UDC 521.1

PROBLEM FOUR STILL CENTRES

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New «model» problem of celestial mechanics as the plane problem of four fixed centres, two of those after crossing axes $O'x'$ *and the other two – on crossing axes* $O'y'$ *at the distance between centres, are considered. The problem is given like quadratures and the number of its applications to the – theory of both artificial and natural heavenly bodies' motion are introduced.*

Proposed below problem refers to class named model problems of heaven mechanics, allowing to execute the necessary calculations simply enough (in the first approximations) and they are in ditto time are the main for building exact analytical theory of motion of that or other material objects (heaven bodies).

We shall consider non - stationary problem of four fixed centres and show that problem is reduced to squaring. We shall consider the move of the material point P in non - stationary gravitation field:

$$U = G\left(\frac{m_1(t)}{r_1} + \frac{m_2(t)}{r_2} + \frac{m_3(t)}{r_3} + \frac{m_4(t)}{r_4}\right), \quad m(t) = \frac{m_0}{\sqrt{\alpha+\beta t}} \tag{1}$$

in which $m_1(t), m_2(t), m_3(t), m_4(t)$ - are masses fixed centre, r_1, r_2, r_3, r_4 - are radius-vectors from point P till considered centre and accordingly are equal:

$$r_1 = \sqrt{(\lambda - x_1)^2 + (\mu - d)^2}, \; r_2 = \sqrt{(\lambda - в)^2 + (\mu - c_1)^2}$$

$$r_3 = \sqrt{(\lambda - x_2)^2 + (\mu - d)^2}, \; r_4 = \sqrt{(\lambda - в)^2 + (\mu - c_2)^2} \tag{2}$$

but G – is a gravitation constant.

We shall choose the fixed coordinate system O $\lambda\,\mu$ the fixed centres P_1 and P_3 on axis O'x', and fixed centres P_2 and P_4 on axis O'y' on free distance from friend each other, thereby all these centres we shall put on plane P. Equation of the moving of the material point at presence of additional power proportional to the speed of the body is in the form of:

$$\frac{d^2\lambda}{dt^2} = \frac{\partial U}{\partial \lambda} + \varphi\dot{\lambda},$$

$$\frac{d^2\mu}{dt^2} = \frac{\partial U}{\partial \mu} + \varphi\dot{\mu} \tag{3}$$

where φ is an unceasing function of time.

We shall go to new variable *x* and *y* by formula:

$$\lambda = ax + в, \;\; \mu = ay + d \tag{4}$$

in which

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$$a = \frac{x_3 - x_1}{2},\ b = \frac{x_3 - x_1}{2},\ d = a\delta$$

where δ is constant.

If in equation (1) according to // place

$$c_1 = a(\delta - i), c_2 = a(\delta + i), i = \sqrt{-1}, \tag{5}$$

Then in new variables:

$$r_1 = a\sqrt{(x - i)^2 + y^2}, r_2 = a\sqrt{x^2 + (y + i)^2}$$

$$r_3 = a\sqrt{(x - i)^2 + y^2}, r_4 = a\sqrt{x^2 + (y - i)^2} \tag{6}$$

$$U = \frac{1}{\sqrt{\alpha + \beta t}} G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + \frac{m_4}{r_4}\right) \tag{7}$$

Function

$$T = \frac{1}{2} a^2 (\dot{x}^2 + \dot{y}^2) \tag{8}$$

The differential equations of the motion (3) in new variables x, y will be written as:

$$\frac{d^2 x}{dt^2} = \frac{1}{a^2} \frac{\partial U}{\partial x} + \varphi \dot{x},$$

$$\frac{d^2 y}{dt^2} = \frac{1}{a^2} \frac{\partial U}{\partial y} + \varphi \dot{y} \tag{9}$$

Expecting

$$H = T - U \tag{10}$$

the system (9) we may write in halfcanonical form, which is in the form of:

$$\frac{dx}{dt} = \frac{\partial H}{\partial P_x}, \frac{dP_x}{dt} = -\frac{\partial H}{\partial x} + \varphi P_x,$$

$$\frac{dy}{dt} = \frac{\partial H}{\partial P_y}, \frac{dP_y}{dt} = -\frac{\partial H}{\partial y} + \varphi P_y, \tag{11}$$

where

$$P_x = \dot{x} a^2,\ P_y = \dot{y} a^2, \tag{12}$$

Producing change of variables P_h, P_u in the form:

$$P_x = f(t)\tilde{P}_x, P_y = f(t)\tilde{P}_y, \tag{13}$$

We shall write the equation (11) in canonical form:

$$\frac{dx}{dt} = \frac{\partial \tilde{H}}{\partial \tilde{P}_x}, \frac{d\tilde{P}_x}{dt} = -\frac{\partial \tilde{H}}{\partial x},$$

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$$\frac{dy}{dt} = \frac{\partial \tilde{H}}{\partial \tilde{P}_y}, \frac{d\tilde{P}_y}{dt} = -\frac{\partial \tilde{H}}{\partial y}, \qquad (14)$$

where

$$\tilde{H} = \frac{1}{f} H \qquad (15)$$

when performing the task

$$f = f_0 exp(\int \varphi(\tau)\, d\tau), f_0 = const. \qquad (16)$$

which satisfies correlation

$$\varphi = \frac{1}{2}\frac{\gamma'}{\gamma} \qquad (17)$$

And then equation (15) will be written in evident type

$$\tilde{H} = \frac{f}{2a^2}\left(\tilde{P}_x^2 + \tilde{P}_y^2\right) - \frac{\gamma}{f}\tilde{U} \qquad (18)$$

