3. Results

In order to compute a probability of a given distance between maxima, we have to identify corresponding integrals, evaluate them and sum them up. For example, computing the probability of distance 4 involves evaluation, see equation 4. A simple analytical expression for the sums of integrals in numerators of the above probabilities was presented in [6]:

$$p(l)=2^l\frac{(l-1)(l+1)}{(l+3)!}. \tag{7}$$

In [6] a set of similar problems are studied, e.g. permutation generated random walks, by using a different and more generic approach. However, the equation 7 can be used to to derive the probability of distances between local maxima:

$$f_m(d)=\frac{p(l)}{P_{\max}}=3\cdot 2^d\frac{(d-1)(d+1)}{(d+3)!}. \tag{8}$$

We were not aware of this work, and in absence of a simple analytical expression for a sum of integrals in the PMF equations (such as 4), we wrote a Java program, which does all required work. First, it generates the necessary integrals using our operator notaion, e.g. $\langle UDU^3D\rangle+\langle UD^2U^2D\rangle+\langle UD^3D\rangle$. Next, it evaluates the corresponding integrals and sums symbolically.

The resulting PMF table is shown in table. Variance of this distribution ≈ 1.167168 and the standard deviation ≈ 1.08.

Table 1

Table of PMF of distances between local maxima

Distance d	Probability $f_m(d)$	Decimal Approximation
1	2	3
2	2/5	0.4
3	1/3	0.3333333333333333
4	6/35	0.17142857142857143
5	1/15	0.06666666666666667
6	4/189	0.021164021164021163
7	1/175	0.005714285714285714
8	2/1485	0.0013468013468013469
9	4/14175	2.821869488536155E-4
10	4/75075	5.328005328005328E-5
11	2/218295	9.161913923818686E-6
12	4/2764125	1.4471125582236693E-6
13	1/4729725	2.114287828573543E-7

1	2	3
14	8/278326125	2.8743259368842937E-8
15	1/273648375	3.654324641978963E-9
16	2/4583103525	4.363855167334454E-10
17	8/162820783125	4.913377669887681E-11
18	4/764299911375	5.2335476433640715E-12
19	2/3781060408125	5.289521414950853E-13
20	4/78642438249375	5.0863122876684074E-14
21	2/428772250281375	4.664480965565126E-15
22	8/19566987612046875	4.088518968077954E-16
23	2/58274046742786875	3.432059573325511E-17
24	4/1447106344699640625	2.7641368684830376E-18
25	8/37392513326621578125	2.1394657080470717E-19
26	8/501914364595623354375	1.5938973985025013E-20
27	4/3494761822449632109375	1.1445701318770342E-21
28	8/1008476084418983962031 25	7.932761246003232E-23
29	1/188217886723358757890625	5.31299132834167 1E-24

Total $\frac{272288542793125669748 4374}{2722885427931256697484375}$ 1 — 3.6725746509274224E-25.

We tested validity of a computed PDF table on several random and pseudo-random number sequences. For pseudo-random number sequences we used Java's standard pseudo-random generator java.lang. Random and Daniel Cer's Java implementation [1] of notorious RANDU generator [2]. We used Mads Haahr's True Random Number Service web site [3] as a source of «true» random numbers. We modified the supplied Java client, which connects to the server and retrieves the true random number batches. We generates random number sequences using these methods and compared them with the theoretical PMF using several tests such as Kolmogorov-Smirnov and χ^2 goodness of fit tests, see chapters 1.3.5.15 and 1.3.5.16 in [4].

Also, according to the central limit theorem in large samples the standard deviation of the average distance between maxima should approach [(σ)/(√n)], where σ is the standard deviation of the distances in the population and n is the size of the sample [5]. We used this feature to compare sample average distances to a theoretical average distance 3 [table 2].

Table 2

Table of CDF of distances between local maxima

Distance d	Cumulative Probability $F_m(d)$	Decimal Approximation
1	2	3
2	2/5	0.4
3	11/15	0.7333333333333333
4	19/21	0.9047619047619048
5	34/35	0.9714285714285714
6	134/135	0.9925925925925926
7	4717/4725	0.9983068783068783
8	5773/5775	0.9996536796536797
9	31183/31185	0.9999358666025333
10	184273/184275	0.9999891466558133
11	4729717/4729725	0.9999983085697371
12	16372121/16372125	0.9999997556822954
13	30405374/30405375	0.9999999671110782
14	241215974/241215975	0.9999999958543376
15	32564156609/32564156625	0.9999999995086623

1	2	3
16	36395233873/36395233875	0.9999999999450477
17	343732764373/343732764375	0.9999999999941815
18	3419236445623/3419236445625	0.999999999999415
19	142924083427117/142924083427125	0.999999999999944
20	782679504481871/782679504481875	0.9999999999999949
21	4482618980214373/4482618980214375	0.9999999999999996
22	53596531285171873/53596531285171875	≈ 1.0
23	[534178761808879685 9/534178761808879687 5]	
24	[17307391882607701871/17307391882607701875]	
25	[232984121496642140621/232984121496642140625]	
26	[3253148659416077296871/3253148659416077296875]	
27	[18821788672335875789060 9/18821788672335875789062 5]	
28	[140838901444720174007811 7/140838901444720174007812 5]	
29	[272288542793125669748437 4/272288542793125669748437 5]	

