

ON SMOOTHLY APPROXIMABLE ACYCLIC GRAPHS

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We deal with pseudofinite countably categorical structures [1]-[4], in particular, countable acyclic graphs [6,7].

Theorem 1. [7] Let Γ be an arbitrary countable graph in which each component contains a finite number of cycles. Then Γ is countably categorical if and only if Γ is bounded and finitely many 1-types are realized in it.

A. Lachlan introduced the concept of *smoothly approximable* structures to change the direction of analysis from finite to infinite, that is, to classify large finite structures that appear to be *smooth approximations* to an infinite limit. A more general approach is developed in [8].

Definition. [2] Let L be a countable language and let M be a countable and ω -categorical L -structure. L -structure M (or $Th(M)$) is said to be *smoothly approximable* if there is an ascending chain of finite substructures $A_0 \subseteq A_1 \subseteq \dots \subseteq M$ such that $i \in \omega A_i = M$ and for every i , and for every $\hat{a}, \hat{b} \in A_i$ if $tp_M(\hat{a}) = tp_M(\hat{b})$, then there is an automorphism σ of M such that $\sigma(\hat{a}) = \hat{b}$ and $\sigma(A_i) = A_i$, or equivalently, if it is the union of an ω -chain of finite homogeneous substructures; or equivalently, if any sentence in $Th(M)$ is true of some finite homogeneous substructure of M .

Smoothly approximated structures were first examined in generality in [2], subsequently in [4]. The model theory of smoothly approximable structures has been developed very much further by G. Cherlin and E. Hrushovski [1].

Recall the following class defined in [5] for acyclic graphs.

Let $G_{\text{fin}}(\lambda)$, for arbitrary cardinality λ , be the family of all infinite acyclic graphs consisting of λ connected components of bounded in aggregate diameters.

Theorem 2. [5] Theory T of any infinite acyclic graph Γ from the class $G_{\text{fin}}(\lambda)$, for arbitrary cardinality λ , is pseudofinite.

Let us distinguish a subclass $G_{\text{cc}}(\lambda)$ of the class $G_{\text{fin}}(\lambda)$ as the class of all countably categorical acyclic graphs.

Theorem 3. Any theory $Th(\Gamma)$, $\Gamma \in G_{\text{cc}}(\lambda)$ is smoothly approximable.

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PERFECT JONSSON VARIETIES AND QUASIVARIETIES

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Definition 1. A is an algebraically prime model of theory T , if A is a model of T and A may be isomorphically embedded in each model of the theory T .

Definition 2. The inductive theory T is called the existentially prime if:

1) it has a algebraically prime model, the class of its AP (algebraically prime models) denote by AP_T ;

1) class E_T non trivial intersects with class AP_T , i.e. $AP_T \cap E_T \neq \emptyset$.

Definition 3. The theory T is called convex if for any its model A and for any family $\{B_i | i \in I\}$ of substructures of A , which are models of the theory T , the intersection $\bigcap_{i \in I} B_i$ is a model of T , provided it is non-empty. If in addition such an intersection is never empty, then T is called strongly convex.

Definition 4. Model C of Jonsson theory T is called semantic model, if it is ω^+ -homogeneous-universal.

Definition 5. The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^* , i.e. $T^* = Th(C)$.

Definition 6. Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^* .

Definition 7. Let T be the Jonsson theory. A companion of a Jonsson theory T is a theory $T^\#$ of the same signature that satisfies the following conditions:

1) $(T^\#)_\forall = T_\forall$;

2) for any Jonsson theory T' , if $T_\forall = T'_\forall$, then $T^\# = (T')^\#$;

3) $T_{\forall\exists} \subseteq T^\#$.

The natural interpretations of the companion $T^\#$ are T^*, T^0, T^f, T^M, T^e .

Definition 8. A set X is called a Jonsson set, if hold the following conditions:

1) X is a definable set by some existential formula $\varphi(\bar{x}, \bar{y}) = \exists \bar{y} \psi(\bar{x}, \bar{y})$;

2) $cl(X) = M, M \in E_T$.

Let K be the class of structures of countable signature σ .

Definition 9. We call a K - Jonsson variety if

1) K is a variety in the usual sense [2; 269];

2) $Th_{\forall\exists}(K)$ is a Jonsson theory.

Definition 10. We call a K - Jonsson quasivariety if

1) K - is a quasivariety in the usual sense [2; 269];

2) $Th_{\forall\exists}(K)$ is a Jonsson theory.

Consider the $JSpV(K)$ - Jonsson spectrum of the Jonsson varieties of class K , where K is the Jonsson variety:

$JSpV(K) = \{T/T \text{ is a Jonsson theory, } T = Th_{\forall\exists}(N); N \subseteq K; N \text{ is a subvariety of } K\}$.

Consider the $JSpQV(K)$ - Jonsson spectrum of the Jonsson quasivarieties of the class K , where K is the Jonsson quasivariety:

$JSpQV(K) = \{T/T \text{ is a Jonsson theory, } T = Th_{\forall\exists}(N); N \subseteq K; N \text{ is a subquasivariety of } K\}$.

Then $JSpQV(K) / \bowtie$ is denoting the factor set of the Jonsson spectrum of Jonsson quasivariety of the class K by the relation \bowtie .