

INTERPOLATION PROPERTIES OF ANISOTROPIC LORENTZ SPACES

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In this work we construct interpolation methods with parametric functions that can be used to study the interpolation properties of spaces with mixed metrics and obtain an interpolation theorem for Lebesgue and Lorentz spaces with mixed metrics.

Let (A_i^0, A_i^1) , $i=1,2$ be compatible pairs of Banach spaces. Let $A_{00} = (A_1^0, A_2^0)$, $A_{01} = (A_1^0, A_2^1)$, $A_{10} = (A_1^1, A_2^0)$, $A_{11} = (A_1^1, A_2^1)$ be spaces with mixed metric [1], [2].

Let $\Sigma A = A_{00} + A_{01} + A_{10} + A_{11}$. For $f \in \Sigma(A)$ and $t > 0$ we define the Petre functional $K(f, t)$ by formula

$$K(f, t) = K(f, t; A_{00}, A_{11}) \\ = \inf_{f=f_{00}+f_{01}+f_{10}+f_{11}} (\|f_{00}\|_{A_{00}} + t_1\|f_{10}\|_{A_{10}} + t_2\|f_{01}\|_{A_{01}} + t_1t_2\|f_{11}\|_{A_{11}}),$$

where

$$\|f_{11}\|_{A_{11}} = \|\|f\|_{A_1^1}\|_{A_2^1}, \\ \|f_{00}\|_{A_{00}} = \|\|f\|_{A_1^0}\|_{A_2^0}, \\ \|f_{10}\|_{A_{10}} = \|\|f\|_{A_1^1}\|_{A_2^0}, \\ \|f_{01}\|_{A_{01}} = \|\|f\|_{A_1^0}\|_{A_2^1}.$$

Let $1 \leq \bar{q} = (q_1, q_2) \leq \infty$, $\bar{\varphi}(t) = (\varphi_1(t), \varphi_2(t)) \geq 0$, $t = (t_1, t_2) > 0$. The space $A_{\bar{\varphi}, \bar{q}} = (A_{00}, A_{11})_{\bar{\varphi}, \bar{q}}$ are set of all elements for which the following norm is finite

$$\|f\|_{A_{\bar{\varphi}, \bar{q}}} = \left(\int_0^\infty \left(\int_0^\infty \left(\frac{K(f, t_1, t_2; A_{00}, A_{11})}{\varphi_1(t_1) \cdot \varphi_2(t_2)} \right)^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}}.$$

We define anisotropic Lorentz spaces as follows:

$$\Lambda_{\bar{q}}(\bar{\varphi}) := \left\{ f: \left(\int_0^{+\infty} \left(\int_0^{+\infty} (f^{*1*2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2))^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}} < \infty \right\},$$

where $f^{*1*2} = f^{*1*2}(t_1, t_2)$ is the nonincreasing permutation of a function f [1], [2]. The paper [3] studies one-dimensional generalized Lorentz spaces.

If $\bar{q} < \infty$, then

$$\|f\|_{\Lambda_{\bar{q}}(\bar{\varphi})} = \left(\int_0^{+\infty} \left(\int_0^{+\infty} (f^{*1*2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2))^{q_1} \frac{dt_1}{t_1} \right)^{\frac{q_2}{q_1}} \frac{dt_2}{t_2} \right)^{\frac{1}{q_2}}.$$

If $\bar{q} = \infty$, then

$$\|f\|_{L_\infty(\bar{\varphi})} = \sup_{t_2 > 0} \sup_{t_1 > 0} f^{*1*2}(t_1, t_2) \varphi_1(t_1) \varphi_2(t_2).$$

Let $\delta > 0$ and $\varphi(t)$ be nonnegative function on $[0, \infty)$. Define a function class C_δ :

$$C_\delta = \left\{ \varphi(t): \begin{array}{l} \varphi(t)t^{-\delta} \text{ is an increasing function and} \\ \varphi(t)t^{-1+\delta} \text{ is a decreasing function} \end{array} \right\}.$$

The class C is defined as follows:

$$C = \bigcup_{\delta > 0} C_\delta.$$

Theorem 1. Let $0 < \bar{p}_0 = (p_1^0, p_2^0) < \bar{p}_1 = (p_1^1, p_2^1) < \infty$, $1 \leq \bar{q} = (q_1, q_2) \leq \infty$, $\gamma_i = \frac{1}{p_i^0} - \frac{1}{p_i^1}$, $i = 1, 2$, $\varphi_1, \varphi_2 \in C$. Then the following inequality is true

$$(L_{\bar{p}_0}, L_{\bar{p}_1})_{\bar{\varphi}, \bar{q}} = \Lambda^{\bar{q}}(\bar{\psi}),$$

$$\text{where } \bar{\psi}(t_1, t_2) = \left(\frac{t_1^{\frac{1}{p_1^0}}}{\varphi_1(t_1^{\gamma_1})}, \frac{t_2^{\frac{1}{p_2^0}}}{\varphi_2(t_2^{\gamma_2})} \right).$$

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ON THE CONVOLUTION OPERATOR IN LEBESGUE AND MORREY SPACES

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Annotation. This paper is devoted to the study of upper bounds for the norm of the convolution operator in Morrey spaces. Upper and lower estimates for the norm of the convolution operator in the Lebesgue space are given. The upper bound refines the classical O'Neill inequality. Young-O'Neil type inequalities in Morrey spaces are proved. New results on the boundedness of Riesz's potential in Morrey spaces are established.

О ВЛОЖЕНИИ ПРОСТРАНСТВА ОБОБЩЕННЫХ ДРОБНО-МАКСИМАЛЬНЫХ ФУНКЦИЙ В ПЕРЕСТАНОВОЧНО-ИНВАРИАНТНЫЕ ПРОСТРАНСТВА

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В данной работе вопрос о вложении пространства обобщенных дробно-максимальных функции в перестановочно-инвариантные пространства сводится к вложению соответствующего конуса в другое перестановочно-инвариантное пространство.