As expected, java.lang.Random's and «true» random sequences were consistent with our PMF on any sample sizes varying from 100 to 100,000,000. Surprisingly, RANDU-generated sequences were also compliant with this PMF. When deriving this PMF, we assumed that numbers in the sequences are independent of each other. RANDU generator has a well known deficiency: its numbers are not independent. However, as it was noted before, it fared well in our tests.

The table 3 shows sample statistics for «true» random and RANDU generated sequences compared to theoretical frequencies of distances between maxima. Both samples are distributed as predicted by theoretical PMF $f_m(d)$, they pass χ^2 goodness of fit test with higher than 0.99 probabilities. Their average distances are also within the $3 \cdot \sigma$ area of a theoretical value of 3. The p-value for the latter test is the probability of the deviation from the theoretical average distance greater than of the observed value.

Table 3

Sample Frequency Comparison

Distance	Theoretical Frequency	True Random Frequency	RANDU Frequency
2	40000	39803	40462
3	33333	33544	33003
4	17143	17119	17073
5	6667	6673	6545
6	2116	2139	2157
7	571	549	571
8	135	136	148
9	28	31	36
10	5	3	5
11	1	2	0
12	0	1	0
Average	3	3.00187	2.99447
Std Dev of Mean	0.0034		
p-value		0.584	0.106
χ^2, df		1.006, 10	1.386, 8
p-value		0.9998	0.9944

Conclusion 1. *We constructed a simple method of computation of PMF of the distribution of distances between local maxima in random number series. We confirmed that selected pseudo-random and true random number sequences are distributed according to this PMF.*

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УДК 535.37:535.34:539.19

Фотолюминесценция сульфатов щелочных металлов, активированных ионами редкоземельных элементов

Photoluminescence of alkaline metals sulfates doped by ions of rare-earth elements

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Зерттеу нысандары Sm және Gd иондарымен активтендірілген K_2SO_4 және $LiKSO_4$ кристалдары мен ұнтақтары болып табылады. Калий сульфаттары кристалдарында аталған қоспалық иондар екі түрлі люминесценция орталықтарын түзетіні тағайындалды. Қыздыру барысында бір түрдегі орталық қайтымсыз түрде екіншіге ауысады. $LiKSO_4$ кристалдарында жерде сирек кездесетін элементтердің иондары бір түрлі орталықтар түзеді. K_2SO_4 кристалдарында екі түрлі орталықтардың түзілуі гидратациямен немесе зарядтың локалді механизм бойынша компенсациялануымен байланыстырылады. $LiKSO_4$ кристалдарында зарядтың компенсациялану локальді емес.

The objects of research are crystals and powders K_2SO_4 and $LiKSO_4$, activated by ions Sm and Gd. It is established that in crystals of potassium sulfate these impurity ions form two types of luminescence centers. At heating one kind transforms to another irreversible. In $LiKSO_4$ ions of rare-earth elements form one kind of the centers. Formation of two kinds of the centers in crystals K_2SO_4 contacts hydration or the local mechanism of charge indemnification. In $LiKSO_4$ the mechanism of charge indemnification is not local.

При активации диэлектрических кристаллов гетеровалентными примесными ионами возникают дополнительные вакансии, компенсирующие избыточные заряды. Компенсация может быть локальной, когда вакансии располагаются в ближайшем окружении примесного центра, и нелокальной. Примесные ионы, взаимодействующие с окружением, образуют центр люминесценции. Следовательно, механизм компенсации избыточного заряда оказывает существенное влияние на оптические характеристики примесных центров свечения. Например, при локальной компенсации заряда в галоидах щелочных металлов, активированных двухвалентными катионами, центр свечения рассматривается как диполь $Me^{2+}V_c^-$ [1].

Объектами исследования в данной работе являются сульфаты щелочных металлов, активированные трехвалентными ионами самария и гадолиния. Исходные кристаллы K_2SO_4 и $LiKSO_4$ были выращены из насыщенных водных растворов методом изотермического испарения растворителя при 40°С. Активаторы добавлялись в исходный раствор в виде водорастворимых солей. Выбор редкоземельных ионов (РЗИ) в качестве люминесцентных зондов обусловлен тем, что их оптические свойства формируются f-d переходами [2]. Они экранируются валентными электронами, поэтому только сильное возмущение в ближайшей координационной сфере оказывает влияние на оптические характеристики примесных центров. По всем справочным данным указанные выше сульфаты щелочных металлов не гидратируются. Введение в кристаллическую решетку трехвалентных примесных ионов