Now we shall go to sphere coordinate u and υ by formula[2]:

$$x = (ch\vartheta + cosu), y = (sh\vartheta - cosu), \qquad (19)$$

then in new variable u, υ:

$$r_1 = a(ch\vartheta + cosu), r_2 = a(sh\vartheta - cosu)$$

$$r_3 = a(ch\vartheta + isinu), r_4 = a(sh\vartheta - isinu) \qquad (20)$$

and

$$U = \gamma(t)\frac{1}{Ja}(Ach\vartheta + Bcosu + Csh\vartheta + iDsinu) \qquad (21)$$

where

$$J = ch^2\vartheta - cos^2u = sh^2\vartheta + sin^2u$$

$$A = G(m_1 + m_3), B = G(m_3 - m_1), C = G(m_2 + m_4), D = G(m_4 - m_2) \qquad (22)$$

and equations of Hamilton will take the type:

$$\dot{\vartheta} = \frac{\partial \tilde{H}}{\partial P_\vartheta},\ \dot{u} = \frac{\partial \tilde{H}}{\partial P_u},\ \dot{P}_\vartheta = -\frac{\partial \tilde{H}}{\partial P_\vartheta},\ \dot{P}_u = -\frac{\partial \tilde{H}}{\partial P_u}, \qquad (23)$$

Then in a new canonical variable

$$\tilde{H} = \frac{\psi}{2a^2\gamma}\left(P_\vartheta{}^2 + P_u{}^2\right) - \frac{\gamma}{\psi}\overline{U}(u,\vartheta) \qquad (24)$$

and

$$P_\vartheta = \left(P_x sh\vartheta cosU + P_y ch\vartheta sinU\right)$$

$$P_u = \left(-P_x ch\vartheta sinU + P_y sh\vartheta cosU\right) \qquad (25)$$

The type to power function U and $\tilde{H}$ in new variables are such that system (23) satisfies the condition of the theorem of Luivile, and therefore and it integrated in quadrature.

Gamilitonov (24) corresponds to the equation on Hamilton - Yakobi, which is written in the manner of:

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$$\frac{\psi}{2\tilde{a}J}\left[\left(\frac{\partial S}{\partial \vartheta}\right)^2+\left(\frac{\partial S}{\partial u}\right)^2\right]-\frac{\gamma}{\psi^2}\bar{U}+\frac{\partial S}{\partial t}=0 \qquad (26)$$

Expecting by this expressions

$$\psi^2=\gamma \qquad (27)$$

We write (26) in the following form:

$$\frac{1}{2\tilde{a}J}\left[\left(\frac{\partial S}{\partial \vartheta}\right)^2+\left(\frac{\partial S}{\partial u}\right)^2\right]-\bar{U}+\frac{1}{\sqrt{\gamma}}\frac{\partial S}{\partial t}=0 \qquad (28)$$

Full integral of the equation (28) we search in the manner of

$$S=-h\int\sqrt{\gamma}\,dt+V(\vartheta,u), \qquad (29)$$

where h - is constant, V - is function in the form of:

$$V=V_1(\vartheta)+V_2(u)$$

Really, equation (28) will be a satisfied by expression (29) if we shall place:

$$\left(\frac{dV_1}{d\vartheta}\right)^2-2a^2hch^2\vartheta-2aAch\vartheta-2aCsh\vartheta+2\alpha_2=0,$$

$$\left(\frac{dV_2}{du}\right)^2-2a^2hcos^2u-2aBcosu-2aiDsinu-2\alpha_2=0. \qquad (30)$$

where α_2 - free constant.

Expecting

$$L(\vartheta)=a^2hch^2\vartheta+aAch\vartheta-aCsh\vartheta-\alpha_2, \qquad (31)$$

$$N(u)=a^2hcos^2u-aBcosu-aiDsinu-\alpha_2, \qquad (32)$$

And integrating (30), we shall find V_1 and V_2, but then decision (29) we put down in the manner of:

$$S=\int\sqrt{2L(\vartheta)}d\vartheta+\int\sqrt{2L(u)}du-h\int\sqrt{\gamma}dt, \qquad (33)$$

Knowing S we shall find the general integral of the system (12) by formula:

$$\frac{\partial S}{\partial h}=\int\sqrt{\gamma}dt+\beta_1,\ \frac{\partial S}{\partial \alpha_1}=\beta_2,\ \frac{\partial S}{\partial \vartheta}=P_\vartheta,\ \frac{\partial S}{\partial u}=P_u, \qquad (34)$$

where β_1, β_2 – new free constants

Calculating quotient free (34), we put down general integral of the problem as follows

$$\frac{1}{\sqrt{2}}\int\frac{a^2ch^2\vartheta}{\sqrt{L(\vartheta)}}d\vartheta-\frac{1}{\sqrt{2}}\int\frac{a^2cos^2udu}{\sqrt{N(u)}}du=\int\sqrt{\gamma}dt+\beta_1,$$

$$-\frac{1}{\sqrt{2}}\int\frac{d\vartheta}{\sqrt{L(\vartheta)}}d\vartheta+\frac{1}{\sqrt{2}}\int\frac{du}{\sqrt{N(u)}}du=\beta_2, \qquad (35)$$

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$$P_\vartheta = \sqrt{2\gamma(t)L(\vartheta)},\ P_u = \sqrt{2}\sqrt{\gamma(t)N(u)} \tag{36}$$

Thereby equation (35), (36) give the full decision of the considered non - stationary problem with potential (1) and additional power proportional to the speed.

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