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The voltage concentration in the vicinity of spherical mine workings of deep foundations

A solution for the determination of voltage concentrations in the vicinity of spherical mine workings of deep foundations is given. A system of differential equations of equilibrium in a spherical coordinate system is used for solving the task. The case of spheroidal deformation in axisymmetric loading is considered.

Key words: mining, elastic deformation, movement vectors, differential equations, boundary value problems.

The system of differential equations of equilibrium in a spherical coordinate system has the following form [1]:

$$\begin{gathered}\frac{\partial\sigma_{11}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{12}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{13}}{\partial\lambda}+\frac{1}{r}\left(2\sigma_{11}-\sigma_{22}-\sigma_{33}+\sigma_{12}ctg\theta\right)=0,\\ \frac{\partial\sigma_{12}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{22}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{23}}{\partial\lambda}+\frac{1}{r}\left[\left(\sigma_{22}-\sigma_{33}\right)ctg\theta+3\sigma_{12}\right]=0,\\ \frac{\partial\sigma_{13}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{23}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{33}}{\partial\lambda}+\frac{1}{r}\left(3\sigma_{13}+2\sigma_{23}ctg\theta\right)=0,\end{gathered} \tag{1}$$

where $\sigma_{ij}\left(i,j=1,2,3\right)$ are the components of voltage tensor; r,θ,λ are the spherical coordinates, and r is the radius, θ is the co latitude, λ is the longitude.

Since gasholder and oil storage are arranged usually among rocks of incompressible material, we use appropriate physics` law that establishes the connection between the components of voltage and strain tensors:

$$\sigma_{ij}-\delta_{ij}\sigma=2G\varepsilon_{ij}, \tag{2}$$

where δ_{ij} is the Kronecker sign, σ is the average voltage; ε_{ij} is the components of strain tensor; G is the shear modulus.

The components of the strain tensor and elastic movements are connected by Cauchy correlations in the spherical coordinate system:

$$\varepsilon_{11}=\frac{\partial u_1}{\partial r},\qquad \varepsilon_{22}=\frac{1}{r}\frac{\partial u_2}{\partial\theta}+\frac{u_1}{r};$$

$$\varepsilon_{33}=\frac{1}{r\sin\theta}\frac{\partial u_3}{\partial\lambda}+\frac{u_2}{r}ctg\theta+\frac{u_1}{r};$$

$$\varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_2}{\partial r}+\frac{1}{r}\frac{\partial u_1}{\partial\theta}-\frac{u_2}{r}\right);$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{1}{r\sin\theta}\frac{\partial u_1}{\partial\lambda} + \frac{\partial u_3}{\partial r} - \frac{u_3}{r} \right);$$

$$\varepsilon_{23} = \frac{1}{2r}\left(\frac{\partial u_3}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial u_2}{\partial\lambda} - u_3 ctg\theta \right). \qquad (3)$$

For an incompressible material dilatancy $\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 0.$ And using the formula (3) it can be written as

$$\Theta' = -r\frac{\partial u_1}{\partial r} = r\left(\varepsilon_{22} + \varepsilon_{33}\right) = \frac{\partial u_2}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial u_3}{\partial\lambda} + u_2 ctg\theta + 2u_1. \qquad (4)$$

The first of equations (1) is represented as the formula below

$$\left(\nabla^2 + 2\right)u_1 + r^2\frac{\partial^2 u_1}{\partial r^2} + 4r\frac{\partial u_1}{\partial r} + \frac{r^2}{G}\frac{\partial\sigma}{\partial r} = 0, \qquad (5)$$

where the following formula:

$$\nabla^2 = \frac{\partial^2}{\partial\theta^2} + ctg\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\lambda^2}$$

is the Beltrami differential operator [2].

From the last two equations (1) we have the formula:

$$\nabla^2\sigma - G\nabla^2\frac{\partial u_1}{\partial r} - r^2 G\frac{\partial^3 u_1}{\partial r^3} - 6rG\frac{\partial^2 u_1}{\partial r^2} - 6G\frac{\partial u_1}{\partial r} = 0. \qquad (6)$$

We take the solutions of equations (5) and (6) as given below:

$$u_1 = \sum_{m=v}^{\infty} u_{10m}\left(r\right)P_n^m\left(\cos\theta\right)\cos m\lambda; \qquad (7)$$

$$\sigma = \sum_{n=0}^{\infty} \sigma_{0m}\left(r\right)P_n^m\left(\cos\theta\right)\cos m\lambda, \qquad (8)$$

where *P_n ^ m* (cos θ) is the Legendre function of the first type of the factual argument $x = \cos\theta$ (n is the power, m is the order of this function). Substituting (7) and (8) into (6) and taking into consideration the identity [3]:

$$\nabla^2 P_n^m\left(\cos\theta\right)\cos m\lambda = -n\left(n+1\right)P_n^m\left(\cos\theta\right)\cos m\lambda,$$

We have the following formula without index m

$$\sigma_0(r) = \frac{G}{rn(n+1)}\left\{ -r^3\frac{d^3u_{10}(r)}{dr^3} - 6r^2\frac{d^2u_{10}(r)}{dr^2} + r\left[n(n+1) - 6\right]\frac{du_{10}(r)}{dr} \right\}. \qquad (9)$$

Substituting the equation (9) into (5), we have

$$r^4\frac{d^4u_{10}(r)}{dr^4} + 8r^3\frac{d^3u_{10}(r)}{dr^3} + \left[12 - 2n\left(n+1\right)\right]\times r^2\frac{d^2u_{10}(r)}{dr^2} -$$

$$-4rn\left(n+1\right)\frac{du_{10}(r)}{dr} + \left[n^2\left(n+1\right)^2 - 2n\left(n+1\right)\right]u_{10}(r) = 0. \qquad (10)$$

This is the Euler equation of the fourth rate [4].

The last two equations (1) have the following form:

$$r^2\frac{\partial^2 u_2}{\partial r^2} + 2r\frac{\partial u_2}{\partial r} = -2\frac{\partial u_1}{\partial\theta} - r\frac{\partial^2 u_1}{\partial r\partial\theta} - \frac{r}{G}\frac{\partial}{\partial\theta}\left(\sigma + 2G\frac{\Theta'}{r} - 2\frac{G}{r}u_1\right) + \frac{2}{\sin\vartheta}\frac{\partial\chi}{\partial\lambda},$$

$$r^2\frac{\partial^2 u_3}{\partial r^2} + 2r\frac{\partial u_3}{\partial r} = -2\frac{1}{\sin\theta}\frac{\partial u_1}{\partial\lambda} - \frac{r}{\sin\theta}\frac{\partial^2 u_1}{\partial r\partial\lambda} -$$

$$-\frac{r}{G}\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}\left(\sigma + 2G\frac{\Theta'}{r} - 2\frac{G}{r}u_1\right) - 2\frac{\partial\chi}{\partial\theta},$$

where the quantity

$$\chi = \frac{1}{2}\left(\frac{\partial u_3}{\partial\theta} + u_3 ctg\theta - \frac{1}{\sin\theta}\frac{\partial u_2}{\partial\lambda}\right),$$

corresponds the radial component of the vector`s rotor displacement and characterizes the rotation of element which is normal to the radius relatively to the radial axis.

In the case of ax symmetric loading, the value of χ 0 and torsional deformation are absent. Thus we have the case of spheroidal deformation.

In this case, the system of differential equations of equilibrium (1) considering (4), (7), (8) and also the representation of displacement u_2 in the form

$$u_2 = u_{20}(r)\frac{dP_n(\cos\theta)}{d\theta},$$

reduced to the system of two ordinary differential equations of Euler - homogeneous (10) and inhomogeneous:

$$r^2\frac{d^2u_{20}(r)}{dr^2} + 2r\frac{du_{20}(r)}{dr} = \frac{1}{n(n+1)} \times \left[r^3\frac{d^3u_{10}(r)}{dr^3} + 6r^2\frac{d^2u_{10}(r)}{dr^2} + 6r\frac{du_{10}(r)}{dr} \right]. \tag{11}$$

Particular solutions of the Euler equation (12) is in the form of

$$u_{10}(r) = r^{\rho}. \tag{12}$$

Substituting (18) into (12), we have the characteristic equation

$$\rho^4 + 2\rho^3 - \left(1 + 2n^2 + 2n\right)\rho^2 - 2\rho\left(1 + n^2 + n\right) + n^4 + 2n^3 - n^2 - 2n = 0.$$

The roots of this equation

$$\rho_1 = n+1; \quad \rho_2 = n-1; \quad \rho_3 = -n; \quad \rho_4 = -n-2.$$

The two first roots correspond internal task, the two last roots correspond the external marginal tasks.

The general solution of equation (10) has the form

$$u_{10}(r) = C_1 r^{n+1} + C_2 r^{n-1} + C_3 r^{-n} + C_4 r^{-n-2}, \tag{13}$$

where C_1, C_2, C_3, C_4 — are arbitrary constants of integration.

In the case of an ax symmetric task for the determination of $u_{20}(r)$ it is sufficient to find a particular solution of the inhomogeneous equation (11), since the right-hand side of this equation includes defined function $u_{10}(r)$, which depends on four arbitrary constants of integration.

Solving the internal and external marginal tasks in the ax symmetric case the required quantities contain four arbitrary constants of integration.

We define the expression on the right part of equation (11) taking into consideration (13). We find the partial derivatives of function (21) and substitute them into equation (11). Then we have:

$$r^2\frac{d^2u_{20}}{dr^2} + 2r\frac{du_{20}(r)}{dr} = A_1 r^{n+1} + A_2 r^{n-1} + A_3 r^{-n} + A_4 r^{-n-2}, \tag{14}$$

where

$$A_1 = C_1\frac{(n+2)(n+3)}{n}; \quad A_2 = C_2(n-1); \quad A_3 = -C_3\frac{(n-1)(n-2)}{n+1}; \quad A_4 = -C_4(n+2). \tag{15}$$

Particular solution of equation (14) can be found as follows:

a) If $n \ge 2$, then the decision will be

$$u_{20}(r) = B_1 r^{n+1} + B_2 r^{n-1} + B_3 r^{-n} + B_4 r^{-n-2}, \tag{16}$$

where B_1, B_2, B_3, B_4 — are undetermined coefficients.

Differentiating (16) and substituting in equation (14), using (15) we have

$$B_1 = C_1\frac{(n+3)}{n(n+1)}; \quad B_2 = \frac{C_2}{n}; \quad B_3 = -C_3\frac{(n-2)}{n(n+1)}; \quad B_4 = -\frac{C_4}{(n+1)};$$

б) If $n = 1$, then equation (14) considering (15) takes the form

$$r^2\frac{d^2u_{20}(r)}{dr^2} + 2r\frac{du_{20}(r)}{dr} = 12C_1 r^2 - 3C_4 r^{-3}. \tag{17}$$

The characteristic equation corresponding to (17) of the inhomogeneous differential equation have the roots: $k_1 = 0; \quad k = -1$.

The general solution will be $\left[u_{20}(r)\right]_1 = C_2 + C_3 r^{-1}$.

We define a particular solution of the inhomogeneous equation (17) in the form

$$\left[u_{20}(r)\right]_2 = Ar^2 + Br^{-3}. \tag{18}$$

Substituting (18) into (17), we have $A = 2C_1$, $B = -\frac{1}{2}C_4$. Then the general solution of the inhomogeneous equation (17) will be $u_{20}(r) = 2C_1 r^2 + C_2 + C_3 r^{-1} - \frac{1}{2}C_4 r^{-3}$;

c) The case $n = 0$ was considered earlier [5].

According to Cauchy relations (3) we find the components of the strain tensor, the generalized Hooke's law (the components of voltage tensor). To determine the arbitrary constants of integration the boundary conditions should be used.

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Терең орналасқан сфера түріндегі тау-кен тұтқыр маңайындағы кернеулердің шоғырлануы

Терең орналасқан сфера түріндегі тау-кен тұтқыр маңайындағы кернеулердің шоғырлануын анықтау есептерінің шешімдері берілген. Есепті шешу үшін сфералық координаталар жүйесінде дифференциалдық теңдеулер жүйесі қолданылды. Ось симметриялы жүктеуде сфероидалды сығылу жағдайы қарастырылды.

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Концентрация напряжений в окрестности горных выработок сферической формы глубокого заложения

В статье дано решение для определения концентраций напряжения в окрестности горных выработок сферической формы глубокого заложения. Для решения задачи использована система дифференциальных уравнений равновесия в сферической системе координат. Рассмотрен случай сфероидальной деформации при осесимметричном нагружении.

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